

We find that Carvallo's equations (for a bicycle with massless forks and handlebars) agree exactly with ours. Most quantities are defined in the text, but the reader should note that the wheel inertias are defined relative to their ground contact, i.e. C_1 is for spin about a diameter, A_1 is for lean (i.e., $A_1 = C_1 + \mu_1 R^2$), B_1 is for rolling about the contact point (i.e., $B_1 = I_p + \mu_1 R^2$). $S = \frac{V}{R}$ is the wheel rotation rate. Carvallo makes no reference to other works, which is not surprising as his research was evidently performed in 1898.

Sommerfeld and Klein, 1903

Sommerfeld and Klein (S & K) in 1903 derived the linearized equations of motions for the Basic bicycle model having all the mass and inertia of the front assembly in the front wheel (similar to Carvallo). Somewhat similar to Whipple [1899], they used a Newtonian analysis of the front and rear assembly, and treated the two parts as two trailers attached to the steering axis, deriving the linearized equations of motions using axes parallel to the steering axis. S & K refer to Whipple [1899] and Carvallo [1901] but do not say whether their equations agree.

Their equations are most easily compared to Döhning's [1955], and are found to be a correct subset of his. It is possible that S & K's slight simplification(s) to the model were due to their main interest in determining what effect the wheels as gyroscopes had on the stability (since the article is a chapter in their massive work on gyroscopes). They are critical of Bourlet [1898] (whose book we have not read).