several typographical errors occur in the Appendix 11. As a result the steer equation (the second equation) is incorrect. The algebraic error made by Sharp results in the incorrect cancellation of the following term (in his notation),

$$2[M_f ek + I_{fz}\cos\epsilon + M_f eb]l_1 t\,\ddot{\delta}$$

We also make note of the following typos: the x_1^2 in the lean equation of Appendix II should read $\dot{x}_1^2$; there is an extra parathesis in the ninth term of the fourth equation in Appendix 1 section entitled "Linear equations of motion"; the term $\dot{x}_1\cos\epsilon\ddot{\delta}$ in the expression for $\ddot{\psi}$ in Appendix II should read $\dot{x}_1\cos\epsilon\dot{\delta}$; I_{fy} should read $i_f y$ in the $\dot{\phi}$ term of the steer equation of appendix 2; and finally terms involving $\frac{i_{fy}}{R_f}l_1 t\dot{x}_1\sin\epsilon$ in the $\dot{\delta}$ term of the steer equation can be eliminated as they cancel one another.

Sharp also makes the slightly restrictive assumption that one principal axis of the center of mass moment of inertia tensor of the front assembly is parallel to the steering axis. Thus the equations in his paper, when corrected, are a subset of those derived in Chapter 111. Sharp refers to the work of Whipple [1899], Pearsall [1922] and Collins [1963], but does not compare his equations to theirs.

Roland, 1971

In 1971 Roland published a report written for the Schwinn Bicycle Company containing a extensive nonlinear computer simulation study.[5] In this report Roland derived nonlinear equations that represent the motion of a bicycle with tire side slip

[5] This report was based on work performed for a National Commission on Product Safety research contract. See Roland [1970].

and rider lean. His 8 equations of motion are shown in matrix form on p. **37** of his report. Reading from the top down the first three equations represent force balance for the entire bicycle. The fourth through the sixth equations represent moment balance for the entire bicycle. The seventh equation is apparently moment balance for the front assembly about the steer axis, which can presumbly be used to solve for the steering torque if the tire side force is eliminated. The eighth and final equation represents the rider upper-body lean degree of freedom, and can be used to solve for the tilting moment of the bicycle on the rider when rider motion is prescribed. These equations are written so that the second time derivatives are all on the left side of the matrix equation, while all the lower order terms are on the right hand side.

Roland used axes parallel and perpendicular to the steering axis in the plane of the rear frame, and perpendicular to the rear frame. However, his report to the Schwinn bicycle company is missing an important figure describing the orientation of the body-fixed axes. This figure is contained in a later publication Mechanics and Sport [1973]. In the later publication, Roland also corrects some typos that were in the 1971 publication.

In the 1971 report, the seventh equation, the steer moment equation, is given on **p.** 13 as eq. (2.3.30). We took this equation and assumed e $= x''_{TF} - x''_{F}$ in order for it to agree with the seventh equation in the matrix on page **37.** We then made simplifications to the equation to see if it agreed with our Lagrange equation for ψ, $(3.10g)$.

First we set any term multiplied by y_F or y''_F equal to zero. (This means there is no lateral imbalance.) We then neglected terms multiplied by the pitch rates q and q'', **which** are second order effects. Next we assumed angles (and their time derivatives) to be small and let $\sin\delta = \delta$ and $\cos\delta = 1$, and cancelled any products of the small quantities p, p'', q, q'', and δ (and their time derivatives).

We then linearized the variables $\gamma_{12}, \gamma_{22}, \gamma_{32}$ in the same way. Terms multiplied by γ_{12} become zero, $\gamma_{22} = m_f(rV - g\phi)$ and $\gamma_{32} = m_f g$.

The coefficient of the $\ddot{\delta}$ term seems correct, and the resulting equation appears somewhat similar to our equation (3.14), but we are not able to make the resulting equation agree completely. There is some question as to whether the comparison we are making is correct, because it is not understood if in fact Roland's equation **(2.3.30)** should be equivalent to our equation 3.10g.

An equation equivalent to our lean equation has not yet been constructed from Roland's equations. However, it is probable that an equivalent equation would be obtained by combining the fourth and sixth equations in his matrix to represent rolling moment about the track line, setting the rider lean angle to zero, including the mass of the rider with that of the rear frame and rear wheel, setting pitching motion to zero, and setting tire side slip to zero. The lateral forces, F_{ytr} and F_{ytf}, on the wheels can perhaps be solved for analytically using the first, fourth and sixth matrix equations or taken from other linearized equations studies. Since we have not attempted this, we are not able to judge whether his lean equation reduces to ours.

Roland refers to the works of Whipple [1899], Bower [1915], Pearsall [1922], Manning [1951], Döhring [1955], Collins [1963], Singh [1964], and also Singh and Goel [1971]. However, he makes no comparisons to their equations of motion.

Weir, 1972

In an appendix to his 1972 UCLA Ph.D. dissertation focusing mainly on the control and handling characteristics of motorcycles, Weir derived the equations of motion for the Basic bicycle model with a general Newtonian approach, linearizing as the derivation proceeded. Weir's final 4 equations, eq. [A-851, [A-92], [A-99], [A-108] in his analysis, represent the lateral motion, yaw, lean and steer equations of motion.

Weir was the only author to state explicitly that he compared his equations to another past work.[6] He compared his equations to Sharp's [1971] four equations (before Sharp's simplification to only two nontrivial degrees of freedom). In comparing Weir's 4 equations to Sharp's four equations, we find Weir and Sharp in agreement with one another. Weir, however, is more general than Sharp, in that he did not make the simplifying assumption regarding the principal axes of the front inertia. When Weir's four equation are simplified by adding the zero sideslip constraints we find his equations agree exactly with ours, as long as our nonstandard sign convention for wheel angular momentum (positive for forward rolling) is recognised.

[6] See page 130 of Weir's dissertation.