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The Lateral Dynamics of Motorcycles and Bicycles

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SUMMARY

The paper is a review of the state of knowledge and understanding of the steering behaviour of single-track vehicles, with the main accent on vehicle design, and vehicle design analysis and behaviour prediction.

The body of the paper consists of a chronological account of the steps which have been taken in establishing the current position. Scientific study of the motions of two-wheelers has been in progress for more than 100 years, but progress was slow and many conflicting conclusions were drawn until increasing understanding of tyre mechanics, systematic application of the laws of motion for systems of rigid bodies, digital computation and modern numerical methods, and improved mobile measurement, recording, and data processing capabilities allowed the pace to accelerate.

The current position, which is that a good understanding of the relationship between design and performance has been achieved, but that by no means have all the problems of significance been solved, is described at the end of the paper.

1 INTRODUCTION

In this review, the events which, in the author's view, have led to the current understanding of the steering behaviour of single track vehicles are recounted chronologically. General observations, scientific measurements and data processing, mathematical modelling of tyres, vehicles, and riders, and the collection and use of vehicle data are referred to, and emphasis is placed on the extent to which predicted and measured behaviours agree with each other.

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Those aspects of the behaviour which are of greatest significance are described, and the relationship between machine design and behavioural properties is discussed. Certain aspects of the design turn out to play a central role in determining behaviour, and these are highlighted.

There are several outstanding issues in the subject area, deserving of further research, some of which is under way. These issues are mentioned, and methods for resolving them are discussed.

2 HISTORICAL SURVEY

A theory of the stability of single track vehicles was probably first produced by Rankine [1]. He regarded the rider as a rigid extension of the rear frame of a bicycle, causing the front frame to steer in response to the bicycle beginning to capsize. The tyres at that stage were not viewed as producers of forces and moments, but were seen as constraining the bicycle to move in the direction in which the wheels pointed, so that the motion of the contact points were determined by the steering, and it was argued that "centrifugal forces" resulted sufficient to balance the gravitational overturning moment consequent on the original rolling motion.

Whipple [2] analysed mathematically the problem substantially as seen by Rankine except that the rider was considered not to touch the bicycle handlebars. The steering system was free to steer itself under the influence of gyroscopic moments associated with the spinning front wheel. Whipple's analysis was Lagrangian, and it treated the tyre/road contact as giving rise to non-holonomic constraints on the vehicle motion. Only roll and steer degrees of freedom existed and, with only small perturbations from straight running allowed, a fourth order characteristic equation resulted. The stability was tested by application of the Routh-Hurwitz criteria, and it was concluded that the (then) modern bicycle and rider was self stabilising (without rider control) in a small speed range extending from 16 to 20 km/hr. The implication was that successful operation of a bicycle outside this speed range would depend on rider control of the steering system, or, as could be observed in practice, by body lean control.

The view of the tyre behaviour contained in Whipple's analysis promotes the idea that the self steering of the front wheel is governed entirely by gyroscopic torques, and Klein and Sommerfeld [3] employed a modified Whipple analysis to show that, under the assumptions made, the self stability of the bicycle is indeed dependent on gyroscopic effects. Pearsall [4] took the position that bicycles had developed in a perfectly satisfactory manner, but that motorcycles were significantly different from each other, giving rise to an engineering problem especially in respect of what he called "speedman's wobble" Clearly this referred to an oscillatory instability occurring at high vehicle speed (for the time), but quantitative information on speed or frequency was not provided.

No further progress on the problem appears to have been made until Wilson-Jones [5] considered the matter from the point of view of the experimentalist and practitioner. By this time (1951), knowledge of tyre behaviour was greatly improved and it had become commonplace, although still by no means universal, to regard the rolling wheel as a force producer, rather than as a constraint on the vehicle motion. It was recognised that the sideslip angle is the principal determinant of the sideforce generated by a steady rolling loaded tyre, that the sideforce acts behind the geometrical centre of the contact region for small slip angles, and that a cambered wheel, again in a steady state, will generate a camber thrust. Wilson-Jones discussed the stability of motorcycles in these terms, and devised and utilised rudimentary methods for measuring tyre slip angles and steering torque inputs. He realised that the self steering action of the front wheel of a single track vehicle is crucial to its stability, and that it is influenced by many factors apart from gyroscopic effects. He was able to conclude that the tyre sideslip angles used were normally very small, that the motorcycle corners mainly by virtue of camber thrust, and that counter-steering is usually employed for turn entry or exit. This last conclusion provoked much discussion at the time of its drawing. Wilson-Jones also mentioned the tendency of motorcycles to roll over or capsize at low vehicle speed, despite the rider's best efforts, if the friction pad steering damper, which was then commonly fitted, was tightened down too far.

Döhring [6] applied the kind of theory which Whipple had developed to various motorcycles, and calculated characteristic roots of the equations of motion. He found that the uncontrolled motion was oscillatorily unstable at low speed, that, as Whipple found for the bicycle, there was a small speed range in which the motion was stable, and that, for higher speeds, the motions was slowly divergent.

Döhring also [7] examined what he called steering wobble in motorcycles. He associated it with shimmying in automobile steering and aircraft tail wheels, and high machine speed, going so far as to describe lateral oscillations as often limiting racing machine speeds before engine performance. Döhring carried out a simple and somewhat flawed analysis of the motion of the steering system about an axis assumed to travel in a precise straight line, treating the tyre as giving sideforce and aligning moment proportional to slip angle. He pointed out the need for data on motorcycle tyres, and discussed the introduction of phase lag between tyre tread motion and wheel motion on account of tyre carcass elasticity under time varying conditions. He associated this with the possibility of energy transfer from forward motion to lateral oscillations, and recorded steering displacements on a running 500 cc motorcycle at 17, 25 and 33 m/s with strong contributions at 1.5, 2 and 3 Hz respectively. Although Döhring used the term wobble and attempted to analyse the problem as a steering oscillation problem, it is likely that he was actually measuring weave motions which we now understand to involve the whole motorcycle.

In 1963, Kondo, Nagaoka, and Yoshimura [8] published an analysis of the motions of a complete cycle and rider, treating the vehicle as two rigid frames joined at an inclined steering axis, the rider being part of the rear frame. Free steering was assumed, and the motion was restricted to small perturbations from straight running. The tyres were described as giving side forces linearly dependent on slip angle and camber angle and correct expressions for these angles were derived. The inertial contributions to the equations of motion, however, appear to have been derived by rather piecemeal methods and it is likely that the final equations contain significant errors. Motion time histories showing rapid divergence at 20 km/hr forward speed (too rapid for manual control to be feasible), and oscillatory motion at 5 Hz frequency becoming unstable just above 28 m/s were generated from parameters which represented a motor scooter. Fu [9] studied the steady turn "equilibrium" of a single track vehicle, accounting for the finite cross section of the tyre, the point of contact between tyre and ground moving round the tyre cross sectional perimeter as the wheel cambers. Only aerodynamic drag and pitching moment of all the aerodynamic effects were considered worth including in the analysis. The calculations agreed with Wilson-Jones' finding that tyre sideslip angles are normally very small.

Jones [10] examined by trial many variants of a normal bicycle, concentrating on variations affecting the self-steering action, and showed just how difficult it is to make a bicycle unrideable by non-extreme changes.

The author's first contribution to the subject was published in 1971 [11]. It consisted of a theoretical analysis of the straight line stability properties of a large motorcycle, and a discussion of the rider control and vehicle design implications. The model used was similar in concept to that employed by Kondo et al [8], but systematic procedures (Lagrangian) were used to complete the equations and subsequent work has confirmed their correctness. An important additional feature was included in the tyre model, the side force being taken to be related to the steady state side force for particular slip and camber angles by a first order lag function which took account of the tyre's distance dependent relaxation property. Eigenvalues were calculated as functions of vehicle speed, and the consequences of omitting the tyre relaxation, and also of omitting tyre sideslip, to the eigenvalues were determined. In this last situation, the model is of the form of Whipple's model, and the stability predictions were not unlike those of Whipple [2] and those of Döhring [6]. The model was also reduced from its full form by omitting the steering degree of freedom. This converted the analysis from "free control" to fixed control, and again eigenvalues were found.

It transpired that the most comprehensive model predicted the presence of three important principal mode motions throughout the speed range. These were christened "capsize", "weave", and "wobble", the last two of which were oscillatory modes except at very low speed when the weave mode becomes overdamped. The capsize mode was stable at low speed and marginally unstable beyond 10 m/s, the weave mode, once oscillatory, increased in frequency from zero to 3 Hz over the machine speed range, stabilising at about 6 m/s, being well damped in the mid speed range, and approaching instability at high speeds again, and the wobble mode had an almost constant natural frequency of 9 Hz and became less damped as speed increased approaching instability at the top speed as the weave mode did. Without tyre relaxation, the capsize and weave modes were not much changed, but the wobble mode was stabilised dramatically, while with no sideslip (which is equivalent to taking the tyre cornering stiffnesses to be infinite), the capsize mode and the weave mode at low speed are little altered but the wobble mode and the weave mode higher up the speed range were much affected. It became clear then that the full model employed represented minimum requirements for one which would apply generally. The fixed control stability appeared relatively unattractive, especially at low speeds, suggesting that a lot of steering system friction could lead to low speed balancing problems for the rider as referred to by Wilson-Jones [5], and tending to confrim that the machine problem should be regarded as involving free control.

Steady state steering torque inputs as functions of speed for a lean angle of 10° were calculated also, and these showed the largest torques to be needed at the lowest speeds. The sign reversed from negative (countersteer torque) to positive at the same speed as the capsize mode changed from unstable to stable with increasing speed, and it is now realised that this coincidence is fundamental, both sign reversal and stability change being associated with passing of the static stiffness of the system, represented by the zero order term in the characteristic polynomial, through zero. This same phenomenon occurs with the oversteering automobile, and in that context is very important, but it does not appear so significant in the case of the single track vehicle because the instability is principally a roll and not a yaw instability, and as speed increases the divergence does not become steadily more rapid. The conclusions were originally drawn that the value of steady turning analysis is in connection with (a) the magnitude of control inputs required at low speeds and (b) the requirement that steering friction torques be substantially less than the rather small control torques required at medium and high speeds, and these seem valid still.

Weir [12] confirmed the detailed accuracy of the model above, and used it without tyre relaxation as a basis for examining theoretically likely modes of rider stabilisation of the man/machine system. From a knowledge of human control capabilities and preferences, Weir was able to postulate that the rider senses one or more motion variables and uses the information, after an appropriate time delay, to control either steer torque or body lean inputs, and by examination of the resulting system's closed loop eigenvalues and open loop frequency responses, he was able to conclude that, according to his modelling, stabilisation can easily be achieved by sensing roll angle and using it to control steer torque. He observed that relatively large body lean inputs were necessary in order to achieve responses comparable with those to steer torque, and although he drew conclusions about the value of body lean inputs in respect of path following, it should be noted that, since no rider lean degree of freedom was contained in the model, body lean was supposed to be representable by an externally applied roll torque.

The excellent work of Eaton [13] was finished in 1973. He developed the author's theory a little way by adding some details to the tyre model including self aligning moments and overturning moments, and applied it to the prediction of the responses to steering torque and roll moment inputs of an instrumented machine. The machine responses were also measured, being recorded on a chart for comparison with analog computer simulations, and later, on magnetic tape for digital analysis to identify both machine and rider dynamics. Rider body movements relative to the motorcycle were discouraged by a bracing frame, and test speeds were limited to 20 m/s. Eaton found the theory to represent the responses well except at very low speeds and higher frequencies. His identification of the rider control activity gave consistent results in agreement with the "crossover" model ideas which had been the basis of Weir's work. He thought, wrongly as it turned out, that the chief deficiency in the motorcycle modelling was in connection with the tyre model, and he suggested the elaboration of this along well established "taut string" guidelines.

Roland [14] reported on a comparison of experimentally determined and computer simulated high speed weave motions of a heavyweight and rear laden motorcycle. The simulation model used was conceptually that of Kondo et al [8], with front and rear frames joined at the steering head and tyres responding instantaneously to slip and camber angle inputs, but was derived independently as a reduction of a more general bicycle and rider model. Good agreement between experiment and theory was obtained, and the experimental records showed more clearly than before the mode shape of the weave oscillation, with steer and roll of equal amplitude, the latter lagging the former by 110°. The weave frequency was about 2 Hz. In [15], Roland described the bicycle and rider model, and included some measured steady state bicycle tyre data. Eight degrees of freedom were allowed, six for the bicycle frame, one for steering of the front frame, and one for rider upper body lean, and quite general motions over a flat surface were allowed, with tyre forces being non-linear functions of slip angle, camber angle, and wheel load. The rider model consisted of a stabilisation sector, partly deriving from Weir's work [12], and a guidance sector, employing a sampled data preview-predictor stratagem with a constant preview time. Comparisons between simulated motions and measured motions were used to tune the rider model parameters, after which effective path following was achievable with reasonably general paths. The riderless bicycle was also examined both experimentally and theoretically, a steering torque disturbance being provided by a small rocket motor fixed to the handlebars, and quite good agreement resulted. The rocket motor was also used to provide a roll disturbance with the rider in place. Although Roland recognised that real riders are adaptive depending on the machine dynamics and the task, the rider model, once tuned, had fixed properties, and in simulations involving wheelbase, mass, mass centre height, steer inertia, steering head angle, mechanical trail, wheel diameter, wheel spin inertia, and tyre cornering stiffness changes, vehicle speed had a stronger influence on stability than any other parameter. A short wheelbase appeared advantageous at low speed.

Watanabe and Yoshida [16] carried out obstacle avoidance experiments in which motorcyclists could pass either side of an obstruction but were told which side to pass with minimal notice. Skilled riders could be successful with 15 to 20% less notice than unskilled ones, and achieved success by using higher levels of steering torque control inputs and much greater maximum lean angles.

Cooper [17] became involved in motorcycle problems through attempts to set class speed records. Steady state aerodynamic forces and moments on an elaborately faired machine were measured, and the effects included in the model of [11] in an attempt to explain a very high speed weaving problem. Not being entirely successful, Cooper postulated that the problem may have involved substantial non-steady aerodynamic influences, which he estimated. From his results, it appeared that the effects of aerodynamic side force, yawing moment, and rolling moment on the lateral stability of production motorcycles would not be large, but that drag, lift, and pitching moment would contribute to changing tyre loadings with speed, and thereby influence the tyre lateral forces.

The author's model was extended [18] by the inclusion of a twist degree of freedom between the rear frame and the rear wheel, corresponding to flexibility in the rear suspension swinging arm or pivot bushes. Eigenvalues showed that levels of stiffness which were low compared with standard production practice would contribute to deterioration of the high speed weave mode stability, but that the returns for using higher than conventional stiffnesses would not be great and would diminish with increasing stiffness.

Jennings [19] drew attention to the occurrence of a modified weave oscillation with motorcycles in cornering, in which suspension motions take a systematic part, with the suspension dampers having an important influence. Jennings suggested that pitching motions may be interacting with weaving motions, and following the making of a film showing the phenomenon by Dunlop, the author [20] showed by a simple analysis that typical motorcycle design of the time would result in the speed invariant pitch mode natural frequency coinciding at high speed with the natural frequency of the weave mode (for small perturbations from straight running), at which condition the weave mode is typically very lightly damped. For straight line operation, it was clear that coupling between in-plane (suspension) motions and out-of-plane motions would be weak, but for oscillations taking place around a cornering trim condition, the coupling would become stronger as the trim lean angle increased. Thus it was expected that a full analysis of the cornering problem would reveal a cornering pitch-weave mode, the stability of which would be sensitive to the suspension damping coefficients.

Singh, Goel, and Bhattacharya [21] measured steady state tyre side force, aligning moment, and overturning moment data for new and worn free rolling scooter tyres. The effect of tyre load in these results is somewhat difficult to discern because the inflation pressure was altered with the load.

Segel and Wilson [22] worked on the supposition that an accurate modelling of the wobble mode, particularly at low vehicle speeds, would require a more precise description of the responses of motorcycle tyres than had up to that time been available. They carried out detailed measurements of the transient build up of sideforce generated by a cambered tyre, and studied the overturning moment too. It appeared that a proportion of the steady state camber thrust followed immediately the wheel camber, and that the remainder built up subsequently as the tyre rolled along with a relaxation length about twice that associated with sideslip responses.

The effects of acceleration on the lateral stability of motorcycles were calculated by the author [23]. The constant forward speed assumption formerly employed had made the longitudinal equation of motion unnecessary, but for small perturbations from straight running, it proved to be substantially decoupled from the lateral equations. This implies the possibility of solving the longitudinal equation for the acceleration, and then using it as a parameter in the lateral equations, the acceleration contributing to longitudinal "inertia forces" at front and rear frame mass centres, and front to rear wheel load transfers. The shortage of tyre data at that time made the tyre model assumptions in respect of load sensitivity rather tentative, but it was clear from the results that the capsize mode is significantly stabilised by accelerating and conversely, mainly controlled by the roll angle to yawing moment feedback term arising from the rear frame "inertia force". It is generally recognised among motorcycle riders that the steering, at low speed, feels much better when accelerating, and riders usually develop a low speed cornering technique to take advantage of this. The theoretical results probably represent a kind of explanation.

Roe and Thorpe [24] measured steer angle fluctuations on a machine prone to self-excitation in its wobble mode, of frequency about 7 Hz. The self-excitation was strongest in the speed range 15 to 20 m/s, which confirmed that there was a very clear mismatch between existing theory and at least some real world behaviour. Roe and Thorpe described the improvement in stability of the mode consequent on stiffening laterally the forks, and stiffening torsionally the rear frame. They showed the behaviour in general to worsen with rear loading.

Sharp and Jones [25], following the line of enquiry suggested by Segel and Wilson [22], elaborated the treatment of the tyre, and used parameter trials to determine which aspects of real tyre behaviour are important to describing the behaviour of the motorcycle in straight running. The problem was predicting correctly the damping of the wobble mode as a function of vehicle speed, since the other aspects of the behaviour were already represented with some accuracy.

In the absence of comprehensive experimental data on motorcycle tyres, it was necessary to construct such data based on the idea that the taut string tyre model, which had been validated for model aircraft tyres and car tyres for nominal loadings, would describe motorcycle tyres well in respect of sideslip and turnslip responses over the range of loads which correspond with operation at different speeds. Such tyre data as was available was used to provide the basic string model parameters, the half contact length, the relaxation length, and the lateral stiffness per unit length for nominal load, and the manners in which these change with load were estimated. Transfer functions were then employed to represent the behaviour (a procedure which later turned out to be unnecessary), and the transfer functions, together with aerodynamic load transfer effects and empirical camber responses proportional to load, were incorporated in the motorcycle model. The motorcycle results proved completely insensitive to whether the camber responses were lagged or not, suggesting that the camber force system is of minor importance in respect of the wobble mode, and it became clear that the disagreement between theory and observation in respect of the wobble mode damping would not be removed by simply describing the tyre with greater accuracy.

Verma [26] carried out a comprehensive study of the small perturbation motions of a large motorcycle. His theoretical work extended the then conventional model by including yaw and camber degrees of freedom relative to the rear frame for the rear wheel, a camber degree of freedom for the front wheel, and a twist freedom for the front forks about an axis parallel to be steering axis. Axes of rotation for the first three of these were defined through carrying out static stiffness tests on the motorcycle frame in which the centre section was anchored and lateral loads were applied to the centres of tyre/road contact, and these tests also provided the stiffness data. An elaborate tyre model was used, based on taut string theory, but employing mostly measured tyre force and moment data, and the tyre model adjusted its parameters as the vehicle speed varied to accommodate aerodynamic load transfer effects. Experimental response measurements were conducted over the machine's full speed range, a bracing frame being used to discourage movement of the rider's body relative to the motorcycle. The input in these tests was a steering torque impulse, and the responses were recorded on a small FM tape recorder. They were subsequently played back and either band pass filtered, or digitised and Fourier analysed to characterise the resonant response peaks which occurred, and, on the assumption that in such conditions, one modal response component would dominate the others, the characteristics of the peaks were used to derive the modal frequencies and damping factors as functions of speed for direct comparison with calculated eigenvalues. The theoretical results were altered very little by the inclusion of the frame flexibilities, the main change being to the damping of the wobble mode at low speed, and the experimental and theoretical results were close except in respect of this aspect. Here there was a clear discrepancy both qualitative and quantitative, the theoretical results suggesting the mode to be much more damped than it was found to be. Surprisingly, the model with frame flexibilities predicted more damping than that without, and was therefore less accurate. In one test at 18 m/s, an obvious wobble oscillation was excited with roll velocity and steer velocity respectively 190° and 210° ahead of yaw velocity, and amplitudes in the ratios 1.3:8.5:1.0, but the theoretical eigenvector, presented without the phase information, was very different (4.5:8.1:1.0) in respect of the roll velocity component. The indications were that frame flexibilities were not responsible for the inaccurate prediction of the wobble mode damping, but this was soon to be further questioned.

McKibben [27], in a discussion paper, referred to what appears to be a fundamental conflict in motorcycle design. This concerns steering geometries which, on the one hand, lead to good low speed turning behaviour but which, on the other, give poor high speed directional stability. He implied that small steering rake angles and trail values are ingredients of such geometries. At the same meeting, Rice [28] presented measured steering torques, steer angles, roll angles, yaw rates and rider lean angles from tests in which four different riders performed lane change manoeuvres. All the riders employed lean angles of several degrees at some stage, and their steering torque inputs usually preceded the lean significantly. One can not be sure whether the body lean should be regarded as a response or as partly the result of body lean torque control inputs. Zellner and Weir [29] also showed response data, in this case for several motorcycles, concentrating on steady turns. Conventional linear theory predicted roll angle responses quite well, but not steer angle. They referred to the coincidence of the sign change in the steady state responses with the transition in the capsize mode from unstable to stable, and the sign changes in the measured responses are unmistakable in some cases, and occur at the expected vehicle speeds. Schwarz [3] used conventional theory to examine the controllability of a long wheelbase, front drive, rear steered motorcycle which it was proposed to build as part of a research program. It was predicted as having a statically unstable "flop" mode with a time constant sufficiently short for rider control to be difficult or impossible, and this was subsequently verified experimentally. Basically the same theory was applied by Zellner and Weir [31] to calculating the response properties of four mopeds for which mass, inertia, geometry, and tyre data were all measured. Eigenvalues, eigenvectors, frequency responses, and steer torque pulse responses were shown, and the effects of design changes calculated.

Aoki [32] used a comprehensive measuring system and data processing procedures to determine the frequency responses of four large motorcycles, Several test inputs were used, the steer torque pulse perhaps providing the best combination of frequency range and repeatability. The weave mode resonance condition was clear in the results, the maximum gain typically increasing with speed, with the peak narrowing and the rate of change of phase with frequency increasing corresponding to reduced damping of the mode predicted by theory. With increasing speed the natural frequency of the mode clearly increased too as expected. In one test at high speed, the relative response magnitudes and phases at 3.4 Hz, where the responses peak, can be seen to be steer velocity:roll velocity:yaw velocity, 1.53:1.77:1.0 at 135°:90°:0°. Aoki concluded that when the motorcycle is large, as in his tests, rider lean as a control input has a very small effect, and that the system can be considered subject only to steering torque inputs.

Weir and Zellner [33] used a comprehensive instrumentation system recording on to a digital cassette tape recorder to measure responses of a range of motorcycles to steer torque pulse inputs. Their results include self sustaining (undamped) medium speed wobbles at 5.2 Hz and 18 m/s for a heavyweight machine, at 5.4 Hz and 16 m/s for a middleweight machine, and at 6.2 Hz and 9 m/s for a lightweight machine, but the main point of significance was that they showed many instances of the cornering weave phenomenon with suspension system deflections taking a systematic part in the oscillations. They found the stability of the mode to be worsened by high speed, high steady state roll angles, rear loading, and ineffective rear suspension dampers. Quite comprehensive steady state force and moment response data for several types of free rolling motorcycle tyre were given by Sakai, Kanaya, and Iijima [34].

In 1980, Koenen and Pacejka [35] published details of a mathematical model of the motorcycle which represented a step change in the technology. The model was designed to represent the small perturbation behaviour from a steady state cornering condtion, and it was recognised that it would be essential in this case to include the in-plane freedoms, bounce, pitch, and front and rear wheel hop motions, in the model. The rider was treated as part of the rear frame and the structures were assumed stiff, and in-plane aerodynamic influences were included. The tyres were treated as radially flexible and their cross-sectional shape was accounted for to some extent by a simple geometrical treatment similar to that employed by Eaton [13], except that tyre overturning moments were neglected. Inputs to the tyre model were slip angle, camber angle, turn-slip, and load, with sideforce and aligning moment as outputs, and steady state experimental tyre data for different sideslip and camber angles were prepared. The calculations were carried out in two stages, the first to solve the steady state equations for the cornering condition of interest, thus defining wheel loads, slip angles etc. about which subsequent small perturbations would occur and attitude angles which appear in many inertial terms in the dynamic equations, the second to determine eigenvalues and eigenvectors from the linearised small perturbation equations. The tyres were ascribed a conventional relaxation property, the same relaxation length being associated with each input to the model. Results for two motorcycles were generated, a notable feature being the use of computer graphic output, recorded on film, to display the eigenvectors, which would otherwise be extremely difficult to interpret. The results have been largely superceded by later ones so they are not described further.

The major problem with the straight line stability predictions existing up to the time was substantially overcome by extension of earlier theory to include a twist flexibility of the motorcycle frame perpendicular to the steering head [36], or, what is equivalent, a lateral bending flexibility of the front forks [37]. In [36], Sharp and Alstead calculated the consequences of including, in the rigid framed model, as an alternative to the above, either a lateral flexibility of the front wheel on its spindle, or a twist flexibility of the front forks about an axis parallel to the steering head. The full state of the art of the taut string tyre model as developed by Pacejka to include tread width, tread rubber blocks which can deform longitudinally, and tread mass effects was utilised to give a tyre model consisting of the desired constant coefficient linear equations by virtue of expanding the exact solution of the taut string equations in series and truncating the series, and adjustments of the fundamental parameters of the string model according to the load on the tyre were postulated in order to describe the tyre forces and moments accurately over the loading range of significance. In the absence of comprehensive experimental data, this is still probably the best way of ensuring accurate representation of the very important phase lag between lateral force and sideslip angle at high reduced frequencies, but the parameter adjustments become a little contrived to achieve a good description of the steady state properties (cornering stiffness etc.) over a wide range of loads, and experimental data to fully justify the process is not available. Camber responses were modelled empirically as instantaneous and proportional to wheel load.

Full ranges of frame stiffnesses were covered, associated with parameter sets which represented the mass and geometric properties of four large production motorcycles. Torsional flexibiity in the front forks parallel to the steering axis was found to be of little significance, but common levels of lateral flexibility were found to be sufficient to deteriorate the wobble mode damping to a significant degree. However, lateral flexibility had this effect throughout the machine speed range and could not, on its own, explain the discrepancy between predictions and observations. The torsional rear frame flexibility did however have the desired influence, decreasing the damping of the wobble mode in mid-speed range and increasing it at high speeds, and the results suggested that an optimum stiffness would exist, maintaining a near constant damping factor for the mode over the speed range. Both lateral and torsional flexibilities subtracted from the stability of the weave mode slightly at high speed.

Spierings [37] confirmed the main result above through an independent analysis, employing a comparatively simple tyre model which nevertheless contained the main constituents of the more elaborate ones described. By generating results, not only for different stiffnesses, but also for different axis heights for the lateral fork bending, and by performing calculations with some gyroscopic terms omitted from the equations, Spierings was able to identify separate influences of flexibility and gyroscopic torques which change in relative importance with speed.

Recognising the importance of the frame twist flexibility in the steering head region, Koenen and Pacejka [38] further developed their model by its inclusion. They also refined their tyre model especially in respect of aligning and overturning moment responses to camber angle, and allowed the rider upper body a lean degree of freedom restrained by a spring/damper system. Tyre tread mass effects were also included. An extensive survey of the static stiffness properties of a motorcycle frame was carried out, the rear wheel hub being anchored to a baseplate, loading being applied laterally at the front wheel rim or to provide moments on the front frame perpendicular and parallel to the steering axis, and deflection measurements being made at several locations on both front and rear frames. Only a little of the data generated could be used in the motorcycle model. Stability properties for small perturbations from steady turn conditions (eigenvalues) encompassing the vehicle's capabilities in terms of forward speed and lateral acceleration were generated, with the straight line results changed in the expected manner from those generated by the earlier rigid framed model to show minimum damping of the wobble mode at about 15 m/s. Again, other results have been superceded by later ones from the same model.

The theory of the steering behaviour of motorcycles was reviewed and explained by the author [39]. Calculated eigenvectors for (straight line) weave and wobble modes were shown to compare very well with the experimental results recorded by Roland [14], Verma [26], and Aoki [32], affording further evidence that by then the straight line theory was accurate in general terms. Frequency responses to steering torque inputs were calculated and tentatively interpreted, and the measurement and data processing issues involved in obtaining precise comparisons between theory and observation were addressed.

In response to suggestions that potentially dangerous situations can be created by the addition to motorcycles of handlebar mounted fairings or windshields, Cooper [40] carried out wind tunnel measurements of steady and unsteady aerodynamic forces on machines so embellished. He was able to conclude that steady aerodynamic moments on the front frame of a faired motorcycle are of the same order as rider applied steering torques, and that unsteady aerodynamic influences on the front frame will add to the damping of the wobble mode.

Giles and Sharp [41] compared the results obtained when measuring frame stiffness properties statically, in the manner which Verma [26] had employed, and dynamically. In the dynamic tests, a shaker was used to provide a swept sine wave measured force input and the responses were measured with a roving accelerometer. The results were processed by a two channel spectrum analyser. Remarkable differences were observed between the torsional stiffness of the rear frame in the steering head region when obtained statically by lateral loading at the wheel rim and when obtained at the resonant condition at 12 Hz, not far removed from the wobble mode frequency, and the corresponding axes of rotation, and the differences were shown to be very significant in relation to the prediction of the wobble mode. The full implications of the results were not appreciated at that time, but it is now thought that the lateral translational flexibility at the steering head was sufficient to cause the differences observed, and an accurate modelling of the motorcycle requires the inclusion of both this translational flexibility and the twist flexibility also. Sharp and Alstead [42] employed the model of [36] which included the frame twist flexibility to calculate responses to forcing arising from the rotation of an imperfect front wheel. Dynamic wheel unbalance, tyre lateral force variations (first harmonic only), and wheel swashing were considered. The results revealed that each imperfection could give rise to a wobble mode resonant condition at the road speed at which the front wheel rotation frequency coincides with the natural frequency of the mode. It appears not uncommon for a production motorcycle to possess minimum damping of its wobble mode, governed mainly by frame stiffness, at a road speed at which this coincidence occurs, and this will make such a motorcycle unnecessarily sensitive to imperfections in the front wheel and tyre typically near to 20 m/s. Oscillations observed in practice near this speed may be resonant oscillations of a lightly damped mode, and do not necessarily indicate instability, and the kind of localised dip in the wobble mode damping observed by Verma [26] could be caused by a wheel or tyre imperfection. The desirability of comparing oscillation frequency with wheel rotation frequency in experimental work is indicated, and it is suggested that where these are close together, an examination of the consistency of the phase relationship between wheel rotation and vibration response (via the coherence function) will show whether or not wheel forcing is a substantial determinant of response.

Thomson and Rathgeber [43] briefly described the application of the computer program NEWEUL [44] to the generation of the equations of motion of a motorcycle represented by eight rigid bodies joined by springs and dampers. The equations were linearised for small perturbations from straight running, and theoretical root locus diagrams of familiar form were generated from the equations. Experimental results to check the predictions were also included, without details of motorcycle and tyre parameter evaluations or experimental and data reduction procedures, and very close agreement occurred. The greatest discrepancy was in respect of the damping of the weave mode, and Thomson and Rathgeber attributed this to the treatment of the rider as a rigid extension of the rear frame, which presumes the rider dynamics to detract from the damping of the weave mode in the speed range 22 to 33 m/s. Somewhat surprisingly, they commented on the high sensitivity of the vehicle dynamics to the tyre data used.

Full details of the most recent version of Koenen's model are contained in his doctoral thesis [45]. The vehicle, tyre, and rider model, and the use made of the equations have already been described, results including descriptions of steady turn behaviour, eigenvalues, eigenvectors, and computer graphic modal descriptions. The baseline vehicle in straight running possesses conventional capsize, weave, and wobble modes, an additional mode arising from the inclusion of the rider upper body freedom with natural frequency around 3Hz and lightly damped, depending on the rider parameters chosen, and bounce, pitch, and wheel hop modes which only depend very slightly on the speed. With increasing steady state roll angle, the interaction of in-plane and out-of-plane motions in contributing to the principal modes increases and the root loci and the necessary terminology become complicated. The most significant changes concern the appearance of a cornering weave mode of lower frequency and reduced stability as compared with straight running, and at high lateral acceleration an unstable combined front wheel hop/wobble mode which is believed to occur in practice (patter) especially on the race track. Many root loci are plotted, to test the sensitivity of the results to the model details, parameter values, or the motorcycle design. A very surprising result which deserves further study is that removing the suspension dampers hardly affects the stability of the cornering weave mode.

3. THE STATE OF THE ART

The most comprehensive model of the single track vehicle is that of Koenen. It allows the calculation of steady state responses and modal properties for small perturbations from straight running or cornering conditions, and for straight running it gives results in substantial agreement with the author's highly developed but restricted model, the detailed accuracy of which has been further corroborated by independent studies. The model calculations rely on the availability of inertia, geometric, frame stiffness, tyre, and rider data and it is not yet certain how the frame stiffness data should be measured, and whether or not an additional structural freedom or freedoms need be included in order to obtain precise agreement between theory and practice, and currently rider data can only be estimated. The best approach to obtaining rider properties will probably be to employ parameter identification techniques on measured response data. Roland's model for the bicycle rider [15] is probably the most advanced available relating to single track vehicles, but it is non-adaptive and appears thereby to be limited in its usefulness. The time when human control preferences are understood to the point where a man-machine model can be used to distinguish between good and bad machine design while corresponding machine only studies can not be so used still seems some way off.

In respect of straight line stability and response properties, there is general agreement between the best theory and experiment covering stability and damping, natural frequencies, mode shapes, and frequency response properties over ranges of speed and design parameters for large motorcycles, and there are no areas of obvious disagreement between theory and practice. Establishing precise agreement, and general agreement in respect of lighter motorcycles, scooters, and bicycles requires further work, outstanding questions relating to the method of measurement and respresentation of frame flexibilities, and the structural properties of the rider. These last clearly increase in significance as the ratio of rider mass to machine mass increases. In respect of cornering stability, it is firmly established in practice that the high speed weave oscillation problem usually worsens in cornering, and it typically then interacts with in-plane motions, mostly the pitch mode which when decoupled has a similar natural frequency, with suspension parameters becoming important. It is also suspected that the patter oscillation occurs in practice and involves the same kind of interaction between front wheel hop and wobble motions. Koenen's theory clearly describes the increasing interaction between in-plane and out-of-plane motions, and shows both cornering weave and patter oscillations, but there does appear to be a significant discrepancy between their actual and predicted parameter dependencies.

Conventional single track vehicles possess three main modes of motion in terms of physical significance. These are normally referred to by the names capsize, weave, and wobble, although the names should not be interpreted too literally since the capsize mode will certainly be stable at low speed, and it may be stable through the entire speed range of a vehicle. The weave mode is oscillatory over the complete speed range except at the very low end. It is unstable at low speed and has a natural frequency which grows from zero at the speed where the mode forms to between 2 and 3 Hz at the high speed end. It quickly stabilises with increasing speed, is well damped in the mid-speed range, and becomes very lightly damped at high speed. Cornering usually reduces the damping, depending on suspension properties, and of course complicates the mode. These descriptions appear to be equally true with respect to motorcycles, scooters, and bicycles, where high speed is taken to mean different things in the different contexts. The wobble mode is principally a steering oscillation like wheel shimmy. It has a natural frequency which varies little with speed, which may be as low as 5 Hz for a very heavy motorcycle and as high as 9 Hz for lightweight motorcycles and bicycles. It is clear that high trail and low steering system inertia will contribute to a high natural frequency. The damping of the mode is very strongly dependent on the dynamic stiffness properties which the front frame "sees" at the steering head, which properties have a very significant component from flexibility in the rear frame. If the front frame itself is very flexible, the damping will be similarly influenced. Typical design practices lead to motorcycles of all sizes having minimum (and very small) damping of the wobble mode in mid-speed range. It is suspected that scooters too show this minimum in mid-range, but that the damping is not so slight. Bicycles are most prone to wobbling at high speed.

The capsize mode at low speed is remarkably insensitive to design changes but is made quite rapidly divergent by locking either wheel. When the locked wheel is the front one, no effective mechanism by which the rider might attempt to control the machine is available and the situation is hopeless, but if the rear wheel is locked, control by steering can be attempted and is often successful. At higher speeds, it becomes almost neutrally stable, and sensitive to some parameters but none more than the machine speed, and it is likely that the rider would be rather indifferent to changes because control of the motion in this mode is so easy in any case. The rate at which the weave mode stabilises with increasing speed, starting from rest, is increased by having a short wheelbase, large trail, a low mass centre, a small offset between front frame mass centre and steer axis, a steep steering head, and minimal steer damping, and these features will enhance the controllability of the vehicle at low speed. At high speed, the natural frequency of the weave mode will be raised by increasing the cornering stiffness of the rear tyre and by moving the mass centre forwards.

The damping is most strongly affected by the longitudinal positioning of the machine mass centre, the steering rake, and the rider parameters, a forward mass centre, and a shallow steering head contributing to damping. If a steering damper is addes to a motorcycle, the high speed weave behaviour is destabilised, and it is possible that it is also occasionally destabilised by the rider's control activity. Cornering weave problems can be accentuated by coincidence of the (uncoupled) pitch and weave mode natural frequencies and by lack of suspension damping. The high speed weave phenomenon is normally a little beyond the closed loop control bandwidth of the rider and it is essential for the machine/rider/load combination to have a measure of free control stability.

The mid-speed damping of the wobble mode can be increased by frame stiffening in the steering head region, by raising the rear frame mass centre and lowering that of the front frame, and it depends on the rider parameters assumed. Sufficient damping can easily be provided by a steering damper (at some cost), and as long as the rider holds on to the handlebars, he can to some extent reproduce the action of a steering damper when he needs to do so by a passive control stratagem. Much analytical effort has gone in to representing the free control wobble mode damping correctly, but it seems likely that, as with the car, the free control predictions can give a pessimistic view of a design. The danger in practice may well be that an unstable free control wobble mode will give no trouble until, with the rider not forewarned, a disturbance will provide an initial condition sufficiently large that the rider can not recover the situation, and a significant and possibly dangerous self excitation may become established. Experimental observations of the medium speed wobble should be conducted mindful of the possibility that excitation from an imperfect front wheel may contribute to the observed response.

Excessive system stability generally leads to lack of responsiveness. This is so because the stability information appears in the denominator of each frequency response function, and high modal damping is associated with large values of this denominator. Responsiveness is a subjectively recognised problem with single-track vehicles. To initiate a rapid manoeuvre, counter steering must be employed, and some space is "wasted" as far as the intended result is concerned, in setting up the appropriate lean angle. Thus it may be best to strive for adequate rather than maximal stability. Even in impulse response testing, spectral power in the steering torque input falls off rapidly beyond 6 Hz, so the capsize and weave mode properties are probably paramount in respect of rider control. Dual control by steer torque and body lean can be used, but for a given output, very much higher levels of body lean torque than of steer torque are required, and higher frequency body lean torques are difficult to produce (unless you happen to be a showgirl). Therefore it seems likely that, apart from very low speed balancing, steer torque is the dominant control input. Frequency response calculations show that there is no difficulty in exciting the full range of vehicle responses by means of this input.

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