offset from the steering axis. When linearized, we find this lean equation agrees with our lean equation simplified for an equivalent configuration.

Döhring, 1955

In 1955, in order to more generally analyze the stability of motorcycles and motorscooters, Döhring extended Sommerfeld and Klein's (S & K) [1903] linearized equations for the Basic bicycle model by allowing the mass distribution of the front assembly to be fully general. Just as S & K did, Döhring used Newton's Laws to derive the equations of motion in linearized form, rather than linearizing from nonlinear equations as Whipple had.

Dohring's final equations were found to be in exact agreement with those derived in Chapter 111. In order to compare his equations to ours we made the following substitutions in his equations (29) and (30) of his [1955] paper,

$$\psi = \gamma \cos \sigma$$

$$\theta_1 = \theta_2 - \gamma \sin \sigma$$

where γ is steer angle (our ψ) and θ_2 is lean angle (our χ_r). When these substitutions are made Döhring's equation (30) is exactly our lean equation. Our steer equation results from the linear combination of Dohring's equation (31) and (30). Using Döhring's notation this combination is as follows:

$$\frac{(eq.\,31)}{1} + \frac{c_1 \sin\sigma (eq.\,30)}{1} = -M_d = our\ M_\psi$$

Although not rigorous in how his linearizations are made, Dohring's derivation was fairly easy to follow, and offers a good physical description of the variables and equations of motion. Dohring refers to S & K, but never states explicitly how his equations compare.

Collins, 1963

In his 1963 University of Wisconsin Ph.D. dissertation R. N. Collins, working on a project supported by Harley Davidson Motor Company, studied a Basic bicycle model with the addition of a driving force on the rear tire and an explicit force for aerodynamic drag applied to the front fork/handlebar assembly. He derived the equations of motion using Euler's equations (Newton's Laws) for the 4 rigid bodies of the Basic bicycle model.

Collins derives nonlinear velocity and acceleration expressions for the rear and front center of mass first (see pages 19 and 20 of his dissertation), and then linearizes about the vertical equilibrium position, before deriving the linearized equations of motion. By writing the drive force and aerodynamic drag force as a function of the square of the forward velocity of the motorcycle (see p. 12 in his dissertation), he alters the vertical contact forces on the front and rear wheels. By making the assumptions of no slip angle and constant velocity he has only two degrees of freedom for his model and he is therefore able to write the linearized governing equations as two coupled second order ordinary differential equations in the lean and steer angles (see p. 76 eq. (5.1) and eq. (5.2) in his dissertation). The final equations are complicated in appearance and include over 30 quantities defined in terms of motorcycle parameters. (These quantities often include previously defined quantities,