Comparison: Carvallo, Coleman

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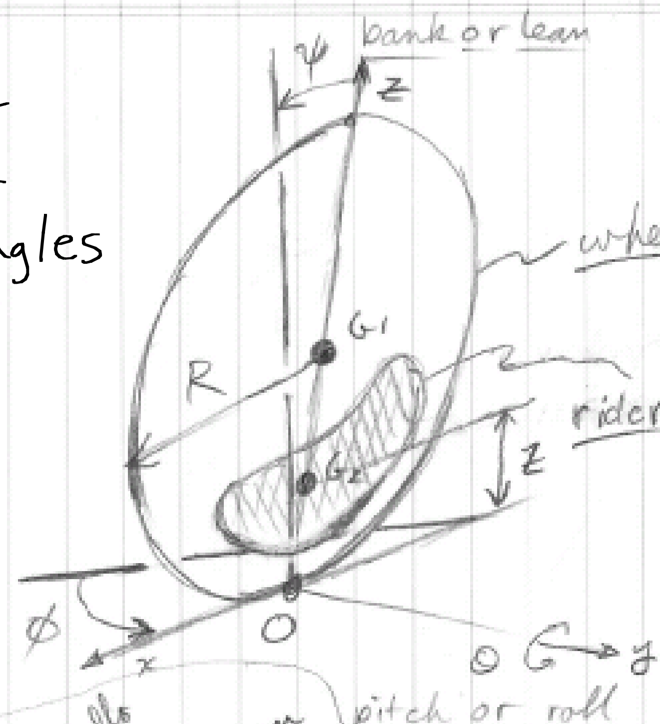
ϕ-ψ-θ
3-1-2
Euler Angles



wheel: m_1, $I^{G_1} = \begin{bmatrix} J_{xx} & & 0 \\ & J_{yy} & \\ 0 & & J_{zz} \end{bmatrix}$

rider: m_2, I^{G_2}

$= \begin{bmatrix} I_{xx} & 0 & I_{xz} \\ 0 & I_{yy} & 0 \\ I_{xz} & 0 & I_{zz} \end{bmatrix}$

$J_{yy} = J_{xx} + J_{zz}$

Looks like: Carvallo coleman

$A = (J_{xx} + m_1 R^2) + I_{xx} + m_2 Z^2$

$B = (J_{yy} + m_1 R^2) + (I_{yy} + m_2 Z^2)$

$C = J_{zz} + I_{zz}$

$L = -I_{xz}$

$\mu = (m_1 + m_2)$

$h = (m_1 R + m_2 Z)/(m_1 + m_2)$

Carvallo	Coleman
θ	ψ
ρ	$\dot{\psi}$
n	$\dot{\phi}$
s	$\dot{\theta}$

I. Non-Linear Equations of Motion

$(c\psi = \cos\psi,\ s\psi = \sin\psi)$
$(c^2\psi = \cos^2\psi,\ s^2\psi = \sin^2\psi)$

Eqn. (1)

Carvallo: $$A\ddot{\psi} + I_{xz}(c\psi)\ddot{\phi} \boxed{-R(m_1R+m_2z)(c\psi)\dot{\theta}\dot{\phi}} + (C-B)(c\psi)(s\psi)\dot{\phi}^2 = g(m_1R+m_2z)(s\psi)$$

Coleman: $$[(J_{xx}+m_1R^2)+(I_{xx}+m_2z^2)]\ddot{\psi} + I_{xz}(c\psi)\ddot{\phi} \boxed{-[J_{yy}+R(m_1R+m_2z)](c\psi)\dot{\theta}\dot{\phi}}$$
$$+\{(J_{zz}+I_{zz})-[(J_{yy}+m_1R^2)+(I_{yy}+m_2z^2)]\}(c\psi)(s\psi)\dot{\phi}^2$$
$$= g(m_1R+m_2z)(s\psi)$$

Eqn. 2

Carvallo: $$\boxed{[C(c^2\psi)+B(s^2\psi)]\ddot{\phi}} + \boxed{R(m_1R+m_2z)(s\psi)\ddot{\theta}} + I_{xz}(c\psi)\ddot{\psi} + \boxed{2(B-C)(c\psi)(s\psi)\dot{\phi}\dot{\psi}}$$
$$-I_{xz}(s\psi)\dot{\psi}^2 + J_{yy}(c\psi)\dot{\psi}\dot{\theta} = 0$$

Coleman: $$\boxed{[(J_{zz}+I_{zz})(c^2\psi)+(I_{yy}+m_2z^2-m_2zR)(s^2\psi)]\ddot{\phi}} + \boxed{m_2R(z-R)(s\psi)\ddot{\theta}} + I_{xz}(c\psi)\ddot{\psi}$$
$$+\boxed{[2(I_{yy}+m_2z^2)-2I_{zz}-2m_2zR+\underbrace{(J_{yy}-J_{xx}-J_{zz})}_{=0 \text{ for } J_{yy}=J_{xx}+J_{zz}}](s\psi)(c\psi)\dot{\phi}\dot{\psi}}$$
$$-I_{xz}(s\psi)\dot{\psi}^2 + J_{yy}(c\psi)\dot{\psi}\dot{\theta} = 0$$

Eqn. 3

Carvallo: $$\boxed{(m_1R+m_2z)R(s\psi)\ddot{\phi}} + \boxed{(m_1+m_2)R^2\ddot{\theta}} + \boxed{2(m_1R+m_2z)R(c\psi)\dot{\psi}\dot{\phi} = 0}$$

Coleman: $$\boxed{[J_{yy}+(m_1R+m_2z)R](s\psi)\ddot{\phi}} + \boxed{[J_{yy}+(m_1+m_2)R^2]\ddot{\theta}} + \boxed{[2(m_1R+m_2z)R+(J_{xx}+J_{yy}-J_{zz})](c\psi)\dot{\psi}\dot{\phi}} = 0$$

II. Characteristic Eqn. $c_2\lambda^2 + c_1\lambda + c_0 = 0$ (from linearization)

c_2

Carvallo : $c_2 = AC - I_{xz}^2$

Coleman : $c_2 = [(J_{xx} + m_1R^2) + (I_{xx} + m_2z^2)](J_{zz} + I_{zz}) - I_{xz}^2$

c_1

Carvallo : $c_1 = -I_{xz}\left[J_{yy} - (m_1R + m_2z)R\right]\dot{\theta}_0 = \boxed{I_{xz}(m_2z)R\dot{\theta}_0}$; if $J_{yy} = m_1R^2$

Coleman : $c_1 = \boxed{I_{xz}(m_1R + m_2z)R\dot{\theta}_0}$

c_0

Carvallo : $c_0 = (m_1R + m_2z)[R\dot{\theta}_0^2 J_{yy} - Cg]$

Coleman : $c_0 = (m_1R + m_2z)[R\dot{\theta}_0^2 J_{yy} - (J_{zz} + I_{zz})g]$

III. Stability Criteria

Carvallo: $m_1 R^3 \dot{\theta}_0^2 - Cg > 0$ $\boxed{I_{xz}(m_2 Z) R \dot{\theta}_0 > 0}$

Coleman $J_{yy} R \dot{\theta}_0^2 - (I_{zz} + J_{zz}) g \geq 0$ $\boxed{I_{xz}((m_1 R + m_2 Z) R \dot{\theta}_0 > 0}$