ON THE DYNAMICAL PRINCIPLES OF THE MOTION OF VELOCIPEDES.

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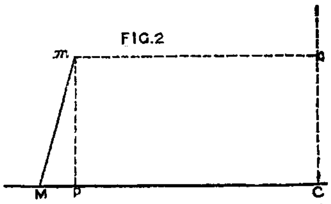
SECTION I. CONTINUED.—BALANCING.

8.—*Deflection of Base-Track.*—In Fig. 2, let the plane of the paper represent a vertical plane, traversing the centre of mass *m*, and the centre *c* of the circular track in which the centre of mass is moving; so that *c m* is the radius of that circle. The vertical line C *c* is the axis about which the velocipede is revolving or sweeping round as a whole. Let C P M be the trace of the roadway, sup-

FIG. 2



posed horizontal, and let *m* P be a vertical straight line through the centre of mass. Lay off the horizontal distance P M, bearing the proportion to *m* P, which the centrifugal force of the curvilinear motion bears to gravity; then the inclined straight line *m* M will be the line of action of the resultant of gravity and centrifugal force; and M will be the proper position for the base-point. Let *v* be the speed with which the centre of mass *m* is moving; then the ratio of the centrifugal force to gravity is expressed by $\frac{v^2}{c\,m} : g$; (g being 32·2ft., or 9·81 metres per second); and, therefore, the horizontal distance P M between the tracks of the base-point and of the centre of mass is given by the following equation:—

$$\mathrm{PM} = \frac{m\,\mathrm{P} \cdot v^2}{g \cdot c\,m} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1.)$$

I shall call this distance, for brevity's sake, the *deflection of the base-track.*

Another way of expressing the same principle is as follows:—let θ denote the angle *p* M *m* which the line of action of the load makes with the vertical; then

$$\tan.\,\theta = \frac{\mathrm{P\,M}}{m\,\mathrm{P}} = \frac{v^2}{g \cdot c\,m} \quad . \quad . \quad . \quad . \quad (1\ \mathrm{A.})$$

The use of Equation 1 is subject to the inconvenience that the vertical height *m* P of the centre of mass above the roadway is not, like the perpendicular *m* M, let fall from the centre of mass on the base, a constant quantity. When the angle θ is very small, the variable difference between *m* P and *m* M may be neglected; but when that difference becomes considerable the following formula is to be used, giving the deflection of the base-track P M in terms of the constant perpendicular *m* M. Calculate tan. θ by means of Equation 1 A, and then make

$$\mathrm{P\,M} = m\,\mathrm{M.}\ \sin.\,\theta = \frac{\mathrm{M}\ m.\ \tan.\,\theta}{\sqrt{(1 + \tan.^2\theta)}} \quad . \quad . \quad . \quad (2.)$$

The following table shows examples of the results of Equations 1 A and 2, as applied to a velocipede in which the perpendicular *m* M = 3·28 ft. = 1 metre.

Velocity 5ft. per second = 3·4 miles per hour = 1·524 metres per second.

Radius of track of centre of mass, feet.	5	10	15	20
Ditto, metres	1·524	3·048	4·572	6·096
Tan. θ	·1553	·0776	·0518	·0388
Deflection, millimetres	154	77	52	39
Ditto, feet	·504	·254	·170	·127
Ditto, inches	6·05	3·05	2·04	1·52

Velocity 10ft. per second = 6·8 miles per hour = 3·048 metres per second.

Radius of track of centre of mass, feet.	10	15	20	30
Ditto, metres	3·048	4·572	6·096	9·144
Tan. θ	·3107	·2070	·1553	·1036
Deflection, millimetres	297	203	154	103
Ditto, feet	·974	·666	·504	·338
Ditto, inches	11·69	7·99	6·05	4·06

Velocity 15ft. per second = 10·2 miles per hour = 4·572 metres per second.

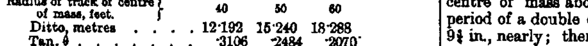
Radius of track of centre of mass, feet.	20	30	40	50
Ditto, metres	6·096	9·144	12·192	15·240
Tan. θ	·3494	·2329	·1747	·1398
Deflection, millimetres	330	227	172	138
Ditto, feet	1·082	0·747	0·565	0·454
Ditto, inches	12·99	8·97	6·78	5·45

Velocity 20ft. per second = 13·6 miles per hour = 6·096 metres per second.

Radius of track of centre of mass, feet.	40	50	60
Ditto, metres	12·192	15·240	18·288
Tan. θ	·3106	·2484	·2070
Deflection, millimetres	297	241	203
Ditto, feet	·974	·790	·666
Ditto, inches	11·69	9·48	7·99

9. *Horizontal Oscillations.*—As it is impossible to find any actual roadway which is absolutely smooth, the tracks of the centre of mass and of the base-point can seldom, if ever, be exact straight lines, or exact circles, but will be curves of a form more or less wavy, according to the roughness or smoothness of the road, and the less or greater skill of the rider. For example, in Fig. 3 the straight line represents the track in which the rider is attempting to move; the plain wavy line represents the actual track of the centre of mass, and the dotted wavy line represents the actual track of the base-point; the proportionate amplitude of the deviations of both points being exaggerated for the sake of distinctness. The deviation of the base-point from a straight track at any point is greater than the deviation of the centre of mass by an amount equal to the deflection of the base-point corresponding to the curvature of the track of the centre of mass at the instant; which curvature, by the known properties of harmonic curves, or wavy curves of small deflection, varies directly as the deviation, inversely as the square of the velocity, and inversely as the height of a pendulum whose swings would keep time with the oscillations of the wavy motion. In symbols, let g denote gravity; y the lateral deviation of the central mass from a straight track at any instant; T, the time of a double oscillation of the wavy motion; r, the radius of curvature of the track of the centre of mass at that instant; then we have, very nearly

FIG. 3



$$\frac{1}{r} = \frac{4\,\pi^2\,y}{v^2\,\mathrm{T}^2} = \frac{g\,y}{v^2\,p}\,; \quad . \quad . \quad . \quad . \quad (3.)$$

in which p is the height of the supposed pendulum. Let y' be the deviation of the base-track from a straight track; so that $y' - y$ is the deflection of the base-track; and let h denote *m* P, the height of the centre of mass above the roadway. Then, by Equation 1, we have, for the deflection of the base-track at the instant in question, the following value:—

$$y' - y = \frac{h\,y}{p} = \frac{4\,\pi^2\,h\,y}{g\,\mathrm{T}^2}\,; \quad . \quad . \quad . \quad . \quad (4.)$$

and, for the deviation of the base-point from a straight track,

$$y' = y\left(1 + \frac{h}{p}\right) = y\left(1 + \frac{4\,\pi^2\,h}{g\,\mathrm{T}^2}\right) \quad . \quad . \quad (5.)$$

There is a tendency for the transverse oscillations to keep time with the alternate pressures of the rider's feet on the cranks. When such is the case, let a be the radius of the fore-wheel; then we have the following value for the periodic time of a revolution of that wheel and of a double transverse oscillation:—

$$\mathrm{T} = \frac{2\,\pi\,a}{v} \quad . \quad . \quad . \quad . \quad . \quad . \quad (6.)$$

When that value of the periodic time is inserted in Equations 4 and 5, it gives for the deflection of the base-track,

$$y' - y = \frac{v^2\,h\,y}{g\,a^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7.)$$

and for the deviation of the base-point from a straight track,

$$y' = y\left\{1 + \frac{v^2\,h}{g\,a^2}\right\} \quad . \quad . \quad . \quad . \quad . \quad (8.)$$

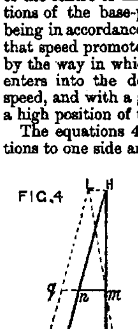
Suppose, now, that no deviations arise from deficiency of skill on the part of the rider, but that unavoidable transverse oscillations of the base-point, of the extent denoted by y', arise from the roughness of the roadway. Then the corresponding extent of the transverse oscillations of the centre of mass is given by the following equation:—

$$y = \frac{y'}{1 + \frac{h}{p}} = \frac{y'}{1 + \frac{4\,\pi^2\,h}{g\,\mathrm{T}^2}} = \frac{y'}{1 + \frac{v^2\,h}{g\,a^2}} \quad . \quad . \quad (9.)$$

This last equation shows that the transverse oscillations of the centre of mass are always less extensive than those of the base-point, and also that the extent of the oscillations of the centre of mass, as compared with that of the oscillations of the base-point, is the less the greater the speed; being in accordance with the well-known fact already stated, that speed promotes steadiness. Equation 9 further shows, by the way in which the height h of the centre of mass enters into the denominator, that steadiness at a given speed, and with a given size of fore-wheel, is promoted by a high position of the centre of mass.

The equations 4, 5, 7, 8, and 9, are applicable to oscillations to one side and to the other of a circular track, as well as of a straight track. The following geometrical construction represents, in a simple way, the relation between the transverse oscillations of the base-point and those of the centre of mass. In Fig 4, let M be the base-point in its undisturbed track; *m*, the centre of mass, also in its undisturbed track. Join M *m*, and produce it to H, making *m* H equal to the altitude of a simple pendulum which would keep time with the oscillations. This is calculated by recollecting that the altitude of a pendulum which makes a double swing in a second is very nearly 0·815ft. = 9·78in. = 248 millimetres, and that the altitude varies as the square of the periodic time. Draw M N horizontally, to represent the extent of the horizontal oscillations of the base-point; join H N, and through *m* draw a horizontal line, cutting H N in *n*; then *m n* will represent the extent of the horizontal oscillations of the centre of mass.

FIG. 4



As an example, suppose that the height *m* M of the centre of mass above the roadway is 39 in., and that the period of a double oscillation is one second, so that *m* H is 9¾ in., nearly; then the oscillations of the centre of mass will have $\frac{9\frac{3}{4}}{39 + 9\frac{3}{4}}$ = *one-fifth* of the extent of those of the base-point. Now, suppose the speed reduced to one-half, so that the period of oscillation is doubled; the altitude *m* H will be increased fourfold, so that it will be equal to *m* M; and the extent of the oscillations of the centre of mass will be increased to *one-half* of that of the oscillations of the base-point.

10. *Effect of Unskilfulness in the Rider upon Oscillations.*—The most probable effect of unskilfulness in the rider is to produce unnecessary additional deflections of the base-track alternately to one side and to the other, keeping time with the pressures of his feet on the cranks. Let y'' be the greatest additional deflection of the base-point produced in this way; then, according to Equations 4 and 7, the corresponding additional extent of oscillation of the centre of mass is

$$\frac{p\,y''}{h} = \frac{g\,\mathrm{T}^2\,y''}{4\,\pi^2\,h} = \frac{g\,a^2\,y''}{v^2\,h} \quad . \quad . \quad (10.)$$

To express the combined effect of deviations of the base-track of the extent y' caused by roughness of the roadway, and of unnecessary deflections of the extent y'' caused by the unskilfulness of the rider, the results of Equations 9 and 10 are to be added together, giving the following value for the extent of oscillation of the centre of mass:—

$$y = \frac{p\,y'}{p + h} + \frac{p\,y''}{h}\,; \quad . \quad . \quad . \quad . \quad . \quad (11.)$$

the value of p being, as before,

$$p = \frac{g\,\mathrm{T}^2}{4\,\pi^2} = \mathrm{T}^2\ (\text{seconds}) \times {\cdot}815\ \text{ft., or } 9{\cdot}78\text{in., or 248 millimetres} \quad . \quad . \quad . \quad (11\mathrm{A.})$$

To introduce the effect of the rider's unskilfulness into the construction of Fig. 4, lay off M K horizontally, to represent the extent of the unnecessary deflections of the base-track; join K *m*, and produce it till it cuts a horizontal line through H in the point L, through which draw L *q* Q parallel to H *n* N, cutting *m n* produced in *q*, and K M N produced in Q. Then *m q* will be the extent of oscillation of the centre of mass, and K Q the extent of oscillation of the base-point.

11. *Effect of a Side Wind.*—To keep the balance when a side-wind is blowing there is required a horizontal deviation of the base-point to leeward of the centre of mass, bearing a proportion to the height of the centre of mass above the roadway equal to the proportion which the pressure of the wind bears to gravity; and that deviation is to be combined with the deflection required by the curvature of the track of the centre of mass. When the pressure of the wind varies the deviation which counteracts it must be varied at the same time and in the same proportion by guiding the fore-wheel.

12. *Concluding Remarks on Balancing.*—The preceding articles of this section relate to the balancing of the mass composed of the velocipede and rider as a whole; and it is assumed throughout, that the rider sits steadily on the saddle, causing, or rather permitting, his body to accompany the lateral movements of the hind-wheel plane, in order that this plane may always traverse the centre of mass. He cannot, by any attitude or movement of his body and limbs, produce any direct effect on the track described by the centre of mass. He may, however, cause the hind-wheel plane to incline to one side or to the other of that centre by leaning over himself to the contrary side; but such inclinations of the hind-wheel plane are useless for purposes of balancing and steering, and they tend to overstrain the wheels and frame, and to make the rider slip off the saddle. The balancing of a velocipede with its rider is in some respects analogous to that of a skater while only one of his skates touches the ice: the skater guides his foot so that the edge of the skate always traverses the resultant of gravity and centrifugal force acting through his centre of mass; and, in like manner, the velocipede rider guides the wheels so that the wheel-base always traverses the same resultant. The force which produces circular motion is derived in the one case from the resistance of the ice to sideward slipping of the skate; in the other case, from the resistance of the roadway to sideward slipping of the wheels. There is a difference, however, in the action by which the guidance is effected; the skater, at the instant of beginning to describe a circle on one skate, impresses on his body, by the aid of the other skate, a rotation about his own vertical axis with an angular velocity equal to his intended angular velocity of revolution about the centre of the circle; and the skate on which he rests, by turning with his body, is guided so as to accompany the circular motion of his centre of mass: the velocipede rider, on the other hand, guides the wheels so as to move in circular tracks about a given centre, by adjusting their planes to a certain angle with each other.

The keeping of the balance in *mounting* a velocipede depends on the fulfilment of one or other of two conditions: either the line of motion of the centre of mass of the rider at the instant when he alights in the saddle must be in the same vertical plane with the line of motion of the centre of mass of the velocipede, and the track of the wheels; or, should that be impracticable, the rider must be prepared instantly to correct the deviation by means of the steering apparatus.

(*To be continued.*)

THE ABUSE OF THE CORD.—At the Hertford County Sessions on Saturday Mr. Joseph Judge Hayes, a gentleman residing at Hertingfordbury, appeared to answer a charge preferred against him by the Great Northern Railway Company of unlawfully signalling a train to stop without reasonable or sufficient excuse for so doing. Mr. Oppenheim appeared for the Great Northern Railway Company. The proceedings were taken under the Act 31 and 32 Victoria, cap. 119, which requires that there shall be provided means of communication between passengers and guards and drivers of all railway trains travelling twenty miles without stopping. It appeared from the evidence that on the 12th inst. the defendant left King's Cross by the 2.45 express to Manchester and Liverpool, which does not stop till it reaches Peterborough. He should have got into a "slip" carriage placed in the rear of the train, which is dropped at Hatfield, at which place he wanted to stop; but instead of doing so he got into a carriage labelled "Bradford," placed nearly in the centre of the train. Just after passing Hatfield Mr. Hayes pulled the cord of communication which runs outside the carriages, and the driver and guard, hearing the gong on the engine sound, looked round and saw him at the window of the Bradford carriage, making signs to the effect that he wanted to get out at the station just passed. The guard signalled back that the train could not be stopped for such a purpose, and the defendant, by a gesture, intimated that he was satisfied. Shortly after the train passed Hitchin the gong sounded again, and on its being brought nearly to a standstill the defendant got out. Mr. Hayes, in defence, said he could not find a "slip" carriage on at King's Cross, and got into the Bradford carriage under the impression made upon his mind, by looking at the time-table, that the train stopped at Hitchin. The magistrates having consulted for some time, the chairman announced their decision. They were, he said, of opinion that the offence charged had been committed—that Mr. Hayes had used the means of communication without reasonable and sufficient cause. It ought never to be used except in grave and important circumstances, such as murderous violence, sudden illness, a carriage being found on fire, or other serious circumstances of a similar kind. As, however, the Great Northern Railway Company had no vindictive feeling in this matter, and as this was the first case of the kind that had come before the bench, they would only inflict a penalty of 5s. and the costs. Mr. Webb (of the firm of Messrs. Johnston, Farquhar, and Leach) said he was afraid that the public would infer from the amount of the penalty that the magistrates looked upon the offence as a very light one. Mr. Oppenheim said the remarks of the bench were of more importance than the amount of the penalty. Mr. Cherry: "The press will give publicity to my remarks." Mr. Hayes is to be prosecuted at Hitchin for stopping the train in that district. A similar case has occurred since on another line.