# Bicycles made to measure 

For almost a century and a half, mathematicians have been racking their brains about the bike. How
can a rolling bicycle be so stable of its own accord? Delft researchers now say they have completed
the model to end all models. Bicycle manufacturer Batavus intends to use it to make better bikes for
the elderly and disabled.

Tomas van DiJk

The conveyor belt passes at speed under Ir. Jodi Kooijman and his bicycle. Kooijman, an enthusiastic off-road cyclist, pedals until his speedometer indicates sixteen kilometres per hour. On the sideline, Dr Ir. Arend Schwab of the Faculty of Mechanical Engineering, Maritime Technology, and Material Sciences (3ME), at the agreed upon moment, yanks a rope attached to the bike's luggage carrier. For a brief moment Kooijman veers to the right, but his bicycle regains its balance within a fraction of a second, appearing to automatically retrace 'the line'.
This video-recorded incident took place on a large conveyor belt at the department of motion sciences of Amsterdam Vrije University. The experiment is just one of those the two 3ME researchers have carried out in the past couple of years to test a mathematical model defining all the forces that act on a moving bicycle. A publication about this bicycle model recently appeared in the 'Proceedings of the Dutch Royal Society', the Royal Dutch Academy of Science.
Schwab shows another video recording. Here, Kooijman is giving a bicycle a hefty push. The bicycle is laden with measuring equipment and the carrier holds a laptop computer that records the bike's every movement. The unmanned bicycle rolls on following a straight line in the sports centre of Delft University. Kooijman runs after it and pushes the bicycle sideways. The bike wobbles a bit, the handlebars move from side to side, but the bike soon regains it's straight course.
"The bike's speed must be between fourteen and twenty seven kilometres per hour," Kooijman says. "At those speeds, the bicycle is inherently stable. If it goes faster, it will wobble less, but if you then push it sideways it will lean over to one side until it topples. The data match our model predictions exactly."

## Balance in motion

Ever since the invention of the pedal-driven bicycle around 1860, researchers have been trying to determine what makes a bike fairly stable of its own accord. They added
formula after formula, each one of them derived from the laws of motion as defined by Newton and Euler, but they never managed to develop a completely accurate model for predicting a bike's riding characteristics.
"Bicycle manufacturers never knew exactly how a bike works either," Schwab says. "They have always had to resort to experiments to improve their products. Not that there's anything wrong with that, but now they can use our model to feed into a computer all the factors affecting a bike's steering properties. The model then calculates how the bicycle will behave at different speeds."
Together with colleagues at Cornell University in the U.S. and at Nottingham University in the U.K. the Delft researchers perused more than fifty publications written by scientists on the subject since the early days of the

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 develop an accurate model that could be used to predict a bike's riding characteristicsbicycle. Many mathematicians claim that the bicycle mainly derives its stability from the fact that it takes effort to change the direction of a rotating mass, the gyroscopic effect.
"The gyroscopic effect certainly plays its part," Schwab says. To demonstrate this, he produces a wheel weighted with lead around the rim, and gives it a mighty jerk. Only with great difficulty can the wheel be made to change direction. "However, mathematicians who took this principle to heart were wrong," Schwab continues. "When we disregarded the weight of the wheels in our model we



Arend Schwab explains bicycle research using a toy bike.
discovered that it was still possible to make the bicycle stable. And there is no truth in the idea that bicycles with small wheels are unstable."

## Countersteering

We all know intuitively the main combination of forces that ensure we stay upright when riding a bicycle. They involve leaning over and steering and they explain why, when we wish to turn to the right, we have to first turn the front wheel slightly to the left. The action, known as counter steer, results in a force that causes the bicycle to
lean over to the other side, which is the direction in which we wanted to go. This also explains why we fall over if we pass too close to a kerb. We just can't manage to get away from it without hitting it.
As for the steering properties, the greater the angle at which the fork of the bicycle points forward, the more stable the bicycle will go in a straight line, but also the more difficult it will be to go round corners. "The distribution of mass is also very important," Kooijman says. "Moving the centre of gravity of a bicycle forward makes it more stable."

## 150 Years of bicycle research

The fact that a velocipede, as the (still front wheel-driven) bicycle was known in its infancy, turns corners by first steering in the wrong direction, was proved in 1869 by Scottish engineer and physicist William John Macquorn Rankine. The fact that countersteer is also used to maintain balance however, wasn't proved until in 1897, when French mathematician Emmanuel Carvallo published his 180-page, award-winning monograph on the dynamics of monocycles and bicycles.
The French scientist was also the first to realise that the amount of trail is extremely important for a stable bike ride. The trail is the distance
between the point where the steering axis intersects the ground and the point where the front wheel touches the ground. The trail causes the wheel to follow the direction of the vehicle. Vehicles with a large amount of trail, such as old-fashioned bicycles and Harley-Davidson motorcycles, give a more comfortable ride, but don't corner as easily as vehicles with less trail. "Unfortunately, Carvallo had disregarded the weight of the fork in his model," Schwab says. "As a result his model wasn't accurate." Around the turn of the century British mathematician Francis Whipple also published a model covering the bicycle's riding properties.
"He came very close," Schwab says, "but his equations have a few minor errors in them. Apparently, some of the plus and minus signs accidentally became transposed when his article went to press, so we cannot draw any firm conclusions from his work."

German researcher Ekkehard Döhring was the first to present a fully accurate model for the selfstabilising properties of a two-wheeled vehicle. He also experimented with a few motorcycles. In 1955 he finished his doctorate thesis on the subject, but he was never able to verify his model experimentally.

The Delft scientists included twenty five such parameters in their model. All of them are relate to the two connected motion equations, one for leaning over and one for steering. "It remains unclear how exactly all these parameters affect the stability," Schwab says. "In the final model these parameters appear in fairly complex forms as coefficients to the motion equations. For practical purposes most researchers used to simplify the equations by disregarding certain parameters, but the results tended to be far from ideal. And scientists who failed to make the connection between leaning and steering certainly were on the wrong track altogether."

## Thoroughbred

A model that indicates whether a design will result in a thoroughbred racing bike or in a stable ride suitable for the elderly, is something the bicycle industry has been eagerly awaiting. Rob van Regenmortel, product development manager of bicycle manufacturer Batavus, is following the Delft research effort with an eagle eye. Van Regenmortel: "Traditionally, when designing a bike, we use three parameters: the general geometry, the distance between the axles, and the angle at which the fork points downwards. Most of these properties were established back in the 1970s. Take the angle of the tube that carries the saddle. On our old-fashioned bikes this tube is mounted almost vertically. On bikes made by Gazelle on the other hand, it is inclined slightly more backwards. These are simple design choices all bicycle

## 'Bicycle manufacturers never knew exactly how a bike works either'

manufacturers made at one point and which they then more or less stuck to for the simple reason that their products kept selling. Now that we have Schwab's model, we hope to be able to start designing bicycles aimed directly at special target groups."
Van Regenmortel would like to collaborate with Schwab and Kooijman on a future project that will also look at the riding behaviour of the cyclist. The ultimate goal of the bicycle research effort is to include the cyclist's riding behaviour in the model so as to be able to investigate the combination of the bicycle and its rider. "We could then actually make a 'tailor-made' bicycle for everyone," Van Regenmortel says. "People who find it difficult to maintain their balance would no longer have to ride a tricycle."
Ultimately, the model is intended to improve customer communications. "Perhaps we could label bicycles
with numbers to give customers an idea of their riding properties. People looking for a bike to carry lots of luggage on holiday could then be recommended a type two bicycle, say, and someone needing transport to work and back might be wanting a slightly more thoroughbred machine, say type four. It's just an idea."
But how do you measure people's riding behaviour? On the conveyor belt in Amsterdam, Kooijman and Schwab have already collected some manned bicycle data through the simple expedient of riding the test bikes themselves. "Scary is the word," Kooijman says. "You're cycling at some speed inside an enclosed space without moving forward. It feels very weird. You're constantly afraid of hitting the wall. We can't ask elderly or disabled people to ride a bicycle that way to collect data. In future we will have to conduct our tests on the road, and then copy the cycling behaviour in a robot bicycle."

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Learning to ride a bike. The principle of leaning and steering is not easy to acquire, but once learnt it is never forgotten.

To make a lefthand turn

## you must briefly steer to the right

## BIKE CHARACTERISTICS 1

Handlebars want to return to straight position
The front fork of the bike is slanted in relation to the vertical (for example 18 degrees). Due to his slant (and the bend of the front fork), the Contact point of the front wheel with the road is located behind the steering axis. Consequently - if the handlebars are turned - the force in this
contact point is exerted in a corrective action on the steering axis, which wants to pull the handlebars back to a straight position. If a cyclist grips the handlebars loosely and cycles straight ahead, small imperfections in the position of the handlebars are self-corrected.


Bike model
A bike model is used to simulate the balance and steering behavior of a bike. This models uses 25 parameters to describe the bike four bodies: ear wheel, rear frame + cyclist, front frame and front wheel) and has speed. The result of the calculation model agrees precisely with the experiments. The model moreover clearly shows that no one parameter dominates the dynamic behavior of a bike. In this way for example the gyroscope effect of the wheels contributes to the stability. But also without this effect: that is, with mass-less wheels, the bike can still be
self-stabilizing self-stabilizing.

The handlebars are further turned to the left, while the bike leans more and more to the left. The bike turns to the left. After 2 seconds, the handlebars and leaning angle are at a maximum, 10 and 14 degrees respectively.

For every leaning angle, there's a corner
When a bike enters a corner, the cyclist leans to the inside. The force of friction from the ground on the tires is perpendicular to the speed. This center-seeking between the tendency to fall over under its own weight and the tendency to break out of the bend. The bike does not fall over as long as the torque from the centripetal force equals that of the force of gravity. The centripetal force depends on the speed of the bike and the diameter of the bend. This delicate balancing act sets a specific leaning angle for every turning circle at a given speed. leaning moment to the left, making the bike in the right turn want to fal to the left. This effect is directly related to the square of the speed and is virtually always greater than the effect of the shifting of the contact
point for speeds greater than $0.2 \mathrm{~m} / \mathrm{s}=0.7 \mathrm{~km} / \mathrm{h}$. As a consequence of the handlebars turning to the right, the bike thus falls/inclines to the left. The leaning angle then increases gradually

## he cycist needs only to momenta

 rily steer to the right, but the bike to the left (radius 16 m )Practically nobody is conscious of the fact that they must steer briefly to the left ion order to make a right-hand turn. But this is not so strange, because the swerve is very small (approximately $\mathbf{3}$ degrees) and happens very quickly $\mathbf{-} 0.5$ seconds. The wet tire tracks from cycling experiments reveal that we all do this. Apparently we learn this unconsciously when we learn to ride a bike. This so-called counter steering is however well known among motorcycle riders.



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