

Gobas, 1978

Using a technique which he calls the Boltzman Hamel method, in 1978 Gobas presented a linearized set of equations very similar in form to Neřmark and Fufaev [1967]. Gobas' equations, (1.4) in his paper, incorporate the forward acceleration of the bicycle, \dot{V} . Setting \mathbf{V} terms to zero and comparing, we think the lean equation **may** be correct, but in the steer equation the coefficient to the χ_r term seems to be in disagreement with the equations in our Chapter III. The variable b is not defined in the paper but we suspect that it is equivalent to our ν .

Gobas refers to Neřmark and Fufaev, but does not compare equations.

Adiele, 1979'

In his 1979 Master's thesis Adiele, who ~~was~~ focusing on design optimization and performance evaluation of two-wheeled vehicles, derived nonlinear equations of motion for the Basic bicycle with tire side slip using Kane's method of generalized active and inertia forces.

His equations, representing lateral motion, lean, steer, and yaw (in that order) are present in matrix form on pages 22-24 of his thesis. His variable V is our \dot{X}_r , λ is our χ_r , θ is our ψ , and r is our θ_r . Because his equations resembled Sharp's [1971] four equations, we expanded Adiele's matrix, linearized his equations for small values of λ and θ , and compared them to the equations in Sharp's [1971] Appendix I.

The results show that Adiele's equations are in error, missing several terms

compared to Sharp and having several sign errors. However, by allowing the front mass to be zero his equations are nearly correct.

Adiele refers to Roland [1971], but does not compare equations. A subsequently published paper by Taylor and Adiele [1980] on stability in large angle steady turns also appears to rely on Adiele's equations, even though the authors evidently knew of earlier linearized studies (by Weir, and others) **which** they could have used to check their equations.

Lowell and Mckell, 1982

In 1982 Lowell and Mckell, using ad hoc arguments similar in style to Pearsall [1922] derive a set of linearized equations for a Basic bicycle model with a point mass in the **rear** part, some steering inertia and front gyroscopic effects, but no front mass, and no tilt of the steering axis. When compared to **our** equations simplified for this case, we find there is significant disagreement. Several terms have been neglected in both the lean and steer equation, however, the terms which are presented are correct. The neglected terms are significant, as a bicycle with vertical steering axis and positive trail *should* return upright if speed is great enough ($E > 0$), and show ever-increasing lean if speed is below a critical value ($E < 0$).⁷ However their approximations make $E = 0$ always, so their bicycle model neither straightens up nor leans further, **but** in fact oscillates about a steady turn.

⁷ For this simple bicycle E varies exactly opposite to E for a standard bicycle. When it is positive at low speeds and negative at high speeds.