MEASURING AND MODELING

THE MECHANICAL PROPERTIES OF BICYCLE TIRES

by

Andrew Erwin Dressel

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ABSTRACT

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The University of Wisconsin-Milwaukee, 2013

Under the Supervision of Professor Adeeb Rahman

It has been shown that tire deformation can play an important role in the stability and handling of a bicycle. It is expected that an accurate understanding of tire behavior is necessary for correct understanding of rider behavior and that correct understanding of rider behavior is necessary for optimizing bicycle design. That certainly has been the case for motorcycles.

Several instances of published bicycle tire stiffness data exist, but they seldom agree with each other, do not all measure the same properties, and often are missing key pieces of test configuration data, such as tire size, rim width, inflation pressure, or vertical load.

In this project, three different test devices specific to bicycle tires were developed. Measured bicycle tire stiffnesses are presented for 14 different tires ranging from 22 to 50 mm wide, at inflation pressures from 2 to 11 bar (29 to 160 psi), under vertical loads from 304 to 731 Newtons (68 to 164 lb); for a total of about 120 different parameter combinations.

Normalized cornering stiffness was found to vary from below 0.15 to over 0.35, which is ±40% from the average, and normalized camber stiffness varies from below 0.0075 to over 0.015, which is ±33% from the average. Based on numerical simulations, this is more than sufficient to influence bicycle stability and handling.

Tires approach the camber stiffness necessary, without slip, for the net ground reaction force to be in the plane of the wheel, obeying the so-called tangent rule, but most tires with most inflation pressures and under most loads presented here fall below that.

A numerical model, based on an analysis developed by Rotta for slender toroidal tire cross sections in contact with the ground, was also developed to provide insight into how the tires generated the forces they do and attempt to predict them from simpler measurements.

Although actual values generated by the model do not exactly match measured values, the trends in contact patch size and lateral stiffness values generated do correspond well with measured data as parameters vary, such as inflation pressure, vertical load, and rim width.

DEDICATION

I dedicate this work to my wife, Anne Dressel, and my mom and dad, Ellen and Erwin

Dressel, for their <u>years</u> of unwavering support and encouragement.

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GLOSSARY

bead: the portion of the tire in contact with the rim.

bead seat diameter: twice the distance parallel to the midplane of the wheel from the center of its axle to where the tire bead contacts the rim. Governed by ISO standard 5775.

Calspan Corporation: a science and technology company located in Buffalo, NY that specializes in aerospace and transportation research and testing. According to their website, "Calspan is known the world over for its unrivaled tire test capability. Our machine can handle all types of tires, including truck, passenger, utility, race and even aircraft tires."[1] It was previously called the Cornell Aeronautical Laboratory.

camber angle: the angle between the midplane of the wheel and the vertical.

camber force: the portion of lateral force generated in the contact patch due to non-zero camber angle. Also known as camber thrust.

camber stiffness: the ratio of camber force over camber angle.

capsize mode: one of two fundamental modes predicted by the Whipple model and named by Sharp, in which the bike slowly leans and steers to one side until the side of the bike contacts the ground.

contact patch: the portion of the tire in contact with the pavement.

cornering force: the portion of lateral force generated in the contact patch due to non-zero slip angle.

cornering stiffness: the ratio of cornering force over slip angle.

DataStudio: the software that accompanies PASCO® instrumentation hardware.

decay length: the length along the circumference of the wheel over which a lateral displacement of the tire centerline decays to $1/e$ (~ 36.8%) of its original magnitude (Pacejka's relaxation length σ [2]).

FastBike: numerical simulation software designed for motorcycles. Its capabilities include modeling frame, fork, wheel, rider, and tire flexibility.[3]

JBike6: software to calculate the stability eigenvalues of the Whipple model for any geometry and mass distribution and at a range of forward speeds.[4]

LabVIEW: software by National Instruments for data acquisition.

lateral force: the force generated in the contact patch in the ground plane perpendicular to the intersection of the wheel midplane and the ground plane.

lateral stiffness: the ratio of lateral force over lateral displacement.

lean angle: the angle between the midplane of the bike and the vertical. Also known as roll angle.

MATLAB®: software by MathWorks for numerical computing.

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PASCO®: brand of instrumentation offering plug-and-play simplicity.

pneumatic trail: the distance by which the net lateral force trails behind the center of the
contact patch.

rim: the outer edge of the wheel that facilitates mounting a tire, characterized by two
parallel flanges to support tire beads.

self-aligning torque: the moment about the vertical axis through the contact patch
associated with non-zero slip angle. Tends to rotate wheel so that it points in the
direction in which it is rolling.

sliding: the movement of tire tread relative to the pavement with which it is in contact.
Requires the friction coefficient between the two materials to be less than infinite.

slip or side slip: the phenomenon by which a tire can roll in a direction other than the
direction in which it is pointed due solely to deformation even if friction is infinite
and no part of the tire tread slides relative to the pavement.

slip angle: the angle between wheel midplane and velocity vector (usually expressed in
degrees). At low angles, this is primarily due to tire deformation and occurs even
if friction is infinite. In practice, there may be a zone, starting at the rear of the
contact patch in which sliding between the tire tread and pavement may occur.
This sliding zone increases in size toward the front of the contact patch as slip
angle increases or friction decreases.

steer angle: the angle between the midplane of the bike and the midplane of the steered wheel.

steering axis: the axis about which the steered wheel rotates.

steering axis angle: the angle between the steering axis and the horizontal, in the case of bicycles, which have steering axis angles near 70º, or the vertical, in the case of motorcycles, which have steering axis angels near 25º.

relaxation rate: rate at which lateral deflection of tire centerline returns to zero.

relaxation distance: the distance long the pavement that a wheel rolls after a step change in slip angle before for the lateral force reaches $1 - 1/e$ (~ 63.2%) of its steady-state value (also Pacejka's relaxation length σ,[2] and Cossalter's L[5]).

relaxation length: either the decay length or the relaxation distance, depending on author and context.

rim width: the distance between parallel flanges of rim that contact the tire beads.

sinkage: the distance that an uncambered wheel moves vertically toward ground plane relative to when it is just touching ground plane and undeformed as it supports a load.

tire: the layer at the outer edge of a wheel that protects the wheel, provides suspension, and improves traction. Usually implemented by enclosing a compressed gas in a flexible torus.

tire arc radius: radius of circular arc formed by a radial cross section of the tire.

tire arc length: the arc length of a radial cross section of the tire, from bead to bead.

tire size: the manufacturer's stated size: approximately the diameter of a radial cross section.

trail: the distance by which the center of the contact patch of the steered wheel trails behind the intersection of the steering axis and the pavement.

TU Delft: Delft University of Technology, located in Delft, The Netherlands. Location of two, side-by-side 2.5 meter tire testing drums and home institution of Prof. Hans Pacejka, author of *Tire and Vehicle Dynamics* and the so-called "Magic Formula" empirical tire model.

twisting torque: the moment about a vertical axis through the contact patch associated with non-zero camber angle. Tends to rotate the wheel toward the direction in which it is cambered.

UWM: University of Wisconsin-Milwaukee. Location of test devices 1 and 2.

weave mode: one of the two fundamental modes predicted by the Whipple model and named by Sharp in which the bike alternately steers and then rolls from side to side.

wheel: a disk that enables a vehicle to roll over ground surface.

Whipple model of a bicycle: simplified model that consists of just four rigid bodies

      connected by frictionless revolute joints: rear frame, front frame, rear wheel, and

      front wheel. The wheels have knife edges that roll without slip.[6]

wobble mode: a fundamental mode of bikes that is not predicted by the Whipple model

      and is characterized by a relatively high frequency oscillation of the steering

      assembly about the steering axis. This mode, also known as shimmy, is only

      predicted by bike models that have some type of compliance: in the frame, in the

      tires, or between a rider and the frame.[7][8]

LIST OF SYMBOLS

x For entire tire: coordinate axis forward from the contact patch at intersection of ground plane and wheel plane. For just tire cross section: coordinate axis to right in plane of section. See **Figure 2-1**.

y For entire tire: coordinate axis to the right from the contact patch, perpendicular to x and in ground plane. For just tire cross section: coordinate axis normal to plane of section and into page. See **Figure 2-1**.

z For entire tire: coordinate axis up from the contact patch, perpendicular to both x and y axes. For just tire cross section: coordinate axis straight up and in plane of section. See **Figure 2-1**.

v velocity vector, direction in which the center of the wheel is moving. See **Figure 2-1**.

α slip angle: the angle between wheel x-axis and velocity vector v (usually expressed in degrees). See **Figure 2-1**.

γ camber angle: the angle between the vertical and the wheel midplane (Rotta's σ) (usually expressed in degrees). See **Figure 2-1**.

F_x rolling resistance force. See **Figure 2-1**.

F_y cornering force if due to non-zero slip angle, or "camber thrust" if due to non-zero camber angle. See **Figure 2-1**.

M_α self-aligning torque: moment about vertical axis through the contact patch associated with non-zero slip angle α. See **Figure 2-1**.

M_γ twisting torque: moment about vertical axis through the contact patch associated with non-zero camber angle γ. See **Figure 2-1**.

M_z total torque about z-axis generated in contact patch: sum of self-aligning torque and twisting toque. See **Figure 2-1**.

C_{Fy} lateral stiffness: ratio of lateral force F_y over lateral displacement δ_h.

$C_{F\alpha}$ cornering stiffness: ratio of lateral force F_y over slip angle α.

$C_{F\gamma}$ camber stiffness: ratio of lateral force F_y over camber angle γ.

w rim width (usually expressed in mm). See **Figure 2-2**.

r tire arc radius. Subscripts 0, 1, and 2 represent entire tire, left side bulge, and right side bulge, respectively (usually expressed in mm). See **Figure 2-2**.

s tire carcass arc length: cross section arc length. Subscripts 0, 1, and 2 represent entire tire, left side bulge, and right side bulge, respectively (usually expressed in mm). See **Figure 2-3**.

t thickness of the tire carcass. See **Figure 2-3**.

θ angle swept by tire arc. Subscripts 0, 1, and 2 represent entire tire, left side bulge, and right side bulge, respectively ($s = r\,\theta$) (usually expressed in radians).

φ angle of tire cross section from vertical (bottom) cross section (usually expressed in radians).

h height of rim: vertical distance between the lowest point on the undeflected tire to the lowest point on the uncambered rim (usually expressed in mm). See **Figure 5-2**.

R Wheel radius, height of axle above undeflected tire (usually expressed in mm).

R_e Effective wheel radius, height of axle above ground plane (usually expressed in mm).

δ displacement of contact patch with respect to rim, or rim with respect to contact patch, depending on context. Subscripts w, v, h, represent in the plane of the wheel, vertically with respect to the ground, and horizontally with respect to the ground, respectively. $\delta_v = R - R_e$.

e horizontal distance to the edge of the contact patch from the center of the tire. Subscripts 1 and 2 represent left side, and right side, respectively.

p inflation pressure.

dy distance between tire cross sections.

dS contribution to horizontal force from one cross section.

dN contribution to vertical force, normal to ground plane, from one cross section.

μ coefficient of friction between the tire and the pavement.

i relaxation rate at which deflection of tire centerline returns to zero.

ℓ decay length (Pacejka's relaxation length σ).

κ relaxation distance (also Pacejka's relaxation length σ, and Cossalter's L).

E modulus of elasticity of the tire carcass.

I area moment of inertia of the tire carcass.

T circumferential tension in the tire carcass.

k foundation stiffness rate on a beam.

f force rate on a beam.

u lateral displacement of a beam.

ACKNOWLEDGMENTS

EPIGRAPH

"If we knew what it was we were doing,

it would not be called research, would it?"

-- Albert Einstein[9]

CHAPTER 1

INTRODUCTION

1.1   Bicycle dynamics

1.1.1   <u>The Whipple Model</u>

The Whipple model of a bicycle is simplified and idealized to facilitate the derivation of

equations of motion without losing key dynamic characteristics, such as self-stability. It

consists of four rigid bodies, two wheels, a rear frame, and a front fork. These are

connected by three frictionless revolute joints, a steering axis between the frame and fork

and two wheel axles. The wheels have knife edges and roll without slip on a flat

surface.[6]
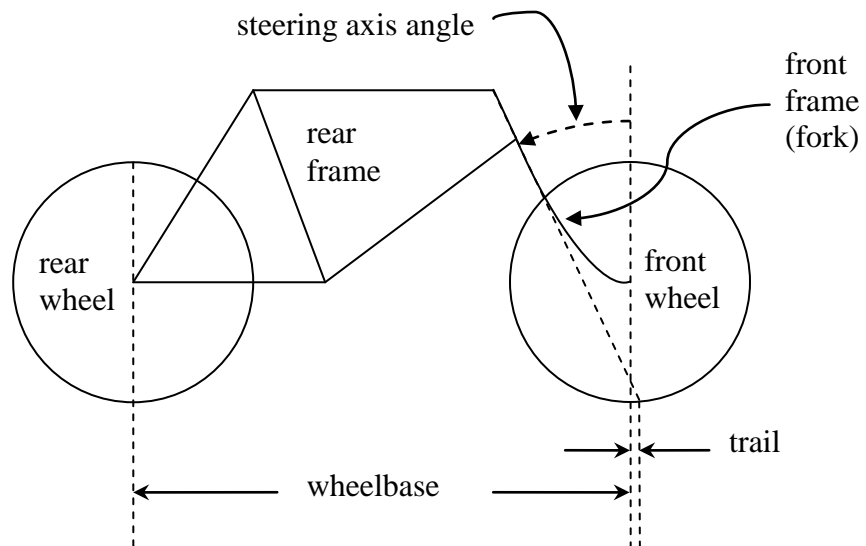
Figure 1-1: Schematic of the Whipple Model bicycle.

The linearized model can be characterized by as few as 17 parameters specifying geometry and mass distribution. The linearized equations have just four state variables, lean angle, steer angle, and their rates, and so can be used to generate four stability eigenvalues, [6] as shown in the right half of **Figure 1-2**.

A bike is self stable at any forward speed at which all its real eigenvalues and the real components of its complex eigenvalues are negative. The bike whose eigenvalues are shown in the right half of **Figure 1-2** is self-stable at forward speeds between about 4.5 and 6.0 m/s.
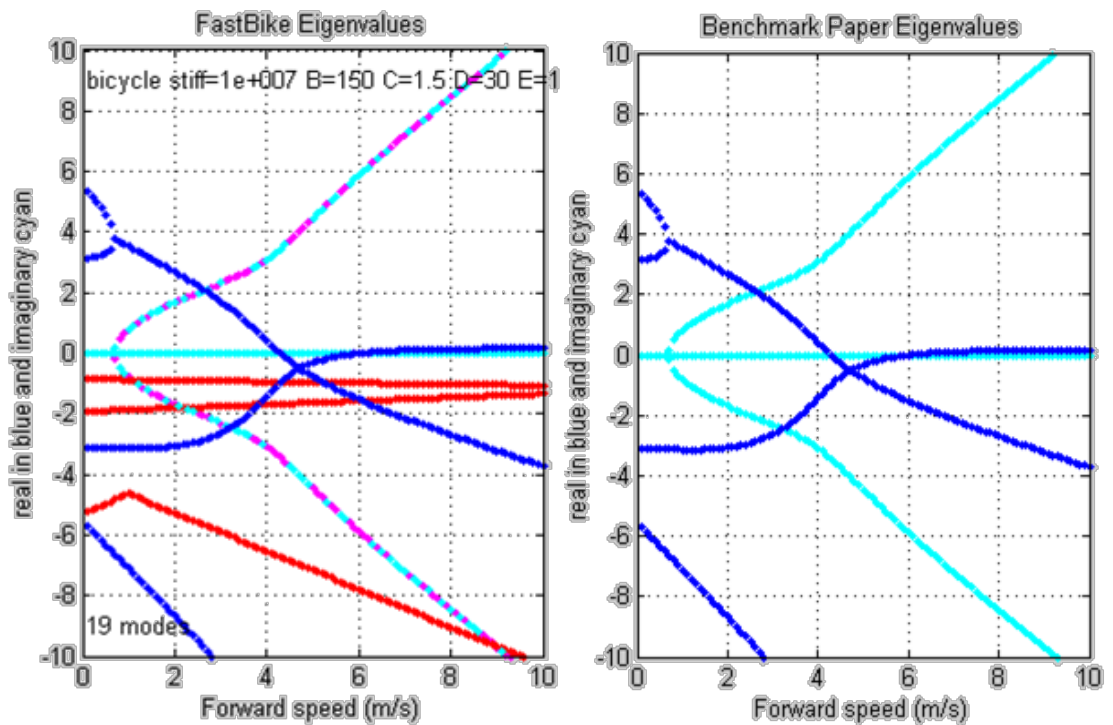
Figure 1-2: Stability eigenvalues for benchmark bicycle from FastBike motorcycle simulation[3] and from JBike6[4] bicycle simulation.

1.2   The role of tires in bicycle dynamics

It has been shown that the Whipple model of a bicycle, with knife-edge wheels that roll without slip, accurately predicts behavior under certain low-speed and controlled circumstances,[10] however it has also been shown that tire deformation can play an important role in more complicated or higher-speed behavior, such as wobble, a well-documented mode not predicted by the Whipple model.[8] Tires are also understood to play a large role in motorcycle dynamics.[11]

When Robin Sharp, author of the landmark 1971 paper, "The Stability and Control of Motorcycles", in which he coins "wobble" and "weave" for two of the fundamental oscillatory modes of bikes,[12] turned his attention to the control of bicycles, he selected bicycle tire data published by Roland.[13] Sharp used an average of the reported cornering stiffness, rejected the reported camber stiffness for being unreasonable, and had to estimate the missing torque stiffness about the vertical axis, M_z. Never-the-less, he reports that "one of the low-speed modes is much altered" and "oscillatory behavior of the bicycle may be predicted badly by the [Whipple model] and rather better by including [tire] slip, as is well known in the case of motorcycles."[7]

It is also expected that an accurate understanding of tire behavior is necessary for correct understanding of rider behavior, and that a correct understanding of rider behavior is necessary for optimizing bicycle design. That certainly is the case for motorcycles.

1.2.1   Previous physical testing

In 2006, Kooijman, *et al.*, showed that the Whipple model does accurately predict weave frequencies for an uncontrolled and very light bicycle at speeds below 6 kph.[10] At 19.2 kg on tires inflated to 3.5 bar, and assuming an even weight distribution, the test bicycle would have a contact patch area under each tire of about 270 mm^2. The measured contact patches presented in Appendix B have a ratio of length to area of about 0.076, so a contact patch of 270 mm^2 would be about 20 mm long, or just under 20% of the average contact patch measured. That is about equivalent to a 32 kg (70 lb) rider on a 6 kg (14 lb) bike with tires inflated to 7 bar (100 psi), which is not a likely combination in the real world. Most real tires support a larger load and so have larger contact patches. Thus the influence of tires was minimized. A more-likely 68 kg (150 lb) rider on a 9 kg (20 lb) bike at the same inflation pressure would have contact patches just about twice that size.
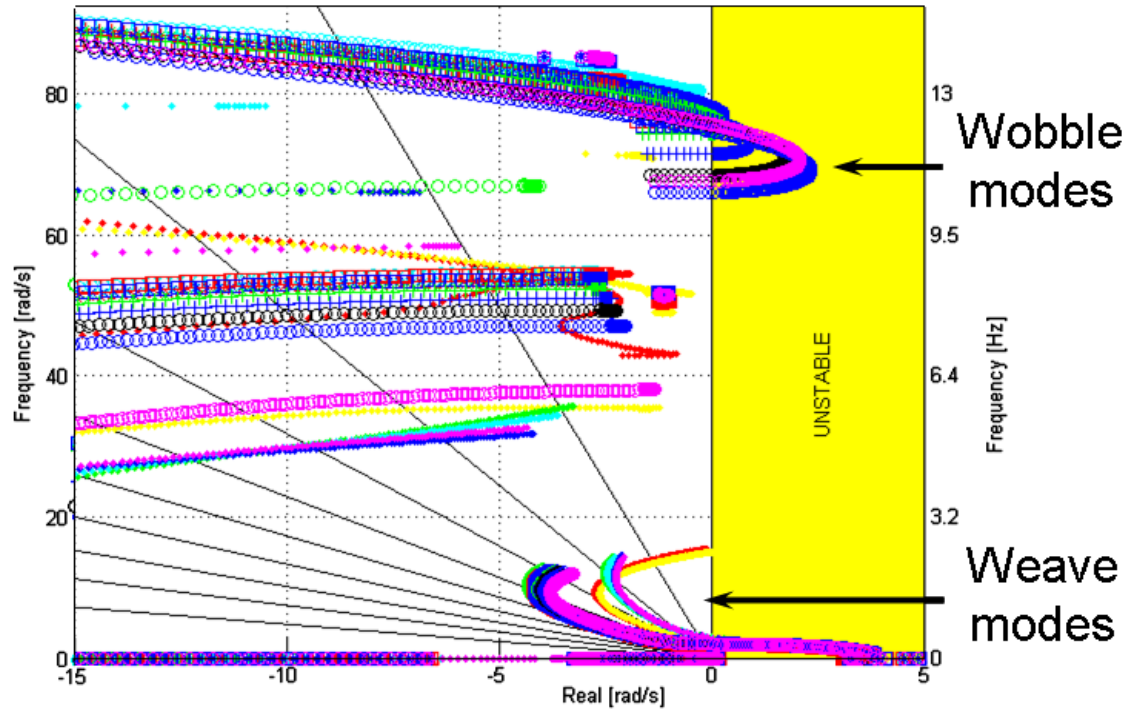
Figure 1-3: Root-locus plot from FastBike[3] showing wobble mode becoming

unstable as tire cornering stiffness varies.

In 2012, Jason Moore, in his National Science Foundation (NSF)-funded Ph.D.

dissertation, reports that in his attempts at bicycle and rider system identification, "the

deficiencies are most likely due to un-modeled effects, with the knife-edge, no side-slip

wheel contact assumptions being the most probable candidate." He also notes that "as

pointed out by many, in particular the motorcycle researchers, there is very good reason

to question [the] assumption [of] knife-edge no side-slip wheels, especially when under a

rider's weight. Specifically, the Whipple model does not predict correct steer torque

magnitudes required for a given steer angle and roll angle."[14]

1.2.2   Previous numerical simulation

Simulations with FastBike,[3] using a bicycle model validated against a benchmark bicycle,[15] predict that small changes in tire stiffness can cause the wobble mode to transition from stable to unstable, as shown in **Figure 1-3**. The bicycle tire stiffnesses used were developed from existing motorcycle tire stiffness provided with the software and augmented with the few bicycle tire stiffness numbers published by Cossalter.[5]

1.3   Problem Statement

Although some bicycle tires have been previously tested, the measured stiffnesses vary by more than two orders of magnitude in some cases, and many test parameters, such as tire size, inflation pressure, or vertical load, are not included with the published results. This makes it difficult to select stiffness values for modeling bicycle stability and handling and to validate the results of that modeling by testing a physical proximity of the model.

1.4   Objectives

The objectives of this project were to determine bicycle tire stiffness values that can be used to improve the fidelity of bicycle numerical modeling. To that end,

- the literature was surveyed to find all available previously existing bicycle tire test data;

- additional tires were tested to broaden the range of tested tires and to provide values for comparison between similar tires;

- all stiffnesses were compared to identify outliers and improve the likelihood that stiffness values used in future modeling represent reality; and

- a model was developed to gain insight into how a tire generates the forces and moments it does.

1.4.1 <u>Additional contributions</u>

In addition to achieving the objectives stated above, it will be shown that:

- The so-called tangent rule, based on the radial brush model, does not accurately predict bicycle tire camber stiffness.

- Bicycle tires exhibit a long decay length beyond the ends of the contact patch as the laterally deflected tire centerline returns to equilibrium and this needs to be considered for accurate modeling.

- The lateral stiffness of bicycle tires depends upon the width of the rim on which they are mounted.

- Bicycle tires exhibit rate-independent and rate-dependent hysteresis, which may contribute to rolling resistance.

1.4.2 <u>Scope</u>

This project is limited to measuring the lateral stiffnesses, and related values, of pneumatic bicycle tires with smooth or semi-smooth tread intended for on-road use. Items and values that were not measured include tires with knobby treads, solid tires,

motorcycle tires, rolling resistance, friction between tire and pavement, loose, soft, or bumpy ground surfaces, operating temperature, or the effects of tread wear.

1.4.3 <u>Setting</u>

Bicycle tire testing was performed both at the University of Wisconsin-Milwaukee (UWM), with devices number 1 and 2, described in sections 3.7 and 3.8; and at Delft University of Technology (TU Delft) with device number 3, described in section 3.9. The testing began in April 2010 and continued through April 2013.

Test device construction occurred at UWM, TU Delft, and at the Milwaukee School of Engineering (MSOE). It began in March 2010 on test device number 1, and modifications for specialized tests continued on test device number 2 through April 2013.

CHAPTER 2

BACKGROUND

2.1   Tire coordinate system

The coordinate system used for a tire is described by Pacejka[2] and Cossalter[5] and has

its origin at the intersection of three planes: the wheel midplane, the ground plane, and a

vertical plane through the hub. The x-axis is in the ground plane and oriented forward, in

the direction of travel; the y-axis is also in the ground plane and rotated 90º clockwise

from the x-axis when viewed from above; and the z-axis is normal to the ground plane

and downward from the origin.
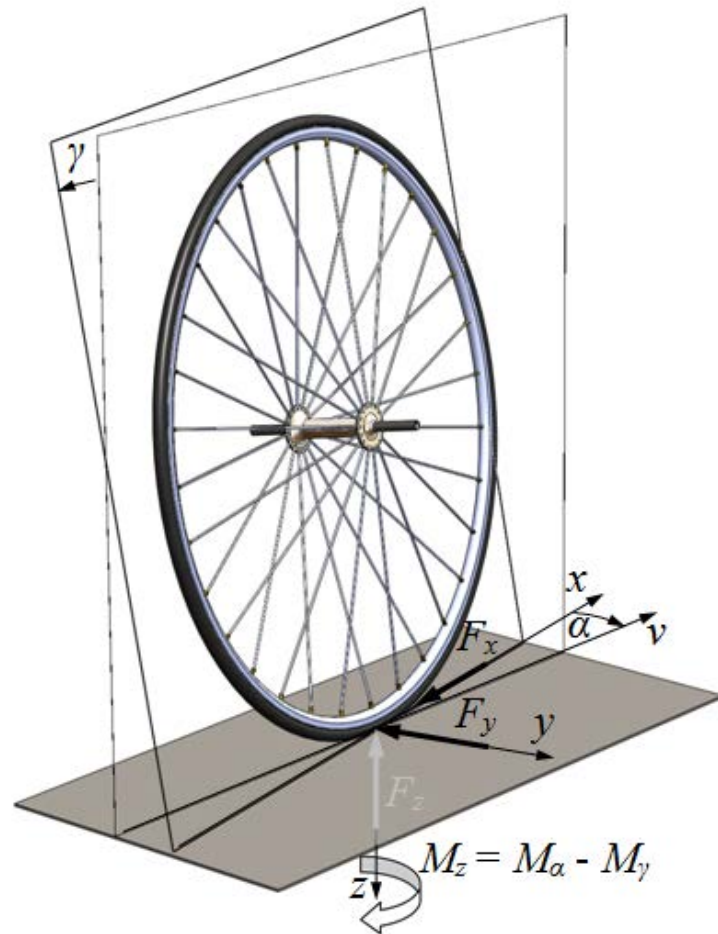
Figure 2-1: Tire coordinate system.

2.2   Tire parameters

A bicycle tire can be characterized by its bead seat diameter (ISO 5775); carcass arc length from bead to bead (s_0 in **Figure 2-3**); ply orientation, either bias or radial; and tread thickness (t in **Figure 2-3**)) and composition (smooth, semi smooth, knobby), flat prevention layer, and tread compound. It has have also been found that the width of the rim onto which a tire is mounted and the actual width of the tire on that rim are both

important parameters. The latter can be calculated from carcass arc length and rim width, *w*, as shown in **Figure 2-2**.
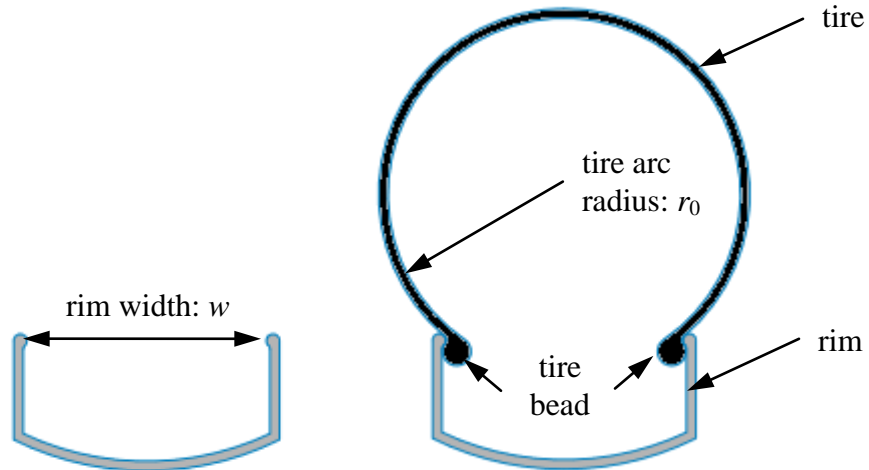
Figure 2-2: Radial cross section of rim and rim with bicycle tire.

Actual rim profiles used for testing are shown in **Table 3-3**.

The carcass circumference, s_0, or arc length was measured by pressing a portion of the tire onto a flat surface and then measuring the distance between the two beads, as shown in **Figure 2-3**.
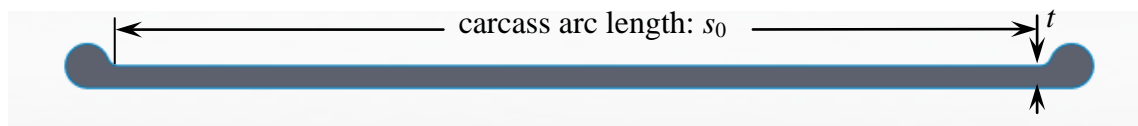
Figure 2-3: Flattened carcass, with outer tread up and inside down, showing arc length s_0 and thickness *t* of a bicycle tire

2.3   Tire mechanical properties

The mechanical properties of a tire that have been found to affect vehicle dynamics include:

i)   lateral stiffness: Pacejka's C_{Fy} on page 234 of 2006 edition [2]

ii)   cornering or lateral slip stiffness: Pacejka's $C_{F\alpha}$ on page 12 of 2006 edition [2]

iii)   camber stiffness: Pacejka's $C_{F\gamma}$ on page 12 of 2006 edition [2]

v)   torques and moments: Pacejka's M_z[2] and the sum of Cossalter's Twisting and Self-aligning torques[5]

vi)   pneumatic trail, the distance between the net horizontal ground reaction force and origin of the tire coordinate system. Pacejka's t, page 5 of 2006 edition [2] and Cossalter's a_t on page 59 of second edition [5].

iv)   relaxation lengths: Pacejka's σ, page 222 of 2006 edition[2] also Sharp[7] and Uil[16]. Separated here for clarity into decay length, ℓ, the distance around the circumference of the wheel over which a lateral displacement decays, and relaxation distance, κ, the distance along the ground required for lateral force to build up in response to a step change in orientation.

In order better to correct for variations in the data caused by the testing devices and to help construct and validate the model, these additional parameters were also measured:

i)   Contact patch size and shape, both on a 2.5 meter drum and a flat plate, as in **Figure 4-1** and **Figure 4-2**

ii)   Hysteresis, both rate-independent and rate-dependent.

(1)   Force due to deformation depending on previous state of deformation, as shown in **Figure 3-12**. This was specifically confirmed with an Instron® Universal Test Instrument, as shown in **Figure 3-28**.

(2)   Force due to deformation depending on speed of deformation: lateral stiffness at various deformation rates, as in **Figure 3-29**.

iii)   Lateral deflection of the tire center line due to a static point load, and as the tire rolls forward under various configurations of vertical load, inflation pressure, slip angle, and camber angle.

2.4   Camber stiffness and the "tangent rule"

For a bicycle tire rolling forward with non-zero camber angle and zero slip angle, the conventional multi-spoke or "brush" tire model, based on a continuous distribution of linear springs, predicts that the net ground reaction force should be in the plane of the wheel, without the need for supplementary cornering force due to a non-zero slip angle.[17] Since the camber force then equals the vertical load times the tangent of the camber angle, Sharp calls this the "tangent rule,"[7] and Sakai *et al.* call this "neutral" camber thrust.[18] There are a variety of different ways to express this condition mathematically.
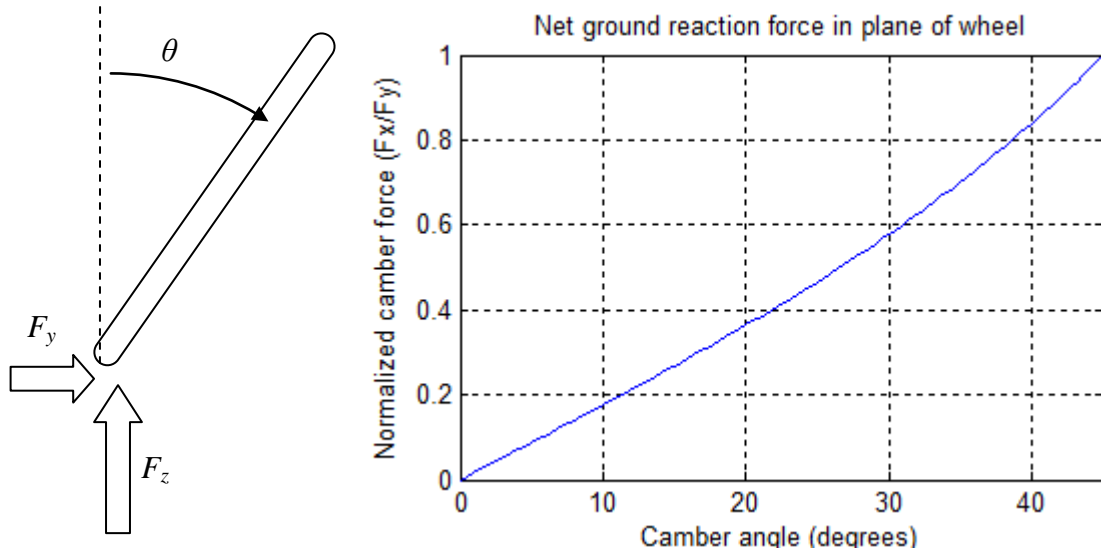
Figure 2-4: A wheel in camber and net ground reaction force orientation.

First, relate forces and angle (in degrees) such that the net ground reaction force is in the plane of a cambered wheel (plotted to right in **Figure 2-4** above):

$$\frac{F_y}{F_z} = \tan\left(\theta\frac{\pi}{180}\right) \Rightarrow \tan^{-1}\left(\frac{F_y}{F_z}\right)\frac{180}{\pi} = \theta \qquad \textbf{Eq. 2-1}$$

Then define camber stiffness and normalized camber stiffness:

$$C_{F\gamma} = \frac{F_y}{\theta} \text{ and } \overline{C_{F\gamma}} = \frac{C_{F\gamma}}{F_z} = \frac{F_y}{F_z\theta} \Rightarrow \theta = \frac{F_y}{F_z C_{F\gamma}} \text{ or}$$

$$\qquad \qquad \textbf{Eq. 2-2}$$

$$\theta\overline{C_{F\gamma}} = \frac{F_y}{F_z}$$

Combine the two expressions above, in **Eq. 2-2**, to eliminate the angle, isolate the inverse tangent and take tangent of both sides:

$$\tan^{-1}\left(\frac{F_y}{F_z}\right)\frac{180}{\pi} = \frac{F_y}{F_z\overline{C_{F\gamma}}} \implies \frac{F_y}{F_z} = \tan\left(\frac{F_y}{F_z\overline{C_{F\gamma}}}\frac{\pi}{180}\right) \qquad \textbf{Eq. 2-3}$$

Plug this back into expressions for normalized camber stiffness from **Eq. 2-2** above:

$$\theta\overline{C_{F\gamma}} = \tan\left(\frac{F_y}{F_z}\frac{F_z\theta}{F_y}\frac{\pi}{180}\right) = \tan\left(\theta\frac{\pi}{180}\right) \qquad \textbf{Eq. 2-4}$$

Thus, the "tangent rule" can be expressed as

$$\overline{C_{F\gamma}} = \frac{1}{\theta}\tan\left(\theta\frac{\pi}{180}\right) \qquad \textbf{Eq. 2-5}$$

Since $\tan(\theta) = \theta$ for small angles in radians, **Eq. 2-5** above can be approximated by

$$\overline{C_{F\gamma}} = \tan\left(\frac{\pi}{180}\right) = 0.017455064928218 \qquad \textbf{Eq. 2-6}$$

This approximation, plotted below, is within about 1% of the true value, also plotted

below, in **Figure 2-5**, for angles up to 10° and is about 21% too low at 45°.
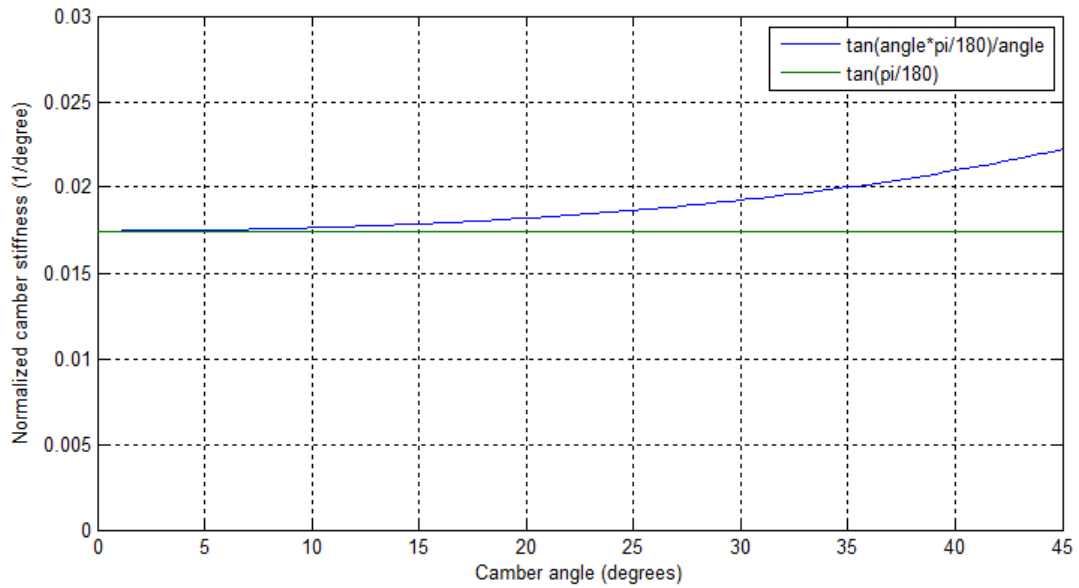
Figure 2-5: Plot of true value vs. approximation of the normalized camber stiffness

that produces a net ground reaction force parallel to the plane of the wheel

2.5   Previous bicycle tire measurements

Several instances of published bicycle tire force data exist, and they are summarized

below in **Figure 2-6**.
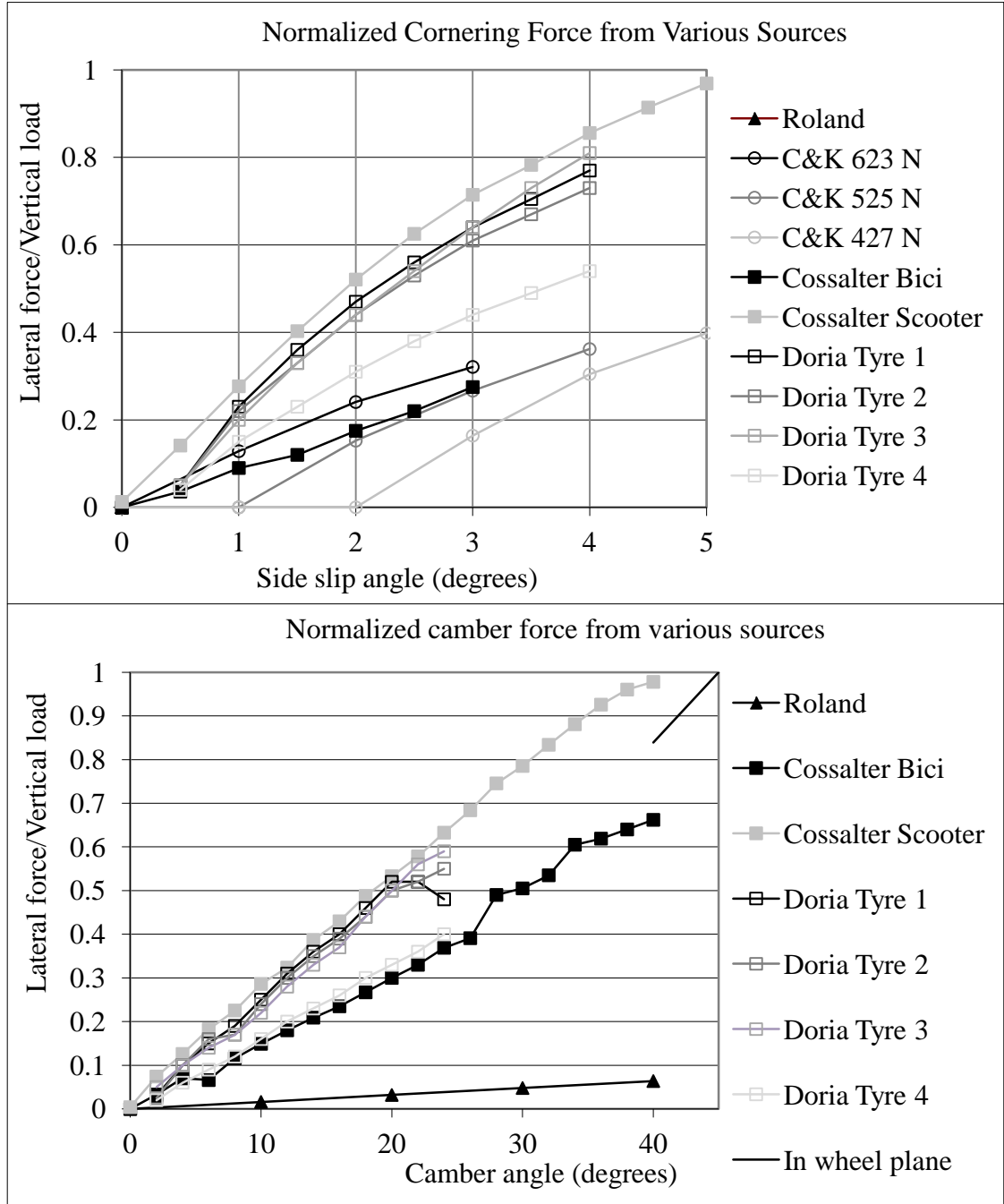
Figure 2-6: Normalized cornering and camber forces from previous bicycle tire

measurements.

The sets of existing tire force and moment data seldom agree with each other, do not all measure the same values, and often are missing key pieces of test configuration data, such as tire size, inflation pressure, or vertical load.

2.5.1   Roland, 1972-1973

Roland reports that "a thorough review of the published bicycle research literature revealed no investigation of the side force characteristics of bicycle tires." He measured bicycle tires at Calspan Corporation with a single-wheel trailer towed behind an automobile.[19][13] Eleven different tires were tested, from 24 to 27 inches in diameter with rated inflation pressures from 55 to 110 psi. He reports a range of cornering stiffness from 0.15 to 0.35 "lb/lb/deg" and a camber stiffness of 0 to 0.1 "lb/lb/deg". The 1972 report contains detailed tire data, and the 1973 report contains sample data for one tire: "normalized side force vs. slip angle and inclination angle for a typical bicycle tire at a vertical load of 75 pounds." This is shown below in **Figure 2-7**.
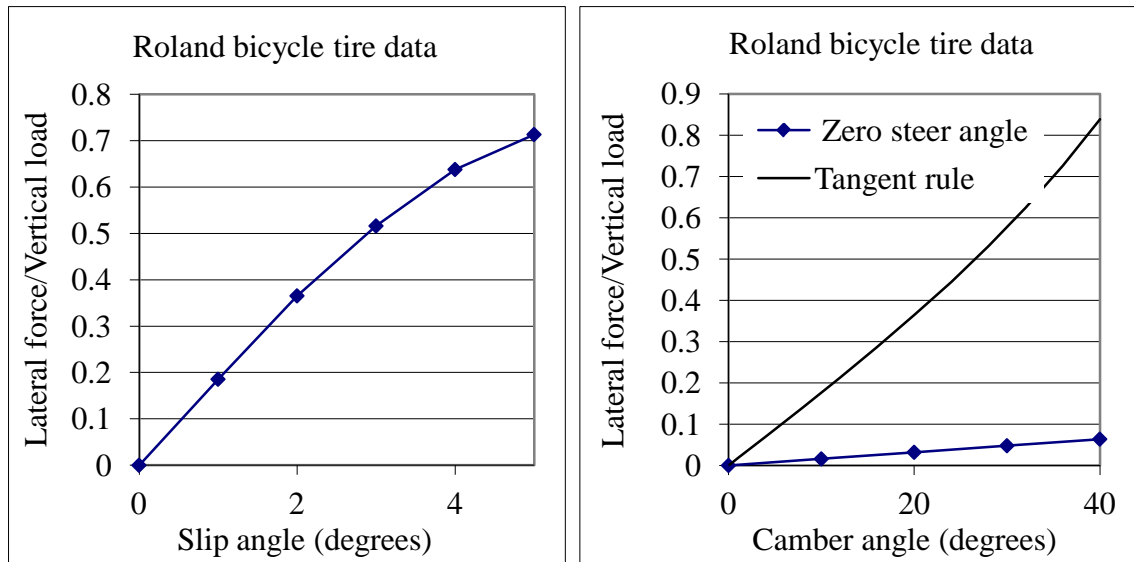
Figure 2-7: Roland, 1973, "typical bicycle tire at a vertical load of 75 pounds".

The size of the particular tire, its inflation pressure, and the forward speed of the testing are not provided. [13]

2.5.2 Davis, 1975

Davis continues the Calspan bicycle tire testing, but on a 4-foot (1.2192 meter) diameter drum, and focuses on wet pavement and low inflation pressures. He reports values similar to Roland under dry conditions for the Schwinn Puff Road Racer 27x1-1/4 (ETRTO 32-630) at 75 psi (5.171 bar) under a vertical load of 100 lb (444.822 N): a cornering stiffness of 0.17 1/° at 1° slip angle, and a camber force of 2 lb (8.896 N) at a 10° camber angle, which converts to a camber stiffness of 0.002 1/°.[20]

Davis also shows an unusual curve for camber force as a function of camber angle. By the time camber angle has increased to 20 degrees, the lateral force is 16 lb (75.62 N),

which increases the normalized camber stiffness to 0.008 1/°, about 48% of the tangent

rule, although by 20 degrees, the tangent rule requires a stiffness of 0.0182 1/°, as

explained in section 2.4.

2.5.3   Man and Kane, 1979

Guy Man and Thomas Kane, at the Mechanical Engineering Department of Stanford

University use values of 12.949 1/rad and 0.1862 1/rad for bicycle tire cornering and

camber stiffness, respectively.[21] That is a cornering stiffness of 0.2260 1/°, which is

similar to other published values, and a camber stiffness of 0.003250 1/°, which is about

twice Roland's value, and Roland's 1973 paper is cited in the bibliography, but only

18.4% of the value predicted by the tangent rule.

Basu-Mandal *et al.* suggest that their surprising results, "that turn radius at a given speed

is independent of the steer angle … may be due to details of tyre slip modelling."[22]

2.5.4   Kyle, 1987, 1988, 1995

Chester R. Kyle, Ph.D., Adjunct Professor of Mechanical Engineering at California State

University, Long Beach, tested a variety of tires, mostly for rolling resistance, at General

Motors facilities in the late 1980s and 1990s. It is not clear that he ever published his

results in a journal or conference proceedings, but Jason Moore at UC Davis has posted

four PDF documents and 3 XLS spreadsheets online at

biosport.ucdavis.edu/blog/bicycle-tire-data.

In a 1987 document titled "General Motors, Detroit 4/13/87", Kyle presents data for a

"small" tire, with an outside diameter of 0.44 m (≈ 20 in), possibly a Moulton on a 1.7 m

drum at 3.5 kph. The data is described as "corrected to flat surface", but without further explanation. The stiffness data is summarized below in **Table 2-1**.[23]

Table 2-1: Kyle, 1987, Small tire

Normalized Lateral Force @ 1 deg slip					
			Load N		
			300	450	600
Press.		75	0.213	0.179	0.157
		100	0.244	0.216	0.183
(psi)		113	0.201	0.173	0.156

Normalized Aligning Torque @ 1 deg slip (mm)					
			Load N		
			300	450	600
Press.		75	2.37	2.78	2.83
		100	2.23	2.4	2.53
(psi)		113	2	2.2	2.33

Normalized Camber @ 1 deg camber				
		Load N		
		300	450	600
Press. (psi)	100	0.0166	0.0156	0.0142

In a 1988 document titled "Society of Automotive Engineers, Warrendale, PA", Kyle

reports on a tire rolling resistance for "the race across Australia." The report includes

some stiffness values for a "17 in x 1¼ in" Moulton tire, which is summarized below in

Table 2-2.[24]

Table 2-2: Kyle, 1988, Moulton 17x1¼

Normalized Cornering @ 1 deg slip				
		Load N		
		300	450	600
Press. (psi)	113	0.202	0.175	0.161

He also reports some observations on how stiffness varies with vertical load and inflation

pressure. In particular "cornering force on a rough stable surface will increase with higher

tire loads… However higher tire pressure can have varying results … Cornering force

can reach a maximum and decline with increasing pressure … Slick treads have a higher

cornering force than patterned treads on dry roads … The effect of tire width on

cornering force is unclear."

In a 1995 document titled "GM Flat Track Rolling Road Test", Kyle presents raw data in

tables for a variety of tires. There is also a spreadsheet with a graph labeled "Lateral

Force Vs. Steering Angle 700C Continental, 180 psi, 3.5 mph". Then some of the data in

the document is incomplete, because the pages were misaligned in the scanner, and data

in the document and the spreadsheet are questionable, because of the surprisingly high

lateral forces reported: up to 2.5 times the vertical load. For example, the last line of page

26 reads: camber angle (deg) = 9.03, slip angle (deg) = 0, radial load (N) = -350, and lateral force (N) = 859. This agrees approximately with the spreadsheet.

Two documents from 1996 and two other spreadsheets appear only to contain rolling resistance data.[25][26]

2.5.5   Cole and Khoo, 2001

Cole and Khoo used a back-to-back tire test device to measure cornering stiffness.[27] They tested 57-406 "20'' diameter and 2.125'' tyre width" at 240 kN/m^2 and under 132-623 N vertical load. A summary of their results, both as presented and normalized, is provided below in **Figure 2-8**. They explain that "the graph is in the form of a carpet plot, where the origins of the six curves are spaced at one degree intervals along the horizontal axis."
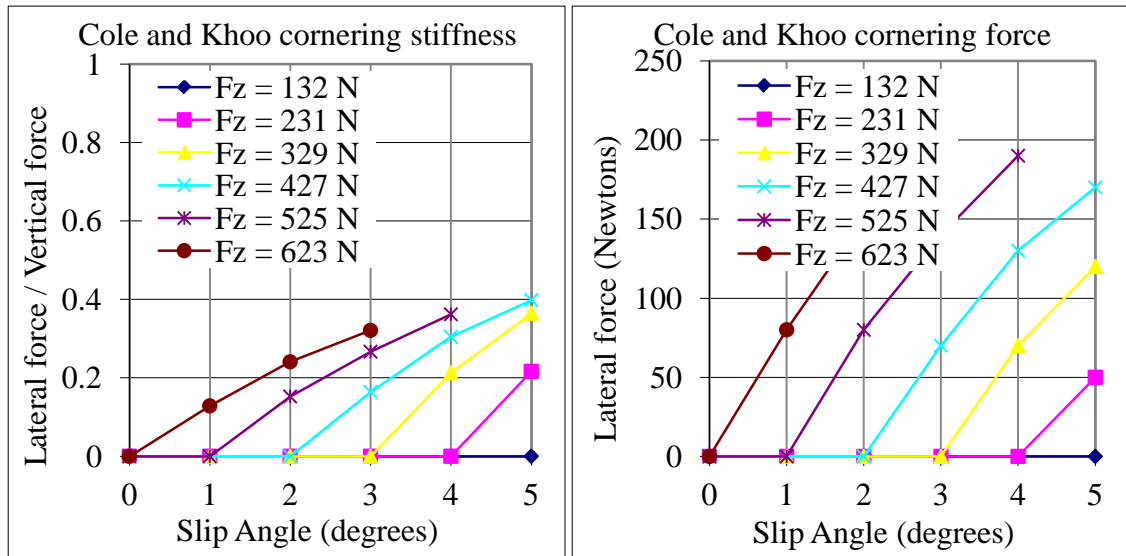
Figure 2-8: Cole and Khoo "bicycle tyres, size 20" diameter and 2.125" tyre width".

The forward speed is not specified, and the camber angle "was set to zero for all the tests." It also appears that for all but the largest vertical load, the lateral force was at or below zero at non-zero slip angles. [27]

2.5.6   Cossalter, 2006

Cossalter measured at least one bicycle tire on his rotating disk test device, [5] designed for motorcycle tires, at the University of Padua.[11] The data he published is presented below in **Figure 2-9**.
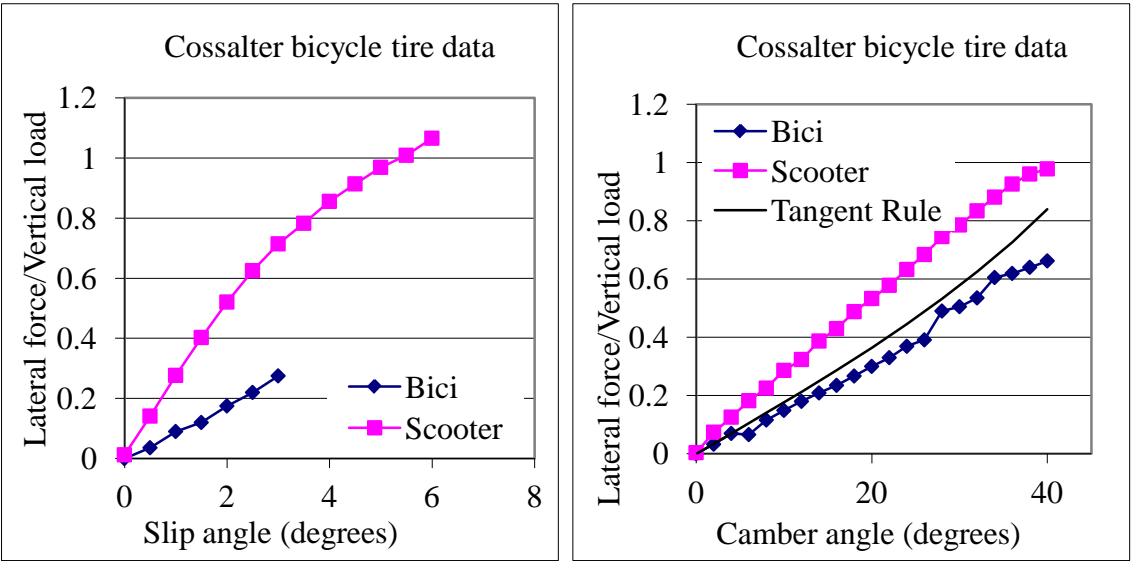
Figure 2-9: Cossalter bicycle tire data

He does not indicate which tire it was, the rim it was on, the inflation pressure, the vertical load, nor the forward speed at which it was tested. [5]

2.5.7   Sharp, 2008

Sharp uses "the side-forces developed by a number of different types of bicycle tires"
measured by Roland and estimates the aligning moments.[7] Sharp uses a value 0.25 for
normalized cornering stiffness. It is less clear what exactly he uses for camber stiffness.
First he observes that "Roland's camber stiffnesses vary from zero to unreasonably
large." Then he invokes the "tangent rule". Finally he states that "C_φ is taken to be equal
to the tire load," where $C_\varphi \varphi$ is a term in his expression for F_y.

2.5.8   Doria *et al.*, 2012

Doria *et al.* measure properties of four bicycle tires, three 37-622 and one 35-622, all on
the rotating disk test device designed for motorcycle and scooter tires at Padua.[28]
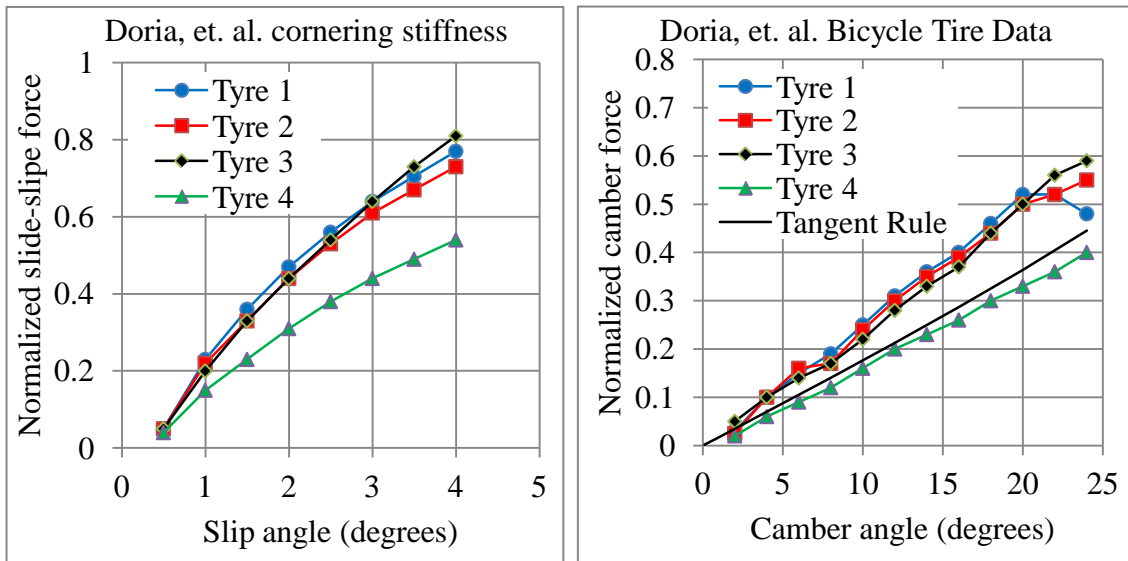
Figure 2-10: Doria *et al.*, 2012, tire force data.

They applied two vertical loads, 400 and 600 N, and two inflation pressures, 2 and 4 bar. They report the results for the tires under "normal conditions", but do not state explicitly what those are. It is likely that they mean 400 N and 4 bar.[28] This is shown below in **Figure 2-10**.

They do not indicate the width of the rim on which the tires were mounted,[28] but responded to an inquiry that it was 18.5 mm.

2.5.9   Motorcycle tires

While motorcycle tires and bicycle tires share many similarities, such as the need to allow large camber angles and an approximately toroidal shape, they also have important differences, such as inflation pressure, vertical load, carcass stiffness and rigidity, and cross section profile. For example, the Rotta model, presented below in section 5.1 assumes a circular cross section profile, but the *FTire* motorcycle tire simulation application allows for multiple radii to define the cross section profile,[29] and the Michelin Power One series of racing motorcycle tires have a pronounced v-shaped profile.[30]

Sharp, in 1971, models a motorcycle with tires that have normalized cornering stiffness of 0.191 to 0.195 1/° and normalized camber stiffness of 0.0161 to 0.0163 1/°. He specifically neglects pneumatic trail.[12]

Rice, in 1975, reported on measuring "trials, universal, and ribbed" motorcycle tires at Calspan Corporation's Tire Research Facility (TIRF).[31] With tires inflated "according to manufacturer's recommendation" and with a vertical load of "nominal value with a 200

lb. rider and and 120% of the nominal value", he measured a normalized cornering stiffness of 0.33 to 0.15 1/º, normalized camber stiffness of 0.009 to 0.21 1/º, and pneumatic trial between 0.5 to 1.0 inches (13 – 25 mm). He also reports that "For a given tire, the value of the cornering stiffness coefficient decreased with increasing normal load; the value of camber stiffness coefficient was relatively unaffected by normal load."

Sakai, Kanaya, and Iijima, in 1979, report normalized camber force to be slightly higher than the tangent rule of 0.0176 1/º.[18]

de Vries and Pacejka, in 1991, report for a 120/70 front tire under 1600 – 3200 N vertical load a normalized cornering stiffness of 0.125 to 0.225 1/º and normalized camber stiffness of 0.0268 to 0.069 1/º.[32]

Kageyama and Kuwahara, in 2002, report experimental normalized cornering stiffness of 0.47 to 0.5 1/º and normalized camber stiffness of 0.0122 to 0.0127 1/º. [33]

Berritta, Cossalter, Doria, and Lot, in 2002, report on their motorcycle tire testing device,[34] and Cossalter, in 2006, presents a variety of stiffnesses.[5] Depending on tire, vertical load, and inflation pressure, normalized cornering stiffness varies from 0.1 to 0.25 1/º, and normalized camber stiffness varies from 0.015 to 0.020 1/º. Cossalter specifically describes how it is common for motorcycle tires to exceed the tangent rule in camber stiffness, up to a camber angle of about 30º, and a negative slip angle is required to achieve equilibrium.
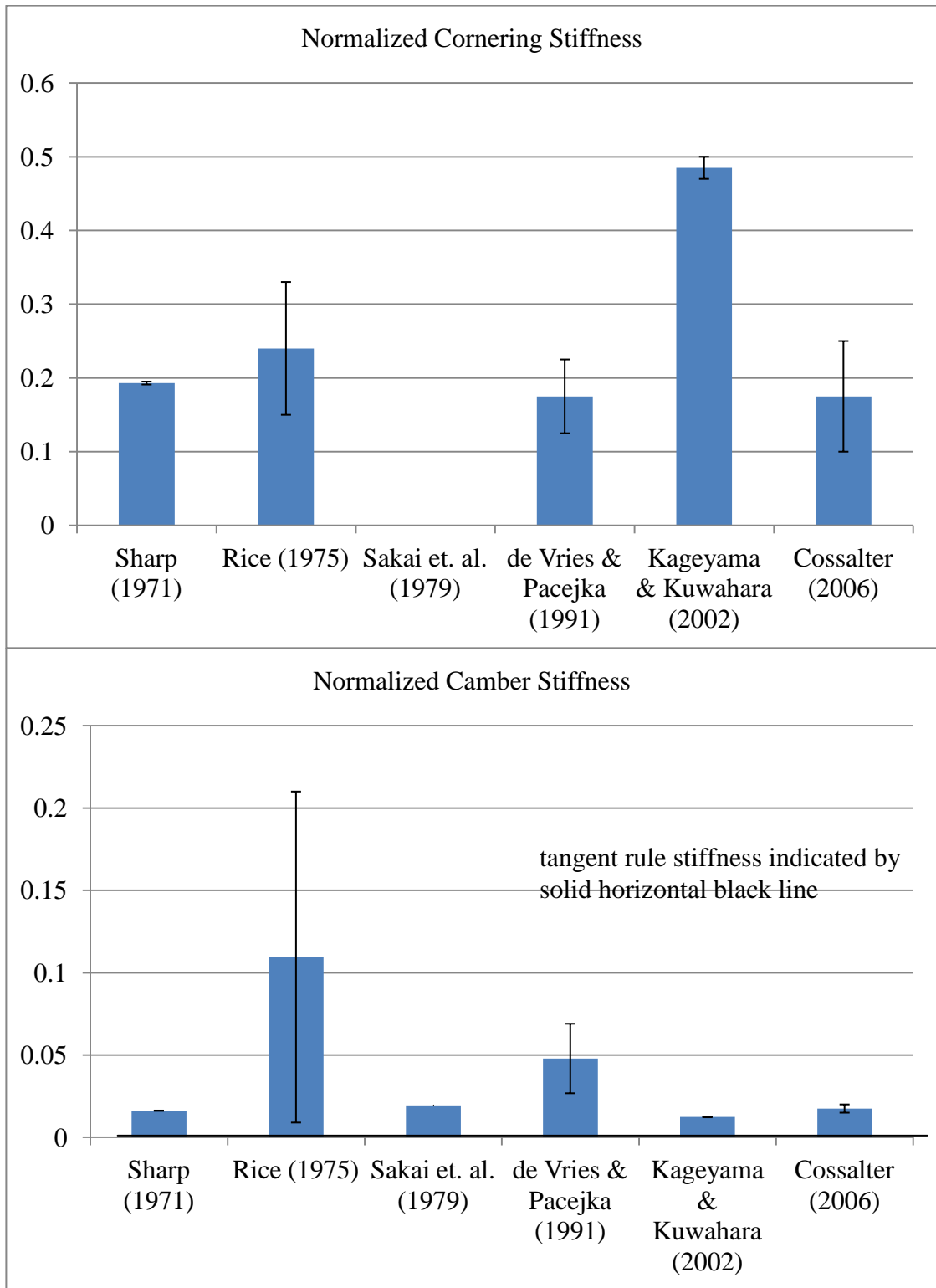
Figure 2-11: Summary of motorcycle tire normalized cornering and camber stiffness

CHAPTER 3

EXPERIMENTATION

Three different test devices were developed, as shown in **Figure 3-4**, one that rolls along

a guide fence on a stationary flat track, one that remains stationary over a moving flat

track, and one that remains stationary over a rotating drum. They were used to measured

bicycle tire stiffness for 14 different tires from 22 to 50 mm in size, at up to four different

inflation pressures per tire from 2 to 11 bar (29 to 160 psi), and under three different

vertical loads from 304 to 731 Newtons (68 to 164 lb): about 120 different parameter

combinations in all.

Figure 3-1: Test devices 1-3

3.1   Tires tested

A variety of tires was chosen, as listed in **Table 3-1**, in order to gain wider understanding

from the test results. Most are bias ply, with casing cords oriented diagonally, at about

45º from the normal to the tire beads, but one is described as a radial, with casing cords

oriented at about 15º from normal to the beads. All have a bead seat diameter of 622 mm

and all were tested with vertical loads between 304 and 736 N (31 and 75 kg). Since the

behavior of the carcass is the subject of this study, thick tread, especially knobby tires,

were avoided.

Table 3-1: Tires tested on third device at TU Delft.

ID	Brand	Model	Size	Tread	Bead
1	Bontrager	All Weather	23	Semi-smooth	Foldable
2	Bontrager	All Weather	25	Semi-smooth	Foldable
3	Bontrager	All Weather	28	Semi-smooth	Foldable
4	Cheng Shin	Classic Zeppelin	50		Wire
5	Continental	Top Contact Winter	37		Wire
6	Maxxis	Radial Prototype	22	Smooth	Foldable
7	Michelin	Dynamic	23	Semi-smooth	Wire
8	Schwalbe	Big Apple	55		Wire
9	Schwalbe	Kojak	35		Foldable
10	Schwalbe	Marathon Plus	37		Wire
11	Vittoria	Randonnear Hyper	37		Foldable
12	Vredestein	Perfect Tour	37	Semi-smooth	Wire
13	Vredestein	Fortezza DuoComp	23	Smooth	Foldable
14	Vredestein	Fortezza TriComp	23	Smooth	Foldable

As the results of testing and modeling were compared, it was discovered that the rim width, w, and the resulting true tire arc radius, r_0, influences stiffnesses. Thus, the arc length of each tire carcass was measured, from bead to bead when pressed flat, as shown in **Figure 2-3**, and calculated a true tire radius from that. In this document, "arc length" is used to mean this measurement, and "size" is used to mean the manufacturer's stated size, which is approximately the diameter of the mounted tire.

Sheldon Brown discusses some practicalities of tire sizes and widths, including a tendency by manufacturers to under report tire sizes during the 1970s and 1980s, the sizing standards of the European Tyre and Rim Technical Organisation (ETRTO) and International Organization for Standardization (ISO), and some guidelines and rules of thumb for tire size and rim width combinations. Specifically, "if you flatten out a tire and measure the total [arc length] from bead to bead, it should be approximately 2.5 x the ISO [size]," and "the tire [size] should be between 1.45/2.0 x the inner rim width."[35]

As reported in **Table 3-2**, the tires tested have a ratio of arc length to size between 1.99 and 2.43, with an average of 2.27, which is 0.23 below Brown's target. The ratio of true size to rim width, w, is between 1.43 and 2.51, with an average of 1.78, which is right in the middle of the target range.

Table 3-2: Dimensions of tires tested, published, measured, and calculated.

ID	Mfr's Size (mm)	Arc length s_0 (mm)	Radius (mm)	Rim width w (mm)	"True size" (mm)	Size diff. (%)	Arc length over size	True size over rim width
1	23	51.5	10.5	13.4	21.0	-8.6	2.24	1.57

2	25	60.0	11.8	13.4	23.6	-5.5	2.40	1.76
3	28	65.0	12.6	13.4	25.2	-10.0	2.32	1.88
4	50	119.5	22.1	18.7	44.2	-11.6	2.39	2.36
5	37	78.0	15.6	18.7	31.2	-15.6	2.11	1.67
6	22	45.0	9.5	13.4	19.1	-13.4	2.05	1.43
7	23	53.5	10.8	13.4	21.6	-6.0	2.33	1.61
8	55	128.2	23.5	18.7	46.9	-14.7	2.33	2.51
9	35	78.5	15.7	18.7	31.4	-10.4	2.24	1.68
10	37	73.5	14.9	18.7	29.8	-19.4	1.99	1.59
11	37	90.0	17.5	18.7	34.9	-5.6	2.43	1.87
12	37	85.7	16.8	18.7	33.6	-9.2	2.32	1.80
13	23	52.5	10.7	13.4	21.3	-7.3	2.28	1.59
14	23	54.0	10.9	13.4	21.8	-5.3	2.35	1.63

All tires sized 22-28 mounted on a 13.4 mm wide rim and sized 37-50 on a 18.7 mm wide rim. Sample calculations provided below in **Table 5-1** and **Table 5-2**

3.2   Rims used

All the tires tested on the third device at TU Delft where mounted on one of two 32-spoke wheels provided by Batavus: one with a narrow "sport" rim, and the other with a wider, touring or commuting rim. Details are provided in **Table 3-3**.

Table 3-3: Rims on which the tires were mounted for stiffness testing.

Model	Size	measure width	profile
Aero XR1 exel	622-13	13.4 mm	
exal	622-19	18.7 mm	

3.3   Wheels used

In nearly all cases, a standard wire-spoked front wheel was used for testing. They naturally have significant lateral compliance, but Prof. Pacejka at TU Delft urged measuring bicycle tires in as realistic a setting as possible. Thus, instead of a solid bicycle wheel made to be especially stiff, a displacement sensor was used to record the deflection of the rim relative to the fork. The resulting change in camber angle due to wheel flex was found to be negligible, as described in section 3.10.3. Twisting of the rim with respect to the wheel midplane, however, was not measured.

The one exception is the carbon tri-spoked rear wheel by HED Cycling. This was used when measuring the lateral displacement of the tire centerline with respect to the rim, as explained in section 3.8.6.3 to eliminate slight lateral deflections of the rim caused by decreasing tension in non-vertical wire spokes over the contact patch.

3.4   Pavement surface

In all cases, when lateral forces were measured, the surface against which the tire was pressed was covered with non-skid tape. For devices 1 and 2 at UWM, that was implemented with National Floor Safety Institute (NFSI) certified "Safety Grit Tape" by LifeSafe® with a grit designation of 60 and an average particle size of 0.265 millimeters.[36] This was intended to insure maximum friction between the tire and the pavement so that tire stiffness can be measured instead of the friction coefficient between the tire and the pavement. Without this, the cornering and camber stiffness decrease

much more quickly as slip and camber angles increase than they do in the data presented in **Figure 4-5**, **Figure 4-6**, and Appendix B.

3.5   Contact patch ink prints

Ink prints were made of several bicycle tires under various vertical loads and inflation pressures. Once inflated, mounted in the test device, and loaded, the wheel was simply lifted off the drum, placed on an ink pad, and then placed back on a sheet of white paper, first on the drum, and then on a flat plate. A description of how the resulting ink prints were analyzed is presented below in section 4.1.1.

Figure 3-2: Ink prints of contact patches of a Maxxis radial.

In **Figure 3-2**, the top ink print is on the 2.5 meter drum at TU Delft and the bottom is on a flat plate, both supporting a 31 kg vertical load and at 100 psi (0.6895 N/mm^2) inflation pressure.

3.6   Test devices

The motivating principles behind all three test devices constructed were:

1. Hold the wheel at some approximate orientation, either fixed or actuated so that a range of orientation angles can be swept during testing. Only an approximate orientation is sufficient because the actual orientation is measured directly from the rim braking surfaces to allow for flex in the bicycle wheel and test device frame.

2. Apply a vertical load (dead weight) that results in a known vertical force in the contact patch: F_z.

3. Roll the wheel forward over the pavement, either by moving the wheel or moving the pavement. The wheel must be constrained from moving laterally relative to the forward direction.

4. As lateral force builds up in the contact patch between the tire and the pavement, measure the force required to do so. This is F_y.

5. As a moment about the steering axis builds up in the contact patch, measure the torque required to constrain the wheel rotating about this axis. This is M_z.

6. Minimize geometries and mechanisms that complicate or interfere with these two measurements.

7. Measure the orientation of the rim with respect to the direction the wheel is moving with a pair of displacement sensors located symmetrically about the center of the contact patch. The angle calculated from arctangent of the difference between the two displacements over the distance between them is the slip angle α.

8. Measure the orientation of the rim with respect to the vertical, either with a pair of displacement measurements or with an inclinometer mounted on the fork and a displacement sensor on an extension of one fork blade to measure the lateral displacement of the rim over the center of the contact patch. The resulting angle is the camber angle γ.

9. Adjust slip angle by pivoting about a vertical axis through the center of the contact patch, either by rotating the wheel above the pavement, or rotating the pavement under the wheel. This minimizes lateral forces due to wheel lateral motion, and keeps the contact patch on the narrow strip of non-skid tape described above in section 3.4.

10. Adjust camber angle by pivoting the wheel about a horizontal axis through the center of the contact patch. This minimizes lateral forces due to wheel lateral motion, and keeps the contact patch on the narrow strip on non-skid tape described above in section 3.4.

11. Vertical load is implemented by the mass of the frame. Additional mass can be mounted near the contact patch, as desired.

3.7 Test device number 1

A description of this device and the data it generated has been previously published in Vehicle System Dynamics by Dressel and Rahman.[37]

3.7.1   Description

The device consists of a wooden cart to hold a bicycle wheel in the desired orientation as it is towed forward along a straight guide fence and a flat and stationary track. The cart can be adjusted to change the camber and slip angles of the wheel. Actual wheel orientation is measured by hand with a separate jig placed on the track for measurements and removed for testing. Setting and measuring wheel orientation are all performed by hand when the cart is stationary. Sweeping of angles is not possible. A schematic of the instrumented test cart is provided in **Figure 3-3**, and a photograph of test device number 1 on its final track is provided in **Figure 3-6**.

The track went through several iterations in an attempt to improve its level, flatness, and straightness in order to remove the influence of imperfections on the measured forces:

1. Flat and level section of floor in hallway adjacent to straight wall.

2. Flat and level section of floor in hallway adjacent to straight aluminum beam against straight wall.

3. Laminate counter top reinforced for stiffness and shimmed to be level.

4. Self-leveling concrete poured onto the web of a large, steel I-beam lying on its side and shimmed to be level.

The towing mechanism also evolved over several iterations:

1. Pulling on a cord directly by hand.

2. Pulling on a bicycle chain with a sprocket and hand crank.

3. Pulling on a chain with a a sprocket driven by an electric motor and gearbox.

3.7.2   <u>Contacts and force transmission</u>

The bicycle wheel and the frame that holds it have a total of six contacts with the external world:

- The tire contact patch.

- Two casters that roll on the horizontal track to provide camber stability.

- Two casters that roll against a vertical surface along the edge of the track to prevent lateral motion.

- One point of attachment for a towing chain.

The lateral force generated in the contact patch is transmitted through the bicycle wheel to the fork to the test device frame. From the frame, the force is transmitted through the force sensors to the casters that roll against the vertical surface along the edge of the track.

force sensor attached to bracket enforcing steer angle to measure torque generated about steer axis

fork free to pivot about steer axis

yoke

bracket to enforce steer angle

steer angle that produces side slip angle

hinge

lateral force sensors

lever arms to deamplify force

wheels that roll against vertical surface to constrain direction of travel

turnbuckle to set steer angle

turnbuckles (one on each side) to set camber angle

caster (one on each side) to support back of cart

hinge

Figure 3-3: Schematic of test device number 1.
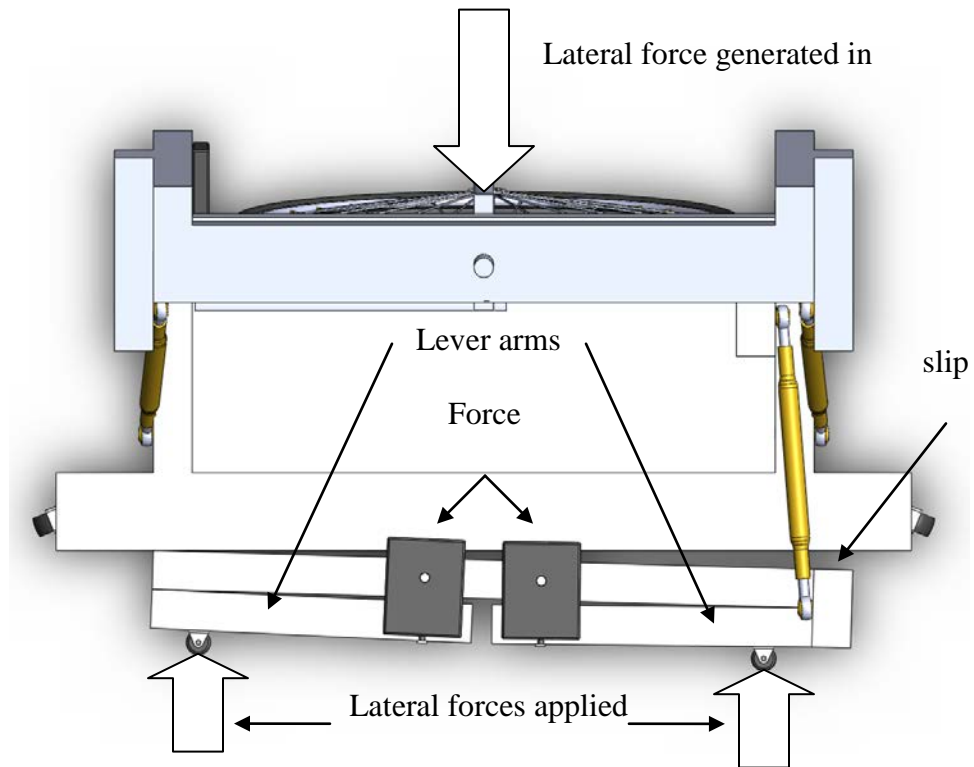
Figure 3-4: Plane view of test device number 1 showing external lateral forces.

3.7.3   Forward speeds and actuation rates

A forward towing speed of only 0.07 m/s was used, and all the angles were always fixed

during an entire test run, so there was no actuation rate.

3.7.4   Instrumentation

The instrumentation and data collection system consisted entirely of proprietary plug-

and-play components from PASCO®. The data is uploaded to a laptop computer to be

read, presented, and saved with DataStudio, a proprietary software application also by

PASCO®. More details are provided about the PASCO® equipment below in section

3.8.3.
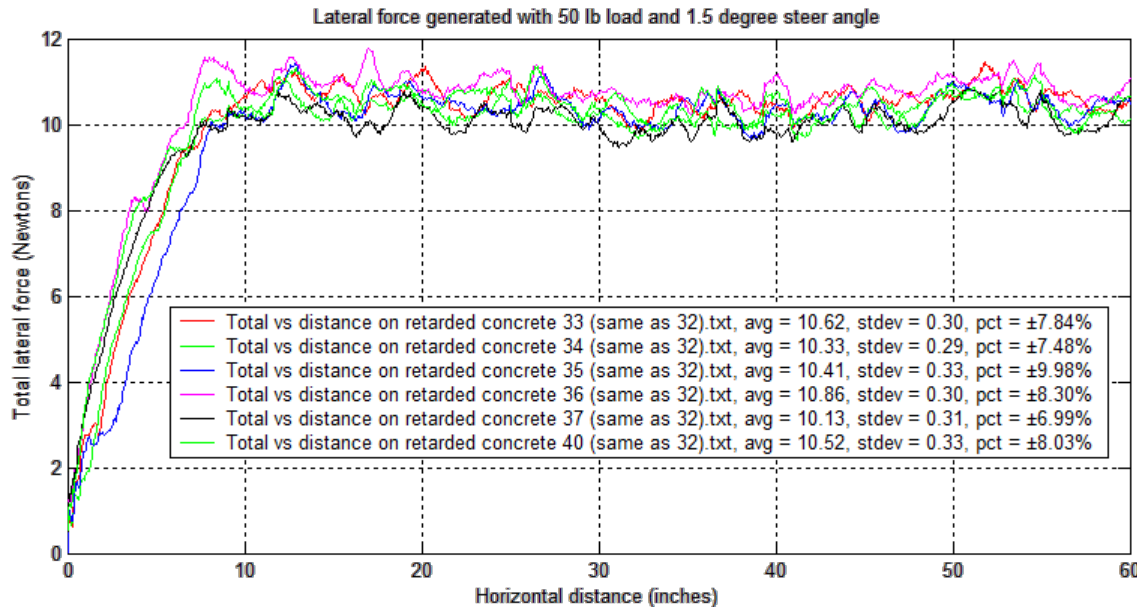
Figure 3-5: Raw force data captured from test device number 1

3.7.5   Advantages and disadvantages of design

The main goal of this design was to be inexpensive and portable, and it was that initially.

After the search for a sufficiently flat, smooth, and level track resulted in a massive I-

beam lying on the floor, the portability was lost.

Testing tires with this device was very time consuming and labor intensive. The only data

automatically recorded was the lateral force. The tire orientation had to be set and

recorded by hand for each combination of slip and camber angles tested. Because

changes in the tire orientation could not be detected as forces built up and the bicycle

wheel flexed, the wheel was supported by small bearings that ran on the braking surface.
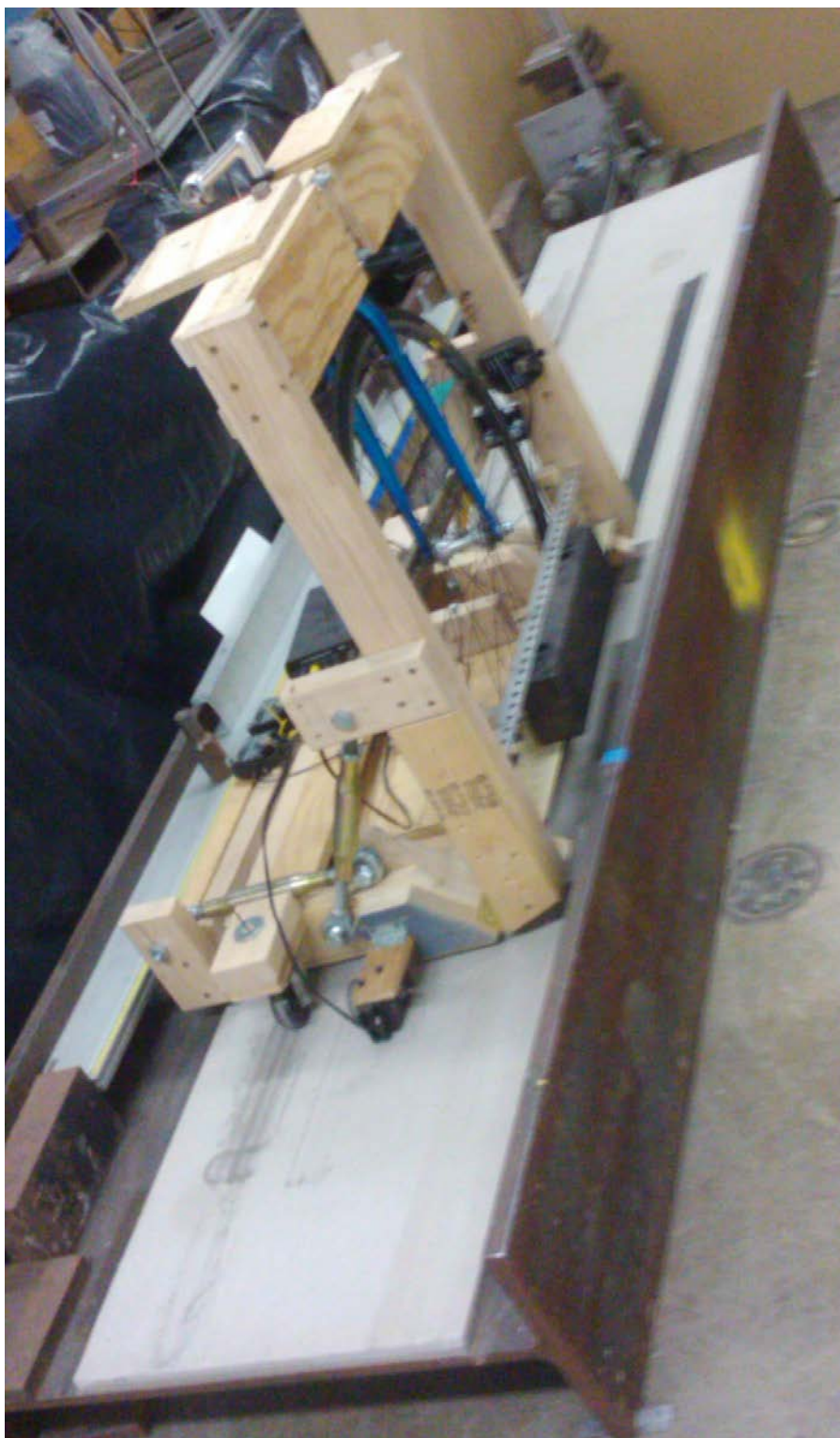
Figure 3-6: Photograph of final test device number 1 on its concrete track.

Finally, test runs were constrained by the length of the track. It was never possible to record the force data generated for an entire rotation of the wheel. Thus, the pattern in the force data with the same period as the wheel circumference was never detected.

3.8   Test device number 2

The second test device consists of a welded steel frame to hold the wheel stationary in a desired orientation, as shown below in **Figure 3-7**, on top of a small track of flat-top chain, as shown below in **Figure 3-9**. It is based on a design recommended by Jim Papadopoulos which has a two-degrees of freedom pivot, implemented with an automobile universal joint, far forward of the bicycle tire so that slight variations in vertical or horizontal position produce negligible variations in orientation angle. Potential sources of variations in vertical or horizontal position include any out-of-roundness of the bicycle wheel or tire and deflections of the lateral force sensor or frame due to lateral force generated in the contact patch. The forward pivot is implemented with needle-bearings in an automotive universal joint approximately 1.3 meters forward of the contact patch, so that any friction in the bearings or seals generates a negligible lateral force at the contact patch.

Figure 3-7: Photograph of test device number 2. The forward pivot is on the far left, and the flat-top chain track is under the wheel on the right.

3.8.1 <u>Geometry</u>

This device allows for sweeping slip and camber angles while measuring the lateral force, F_y, and vertical moment, M_z, generated in the contact patch. It uses one or more force sensors, depending on expected load, to maintain the lateral location of the contact patch and a second force sensor to prevent rotation of the fork that holds the bicycle wheel about its steering axis.
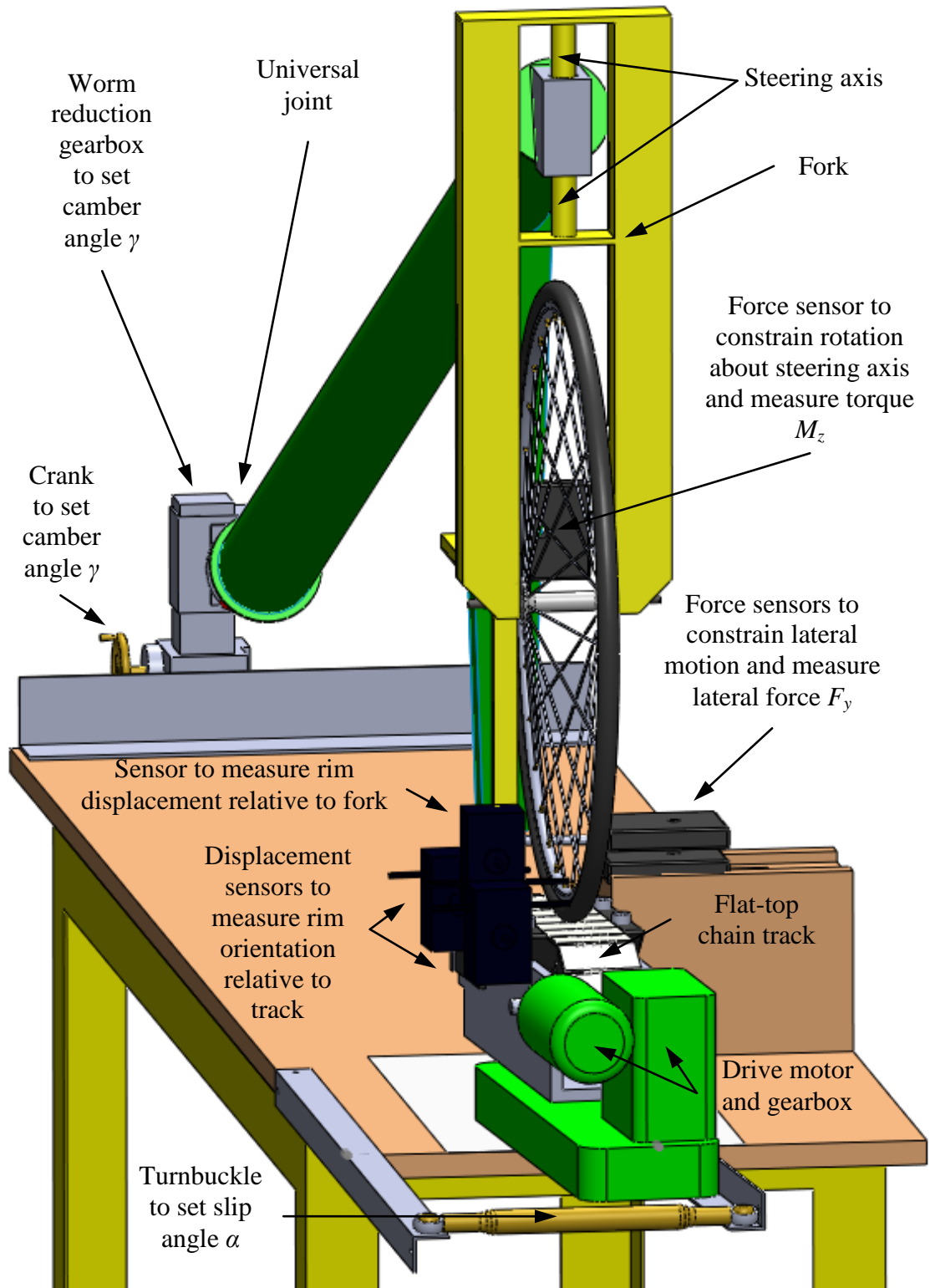
Figure 3-8: Schematic of test device number 2.

In order to allow for the inevitable flexibility of the test frame and the bicycle wheel, the slip orientation of the bicycle rim is measured with a pair of position sensors mounted rigidly to the support platen for the flat-top chain near each end of the contact patch, one of which is shown below in **Figure 3-9**. Similarly, the camber orientation of the rim is measured with an accelerometer on the fork and a position sensor mounted rigidly to an extension of the fork to measure displacement of the rim relative to the fork.
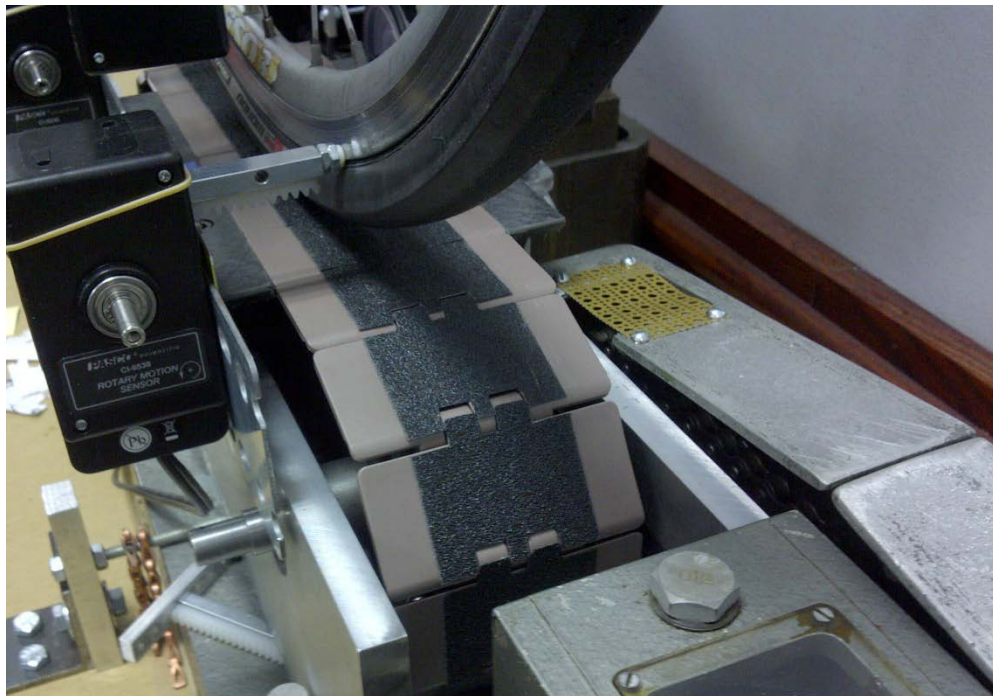
Figure 3-9: Test device number 2 flat-top chain with anti-skid tape also showing one rim displacement sensor contacting the braking surface of the wheel rim.

The forward pivot point is fixed, and slip angle is altered by pivoting the small flat-top chain track about a vertical axis under the center of the contact patch, with a turnbuckle as shown below in **Figure 3-9**. Camber angle is altered by rotating the universal joint about its longitudinal axis, which passes through the contact patch, with a 300:1 worm

reduction gear box. Since the track moves slowly, and the entire device sits on a table top, the angle sweeps are easily implemented by hand cranks.

The vertical load borne by the tire is generated simply by the weight of the frame. Additional mass can be added above the fork as desired. The frame was designed in SolidWorks® and analyzed with the finite element method in ANSYS® Workbench, as shown in **Figure 3-4**, in a effort to create the most stiffness with the least weight. The goal was to have a light enough frame so that the desired vertical loads could be tested without need of some kind of counter-balancing system.
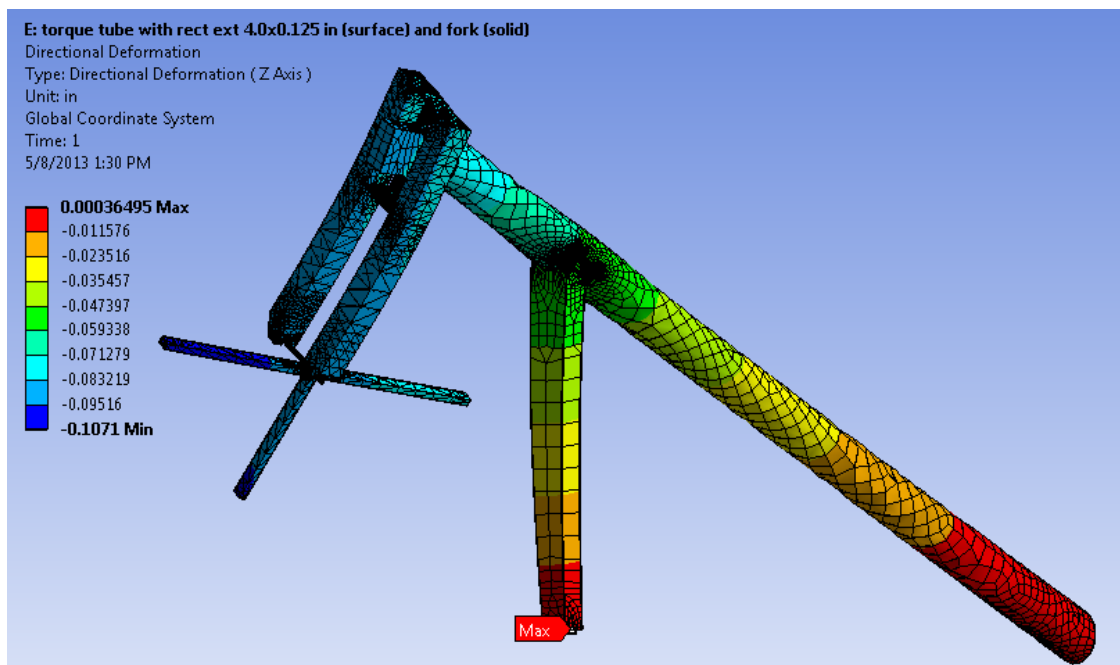
Figure 3-10: Finite Element Analysis of test device 2 frame deformation under load

3.8.2   Contacts and force transmission

The bicycle wheel and the frame that holds it have a total of three contacts with the

external world:

- The tire contact patch.

- A force sensor, which constrains lateral motion of the wheel.

- The universal joint, which allows lateral and vertical motion of the wheel, but

  constrains rotation about the camber axis.

The lateral force generated in the contact patch is transmitted through the bicycle wheel

to the fork to the test device frame. From the frame, the force is transmitted to both the

lateral force sensor and the universal joint. A simple static summing of the moments

about a vertical axis through the universal joint provides a relationship between the

lateral force generated in the contact patch and the lateral force measured by the sensor.

The mounting point on the frame for the lateral force sensor is positioned on the same

axis through the center of the contact patch as the universal joint so that changes in

camber angle have no effect on the lateral force sensor geometry.

3.8.3   Instrumentation

As with test device number 1, the instrumentation and data collection system consisted

entirely of proprietary plug-and-play components from PASCO®.

Table 3-4: Sensors for test device number 2

Sensor	Use.
Force	Measure lateral force generated in the contact patch by constraining frame from moving laterally.
Force	Measure torque about a vertical axis generated in the contact patch by constraining fork from rotating in head bearing.
Acceleration	Measure camber angle of fork.
Displacement	Measure slip angle of wheel relative to flat-top chain by comparing measurements from one sensor near the front of the contact patch and another near the rear.
Displacement	Measure displacement of rim relative to fork.

The data is fed via USB port to a laptop computer to be read, presented, and saved with DataStudio, a proprietary software application also by PASCO®. A PASCO® rotary motion sensor with a linear motion track pressed lightly to the braking surface of the bicycle wheel with a thin rubber band, can be seen in the upper left of the photograph in **Figure 3-9**.

The individual force sensors, described by PASCO® as having a 1% accuracy, 0.03 N (0.006744 lb) resolution, and up to a 1000 Hz sample rate, are rated for only ±50 N (11.24 lb). The force sensors "employ four strain gauges epoxied to a binocular dual-beam made from annealed aluminium. The strain gauges are wired to form a full-bridge

circuit that is driven by a constant voltage source. The voltage across the bridge circuit is proportional to the applied force."[38]

All three displacement sensors require contact and are implemented by converting linear motion to rotary motion with a rack and pinion. PASCO® describes their rotary motion sensor as having an optical encoder with a resolution of 0.25° and accuracy of ±0.09°. The pinion has a radius of 12.72 mm (0.500 in) and thus a resolution of 0.0554 mm (0.0022 in) ±0.02 mm (0.0008 in).
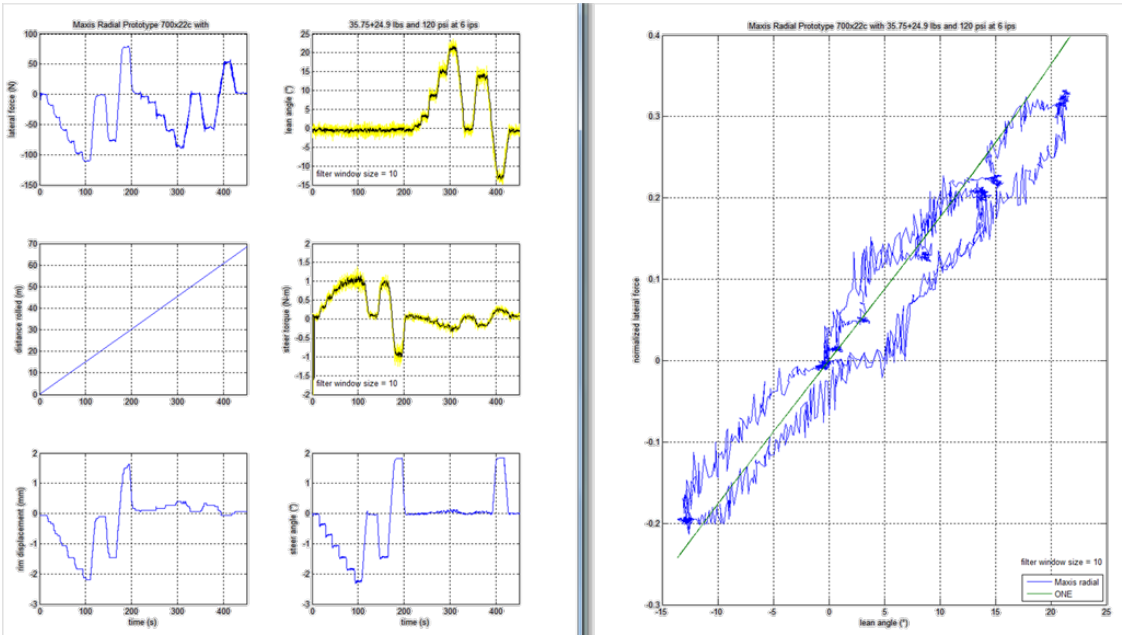
Figure 3-11: Raw force and displacement data captured from test device number 2.

The plot on the right side of **Figure 3-11** shows camber force vs. camber angle, and the source of the apparent hysteresis was not discovered. It was not observed in the same data collected from device number 3, as shown below in the bottom right plot in **Figure 3-23**.

3.8.4 Uncertainty propagation

Propagation of uncertainty from the sensors to a final value can be calculated with **Eq. 3-1**, where f is the function to evaluate the final value y, the x_i are the measured values that contribute to it, Δx_i is the uncertainty in x_i, and Δy is the uncertainty in the final value.

$$\Delta y_{max} = \sum_{i=1}^{n} \left| \frac{\partial f}{\partial x_i} \Delta x_i \right| \qquad \textbf{Eq. 3-1}$$

For lateral stiffness, **Eq. 3-1** expands into **Eq. 3-2**.

$$C_{Fy} = \frac{F}{\delta_h} \Rightarrow \Delta C_{Fy} = \left| 1 \cdot \delta_h^{-1} \cdot \Delta F \right| + \left| -F \cdot \delta_h^{-2} \cdot \Delta \delta_h \right| \qquad \textbf{Eq. 3-2}$$

For a lateral stiffness of 41 N/mm, **Eq. 3-2** yields **Eq. 3-3**.

$$\Delta C_{Fy} = \left| 1 \cdot (1 \text{ mm})^{-1} \cdot 0.01(41 \text{ N}) \right| + \\ \left| -(41 \text{ N}) \cdot (1 \text{ mm})^{-2} \cdot (0.02 \text{ mm}) \right| = 1.23 \text{ N/mm} \qquad \textbf{Eq. 3-3}$$

Thus, the lateral stiffness is 41 N/mm $\pm$ 1.23 N/mm (3%).

3.8.5 Forward speeds and actuation rates

A forward speed of only 0.15 m/s was used, approximately twice the speed of test device number 1. Higher speeds are prohibited by the limited power of the electric motor used to drive the flat-top chain. Camber angle was varied at 1 deg/sec and slip angle at 0.25

deg/sec. Camber angle could be varied more quickly, but slip angle variation rate is limited by the rate at which the controlling turnbuckle can be rotated by hand.

Thus the tires experienced change rates of 1.5 deg/m in slip angle and 6.5 deg/m in camber angle on test device number 2.

3.8.6 Additional tests performed with device number 2

Test device number 2 was also repurposed to perform several additional tests, as described in the following sections.

3.8.6.1 *Radial stiffness*

To compare with the model, presented below in CHAPTER 5, radial stiffness data was collected, in the form of vertical sinkage due to a sequence of increasing versus decreasing dead weights, as shown below in **Figure 3-12**. Slopes of horizontal segments reflect creep or relaxation (rate dependence).

The rate-independent hysteresis, described in section 2.3 can be seen in the difference in deflection under the same load depending on whether the previous load was larger or smaller. The difference in heights could be because of additional creep taking place between loading and unloading.

Vertical displacement was measured by a PASCO® rotary motion sensor with a small cord wrapped around its axle, over a pulley above the rim between two spokes, and straight down to a single spoke nipple.
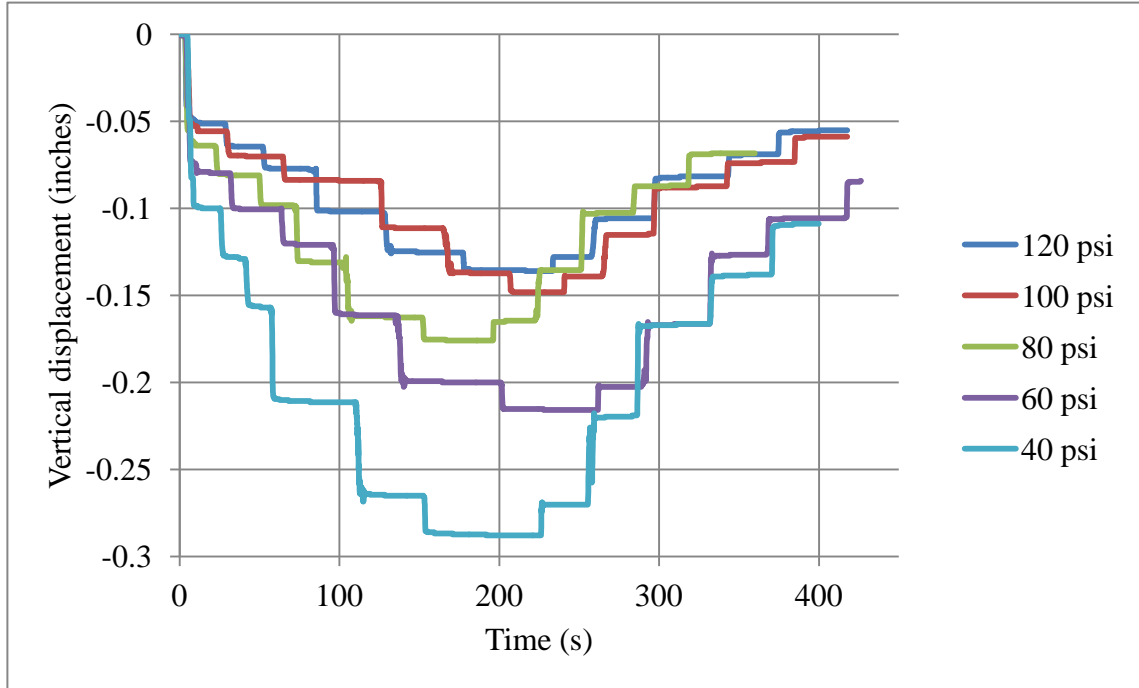
Figure 3-12: Radial stiffness of a Bontrager All Weather 28-622 mm tire at various inflation pressures and under vertical loads of 30.3, 42.2, 54.2, 78.5, 102.7, and 114 lb.

3.8.6.2 *Static lateral stiffness*

To improve accuracy of the static lateral stiffness values obtained, a method was devised to pull directly on the rim slowly and precisely while measuring the force required and the resulting deflection, as shown in **Figure 3-13**. The results were used to measure the sensitivity of tire stiffness to rim width presented in **Figure 6-4** below.

The lateral displacement was provided by a PASCO® Stress-Strain Apparatus, the force was measured with a PASCO® force sensor, and the lateral displacement was measured with a pair of PASCO® rotary motion sensors equipped with the PASCO® linear motion accessory, as shown in **Figure 3-13**.
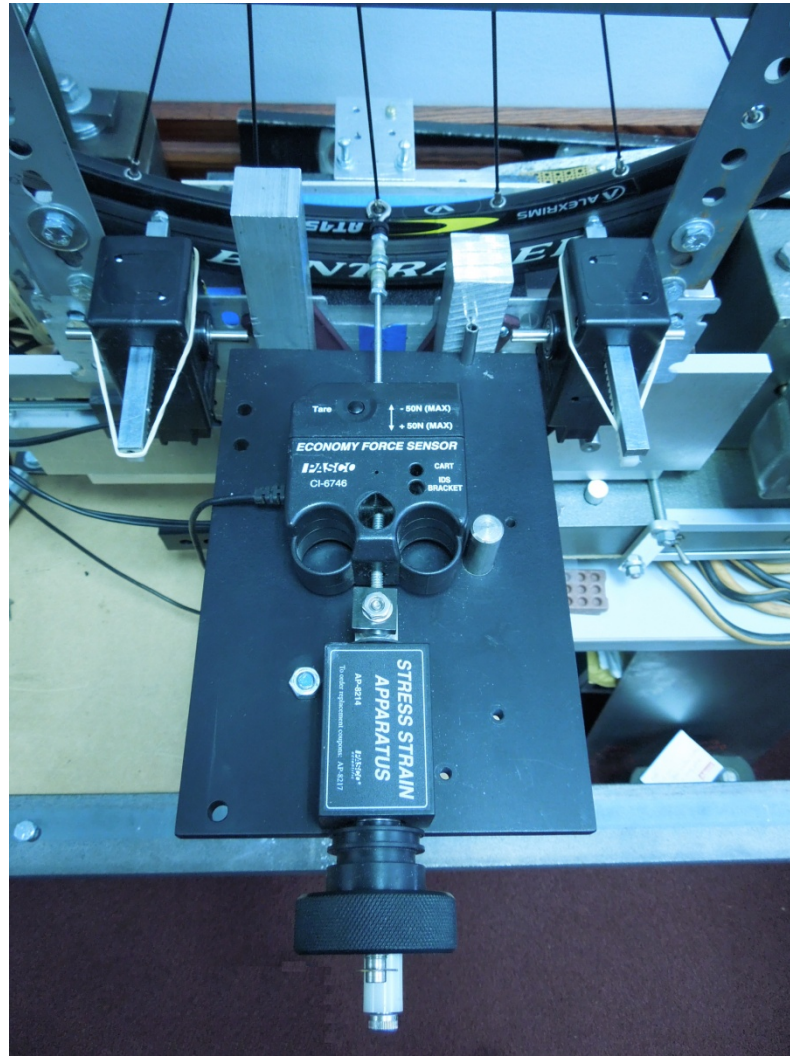
Figure 3-13: Measuring static lateral stiffness with test device number 2.

3.8.6.3   *Decay length*

In order to measure deflection of the tire centerline beyond the contact patch, a PASCO®

rotary motion sensor was attached to the rim so that it rotates with the wheel, as shown in

Figure 3-14. A fine cord, dental floss, is run from a pin-hole in the centerline of the tire,

over a pulley, a common "608" inline skate or skateboard bearing, to the rotary motion

sensor. The bracket that holds the pulley extends 150 mm (nearly 6 inches) so that vertical fluctuations as the tire compresses above the contact patch, will have a minimal effect on the length of the cord and therefore the measured location of the centerline of the tire.

Figure 3-14: Measuring tire centerline lateral deflection beyond the contact patch.

As can be seen in **Figure 4-11**, the lateral deflection looks as might be expected from a non-zero slip angle, but is oddly S-shaped when the tire is not generating a side force.

A wire-spoked wheel, however, under a vertical load experiences a vertical deflection of the rim over the contact patch, it will also deflect laterally "away from the spoke where the load is applied."[39] To confirm that this effect is contributing to the tire centerline lateral deflection data, the same tests were performed with the same tire mounted on a HED Cycling carbon tri-spoke wheel, as pictured below in **Figure 3-15**.

Figure 3-15: Measuring tire centerline lateral deflection on a HED Cycling carbon tri-spoked wheel.

The sensor of **Figure 3-14** and **Figure 3-16** can be seen at the 6 o'clock position of the wheel in **Figure 3-15**.

The particular wheel available for testing was a rear wheel with a 130 mm wide hub. The drive side is implemented as a cassette hub, which integrates the axle bearing into part of the ratchet mechanism and cannot be trivially removed or replaced. A spacer from the non-drive side could be removed, however, which reduced the hub width to about 105 mm. This is only slightly wider than the 100 mm width of front wheels that the test device fork was designed to accommodate. The fork blades could be spread by unbolting the connection at the top of the fork tubes, but this reduced lateral stiffness, and probably caused the actual camber angle of the wheel to be significantly more than the camber

angle of the frame. In **Figure 4-12**, multiple test runs are superimposed to demonstrate the robust repeatability of the measurement.

Finally, to gain a better understanding of what exactly is happening in the contact patch with zero slip angle, a pair of sensors were mounted directly to a spoke of the carbon tri-spoke wheel, one on each side of the wheel, to confirm either the symmetry or asymmetry of the measured displacement.
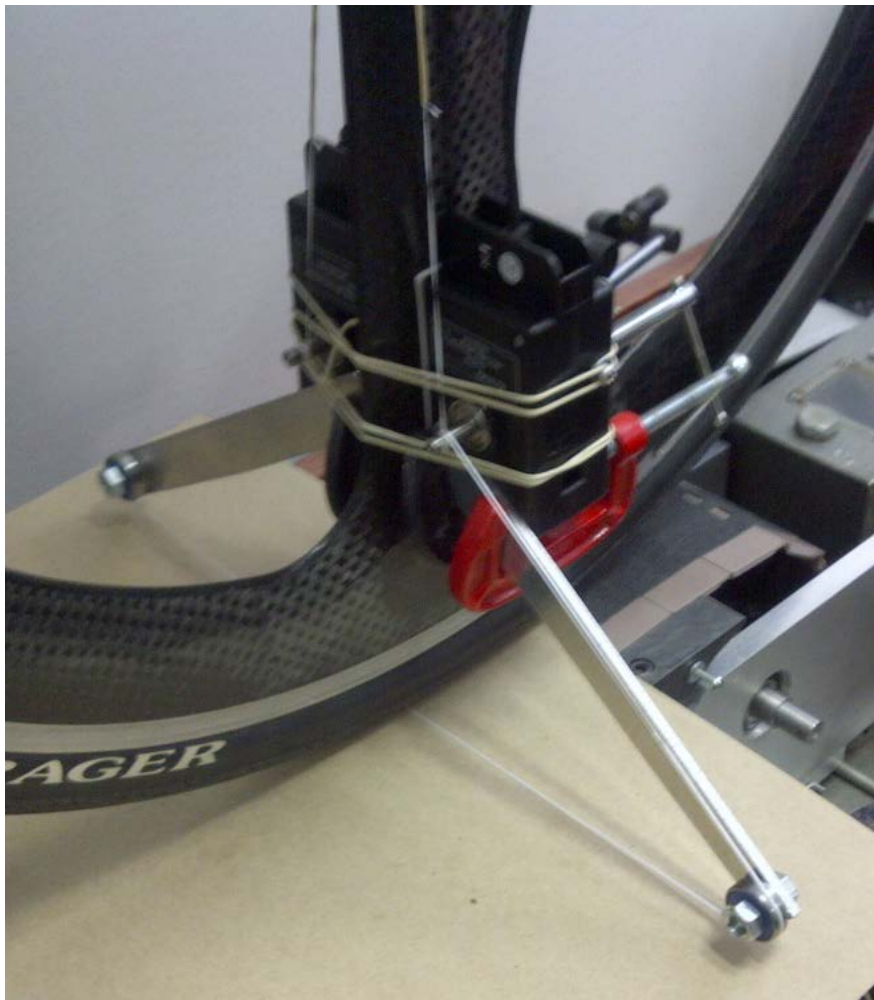
Figure 3-16: Paired displacement sensors.

The data presented in **Figure 4-14** confirm that the small, ~0.1 mm, lateral displacement of the centerline actually occurs, is asymmetric, and is not just some artifact of the measuring technique. Instead, it is likely due to slightly non-zero camber or slip angles, even tire or rim non-uniformity, or some combination of these conditions.

The sensors mounted on the wheel to measure lateral displacement of the tire centerline prevent the use of the pair of sensors mounted to the track and that measure the displacement of the rim, which are used to measure slip angle. Instead, zero slip angle was set for these tests by adjusting the track orientation until the measured lateral force generated by the tire was zero. It is possible that modifications to the test apparatus fork assembly, which were necessary for it to accept the wider hub of the rear tri-spoke wheel, could have caused the wheel to be slightly out of vertical alignment with the fork, and zero camber angle is set with a level mounted to one of the fork blades. Thus, even though the measured lateral force was zero, the camber angle might have been slightly non-zero, and that was countered by a slightly non-zero slip angle in the other direction. Finally, the fact that slip angle has a larger effect on centerline lateral displacement could explain the small lateral displacement measured when the measured lateral force was zero.

3.8.7   Advantages and disadvantages to design

Being able to record wheel orientation automatically and thus sweep slip and camber angles during a single test run was a huge improvement over the previous design. The wheel and frame could be allowed to flex because the true orientation was being recorded along with the force data. Being located on top of a table make working on it far less

arduous. The design also proved to be quite adaptable, as evidenced by the variety of additional experiments performed with it.

The electric motor that drives the flat-top chain is too little power to drive it at speeds anywhere near to those commonly experienced by bicycle tires, however, and it is not clear that the flat-top chain would perform well at those speeds if it could.

The system by which the steering axis is adjusting to be vertical over the contact patch is awkward. Placing the additional weights at the top of the fork ensures that they are centered over the contact patch, but is an ungainly operation to perform.

The PASCO® sensor system is at its limits to capture all the desired signals. The force sensors are limited to 50 Newtons, and so a group of three are used to measure the lateral force. Altogether, 4 force sensors, an accelerometer, and three rotary motion sensors were used. This required using two separate PASCO® interfaces that connect to the PC via two separate USB ports. The only way to collect data through the two ports with the PASCO® DataStudio software is to run two separate instances of it, and there is no way to start them recording data simultaneously. Instead, a synchronizing signal must be generated in the recorded data.

3.9   Test device number 3

Upon the invitation of Prof. Arend Schwab, at Delft University of Technology (TU Delft) in Delft, the Netherlands, three separate visits were made to the tire testing facility there to design, build, and operate a bicycle tire test device that works with one of the existing 2.5 meter diameter tire testing drums.

The device consists of a welded steel frame to hold the wheel stationary in a desired orientation on top of the drum, as shown below in **Figure 3-17** and **Figure 3-18**, and is based on the same design recommendation by Jim Papadopoulos as was used for test device number 2 described above in section 3.8.

The wheel is mounted on a steerable fork held by a frame whose roll is prevented by a front universal joint. The slip angle is achieved by holding the contact patch in a fixed horizontal location, while the universal joint assembly is displaced to alter the angle. Steering torque is measured by a load cell constraining rotation about the steering axis. Only a small section of the large drum penetrates through the laboratory floor.

Track to keep contact patch centered on top of drum

Cart to follow track

Universal joint

Actuator to set slip angle α

Force sensor to constrain rotation about steering axis and measure torque M_z

Yoke to support fork

Steering axis

Actuator to set camber angle γ

Fork

Sensor to measure rim displacement relative to fork

Force sensors to constrain lateral motion and measure lateral force F_y

2.5-meter drum

Displacement sensors to measure rim orientation relative to drum

Pivot point between frame and yoke

Figure 3-17: Schematic diagram of test device number 3

3.9.1 Geometry

In order to create the desired slip angle between the bicycle tire and the drum, which rotates on a fixed axle, the forward pivot point moves on a curved track, just visible in the lower left corner of **Figure 3-18** above and at the top of **Figure 3-17** below, that keeps the tire contact patch stationary on the crown of the drum. Camber angle is implemented by a yoke that holds the bicycle fork and pivots relative to the rest of the frame about an axis that passes through the contact patch, as shown below in **Figure 3-19**. As with test device number 2, there are only three contact points with the fixed ground, the tire contact patch on the drum, the forward pivot point, and the ball-jointed mount for the lateral force sensor. The forward pivot is implemented with needle-bearings in an automotive universal joint approximately 1.3 meters forward of contact patch, so that any friction in the bearings or seals generates a negligible lateral force at the contact patch.

As with test device number 2, the frame was designed in SolidWorks® and analyzed with the finite element method in ANSYS® Workbench in an effort to create the most stiffness with the least weight.

Figure 3-18: Photograph of test device number 3.

3.9.2   Actuation

Because access to the room that contains the drum is prohibited for safety reason when it is rotating, actuators were installed to sweep slip and camber angles. LiftMaster® ECO400K electric gate openers were found with the necessary 400-mm travel, 250-N force, 12-13-mm/s rate, and dimension and used as actuators. They were set to run in a "manual" mode and connected to a pair of switches in the control room. In hind-sight, their 8 cycles/hour duty cycle proved to be on the light side, and care had to be taken to prevent them from over heating. The camber angle actuation is pictured below in **Figure 3-19**.

Figure 3-19: Camber angle actuator installed on test device number 3 before non-skid tape is installed on the drum.

3.9.3   Instrumentation

The instrumentation and data collection system consists of an ad-hoc arrangement of

sensors connected to a National Instruments DAQ controlled by National Instruments

LabVIEW running on a laptop computer, as pictured in **Figure 3-20**.
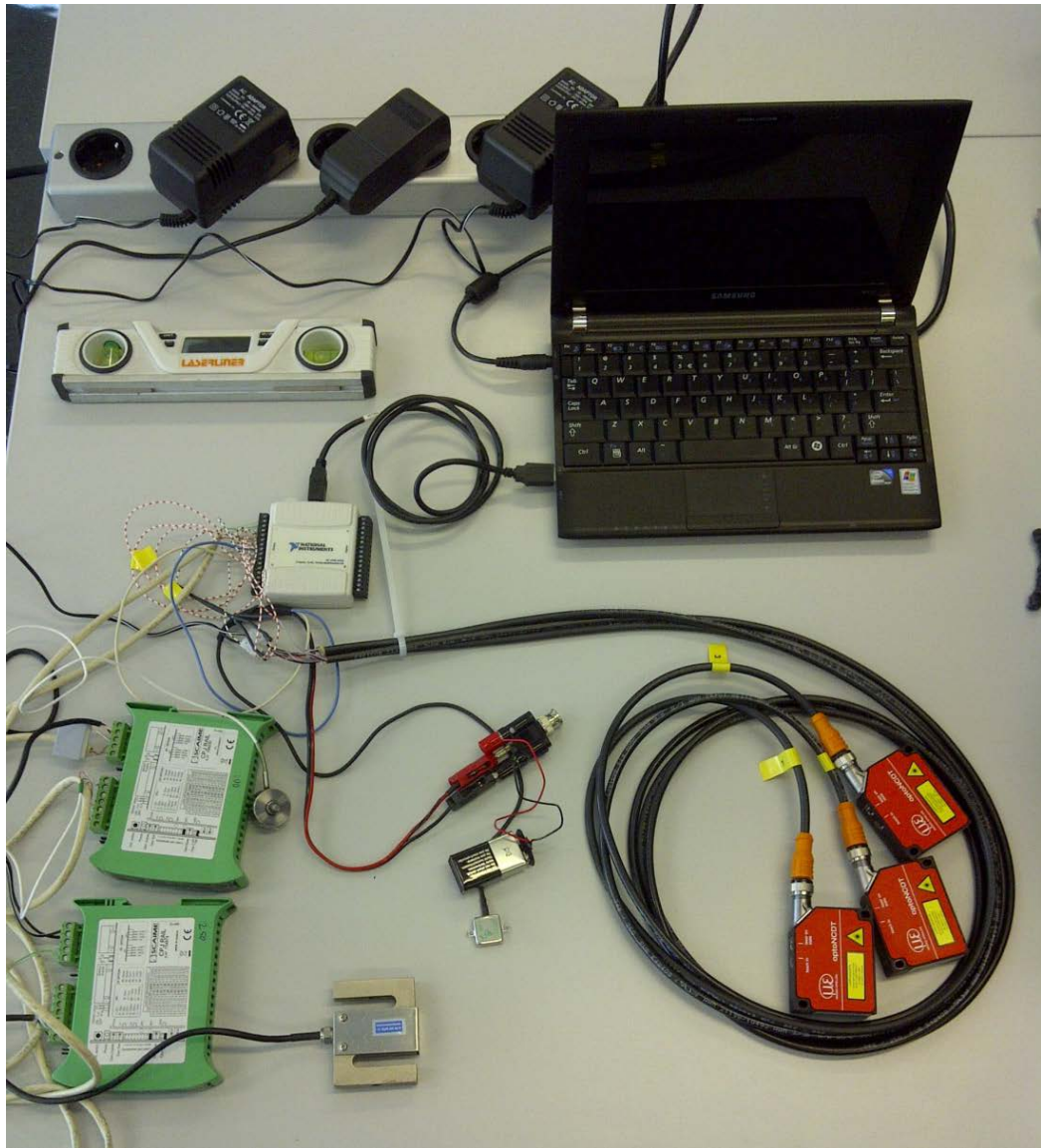
Figure 3-20: Instrumentation before installation on test device number 3.

The lateral force, generated by either slip angle or camber angle, is measured with a single force sensor mounted collinear with the center of the contact patch between two ball joints to minimize torques and off-axis loads. It is in either tension or compression, depending on the sign of the applied slip or camber angle.

The camber angle is measured with an accelerometer mounted on the fork blade. Visual observation during testing was in agreement with recorded values. A laser displacement sensor mounted rigidly to the fork and aimed at the rim directly above the center of the contact patch shows only small displacements during camber angle sweeps that would produce only small variations in actual camber angle experienced by the tire and rim.

All three displacement sensors are contactless and use a laser beam which can be aimed precisely at the point whose displacement needs to be measured. That is the braking surface of the wheel rim in all three cases.

Table 3-5: Sensors for test device number 3.

Sensor	Make and model	Use
Force	Scaime ZFA 100 kg Loadcell with "combined sensor error : 0.03 % full scale" so ±0.29 N	Measure lateral force generated in the contact patch by constraining frame from moving laterally.
Force	Scaime ZFA 25 kg Loadcell with "combined sensor error : 0.03 % full scale" so ±0.074	Measure torque about a vertical axis generated in the contact patch by constraining fork from rotating in

	N	head bearing.
Acceleration/ Inclination		Measure camber angle of fork.
Displacement	Micro-Epsilon OptoNCDT ILD 1300-50 Laser displacement sensor with 50 mm range, 10 μm resolution, and "linearity ±0.2% FSO" so ±0.1 mm	Measure slip angle of wheel relative to drum by comparing measurements from one sensor near the front of the contact patch and another near the rear.
Displacement	Micro-Epsilon OptoNCDT ILD 1300-20 Laser displacement sensor with 20 mm range, 10 μm resolution, and "linearity ±0.2% FSO" so ±0.04 mm	Measure displacement of rim relative to fork.

The vertical load is applied simply with the mass of the test frame and additional masses attached to the frame near the contact patch. The vertical force at the contact patch is measured by placing the tire directly on a scale located immediately beside and at the same level as the drum. See readings for three different vertical loads in the three images shown in **Figure 3-21**.

Figure 3-21: Vertical load created by attaching additional masses to frame of test device number 3.

The tire inflation pressure is measured by two different gauges on two different tire pumps. See reading of second gauge, on the second pump in the image shown in **Figure 3-22**.

Figure 3-22: Inflation pressure gauge integrated with tire pump.

3.9.4 Uncertainty Propagation

For a lateral stiffness of 41 N/mm, **Eq. 3-2** from section 0 yields **Eq. 3-4**.

$$\Delta C_{Fy} = \left|1 \cdot (1 \text{ mm})^{-1} \cdot (0.29 \text{ N})\right| + $$

$$\left|-(41 \text{ N}) \cdot (1 \text{ mm})^{-2} \cdot (0.1 \text{ mm})\right| = 4.39 \text{ N/mm}$$

Eq. 3-4

Thus, the lateral stiffness is 41 N/mm ± 4.39 N/mm (10.7%).

3.9.5 Forward speeds and actuation rates

The 2.5 meter drum at TU Delft can be driven at nearly any forward speed likely to be experienced by an automobile tire. A speed was selected compatible with the fixed actuation rate of the electric gate openers used to sweep slip and camber angles: 22 m/s (~8 kph, ~5 mph), far faster than the two test devices at UWM.

Slip angle was swept at 0.33 deg/sec, and camber angle was swept at 2.33 deg/sec. The difference in rates, from identical actuators, is caused by the different geometries of their installation on the test device frame. Thus the tires on test device number 3 experienced change rates of 0.015 deg/m in slip angle (about 1/100 the rate of test device number 2) and 0.11 deg/m in camber angle (about 1/60 the rate of test device number 2).
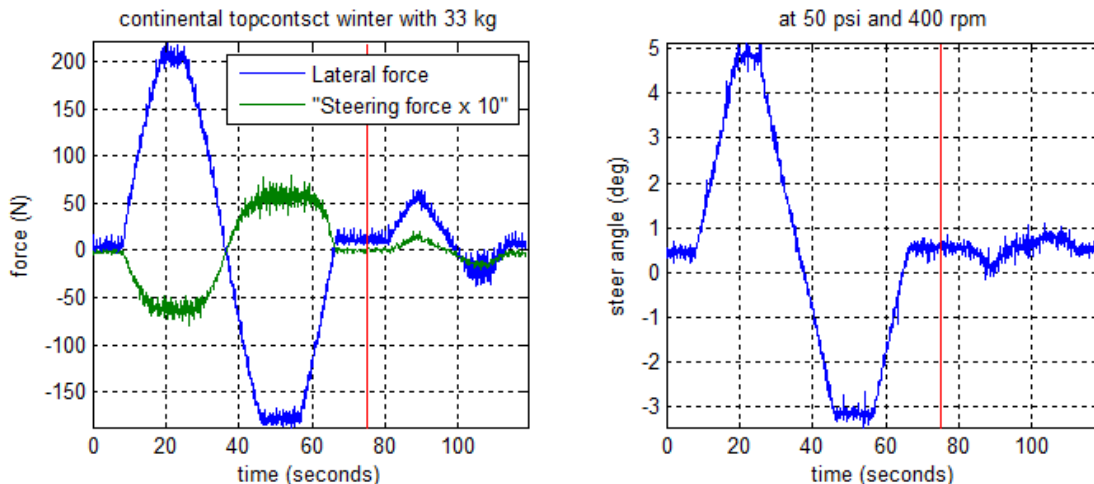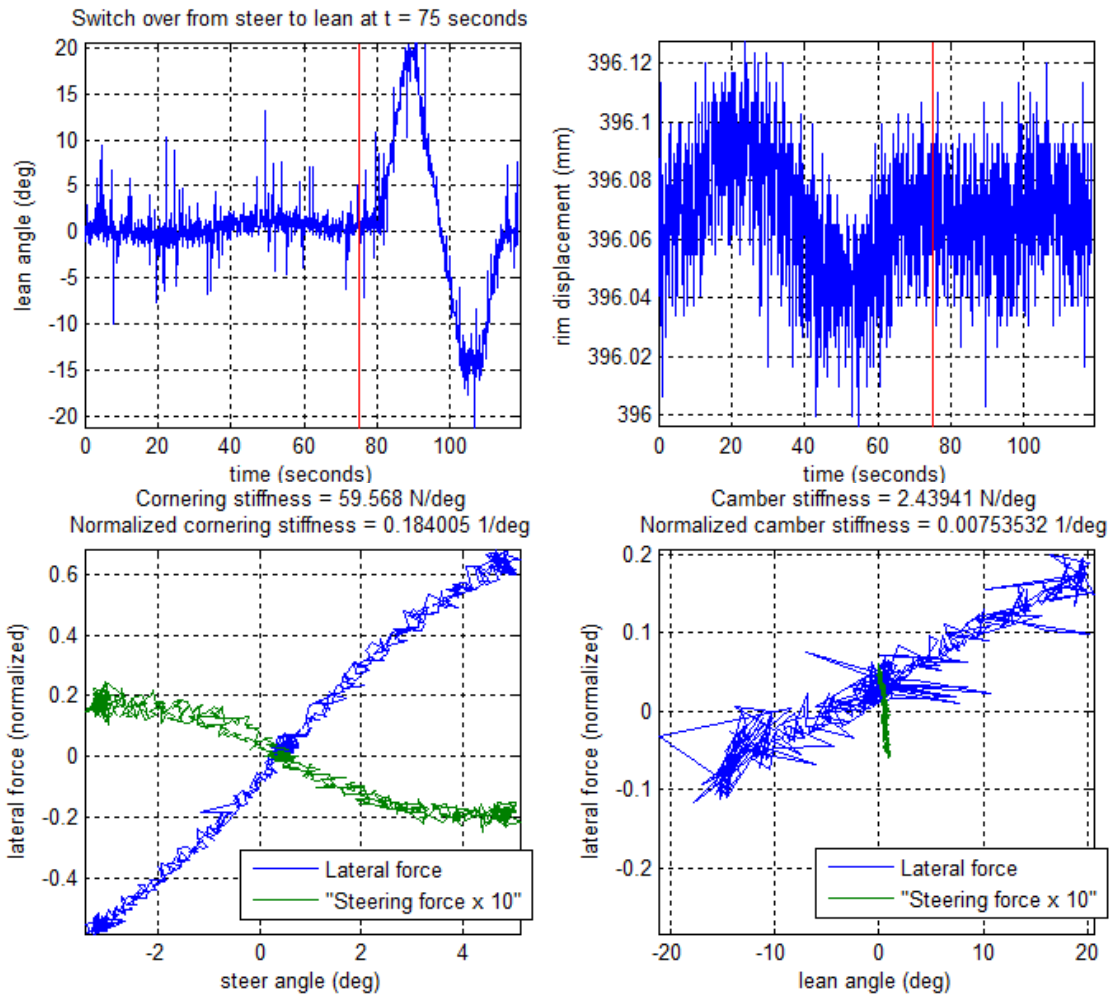
Figure 3-23: Raw force and displacement data captured from test device number 3.

3.9.6 Advantages and disadvantages of design

The 2.5 meter drum at TU Delft was designed for testing automobile and even truck tires, and so is well suited to this task. Any forward speed that a bicycle tire might reasonably encounter is easily producible. Its curvature of the drum, however, changes the length and width of the contact patch slightly. As with test device number 2, the wheel and frame

could be allowed to flex because actual tire orientation was recorded along with the force data.

Also, the safety precautions required for working with such a powerful system can be limiting. All testing must be performed from an adjacent room, behind a glass window. Close examination of the tire or test device during operation was not possible. The room is not well ventilated, and electronic equipment would sometimes fail when the air temperature rose during testing.

The drum is nearly flush to the floor, so working on the test frame, especially raising and lowering it repeatedly to make contact patch ink prints, was exhausting. The simple, manual control developed for the angle actuators meant that it was difficult to zero one angle exactly while sweeping the other.

3.10   Test data capture and processing

The first two systems, both at UWM, use an integrated system of proprietary sensors and data acquisition hardware and software by PASCO®. The third system, at TU Delft, uses an ad-hoc arrangement of sensors connected to a National Instruments DAQ controlled by LabVIEW.

3.10.1   MATLAB®

In all cases, data from force, displacement, and acceleration sensors was captured on a laptop computer and then manipulated with MATLAB®.

Read data from the *.xls file with a MATLAB® application and extract tire name,

inflation pressure, and vertical load from the file name.

Apply calibration to sensor data.

```
% calibrate lateral force

masses = [0 2 2 2 2 2 2 2 2];

masses = cumsum(masses);

volts = [0 0.2 0.4 0.6 0.79 0.99 1.19 1.39 1.59];

p = polyfit(volts, masses, 1);

f1 = f1*p(1)*9.81;

% + p(2); %zeroed amplifier 'in situ' after calibration

% calibrate lean angle: 0.811 = -12 and %-1.706 = 25.5

% 0.7290 = -(90-77.2) and -1.8390 = +(90-59.3)

p = polyfit([0.7290 -1.839],[-(90-77.2) +(90-59.3)], 1);

a1 = a1*p(1) + p(2);
```

Prompt for time that separates slip angle sweep from camber angle sweep.

Combine appropriate force data with corresponding displacement or angle data.

Plot "raw" data, as seen in a sample image in **Figure 3-23** above.

Fit a first-order polynomial to data near the origin to extract stiffness and offset values.

```
p1 = polyfit(a2s, f1s, 1);

cornering_stiffness = p1(1);

cornering_stiffness_offset = p1(2);

p1 = polyfit(a1s, f1s, 1);

camber_stiffness = p1(1);
```

```
camber_stiffness_offset = p1(2);
```

Smooth data and shift data to so that zero force occurs at zero angle.

Plot smoothed and shifted data along with straight lines to indicate extracted stiffness values.

```
window_size = 100;
sorted_data = sortrows([(a2(1:switch_over_time_idx)) , ...
                        (f1(1:switch_over_time_idx))]);
filtered_data(:,1) = filter(ones(1,window_size)/window_size, ...
                            1, sorted_data(:,1));
filtered_data(:,2) = filter(ones(1,window_size)/window_size, ...
                             1,sorted_data(:,2));
slip_angle = abs(filtered_data(window_size:end,1));
normalized_cornering_force = ...
                abs(filtered_data(window_size:end,2) - ...
                    cornering_stiffness_offset)/w;
plot(slip_angle, normalized_cornering_force, '.-k', ...
    slip_angle_disp_range, ...
    slip_angle_disp_range*cornering_stiffness/w, '-k');


window_size = 50;
sorted_data = sortrows([(a1(switch_over_time_idx:end)), ...
                        (f1(switch_over_time_idx:end))]);
filtered_data(:,1) = filter(ones(1,window_size)/window_size, ...
                            1, sorted_data(:,1));
filtered_data(:,2) = filter(ones(1,window_size)/window_size, ...
                            1, sorted_data(:,2));
```

```
normalized_camber_force = ...
                    abs(filtered_data(window_size:end,2) - ...
                 camber_stiffness_offset)/w;
plot(abs(filtered_data(window_size:end,1)), ...
                    normalized_camber_force, '.-k', ...
                 camber_angle_disp_range, ...
           camber_angle_disp_range*camber_stiffness/w, '-k', ...
           [0 5 10 15 20 25], tand([0 5 10 15 20 25]), '--k' );
```

Perform similar manipulations on data from second force sensor, multiply it by the fixed

moment arm, and plot self-aligning torque and twisting torque. Plot straight line to

indicate stiffnesses found for both forces and both torques. Look for lateral stiffness data

for same tire configuration saved in separate file. Calculate pneumatic trail as simply self-

aligning torque divided by cornering force and plot that. Plots of the same data used in

the image above can be seen in the sample image below.

Self-aligning torque stiffness =
-0.00393 m/° (-1.27 N•m/°)

Twisting torque stiffness =
9.94e-005 m/° (0.0322 N•m/°)

Mean pneumatic trail = 16.7 mm

Static lateral stiffness = 0.084/mm (27.2 N/m
Calculated relaxation length = 126 mm
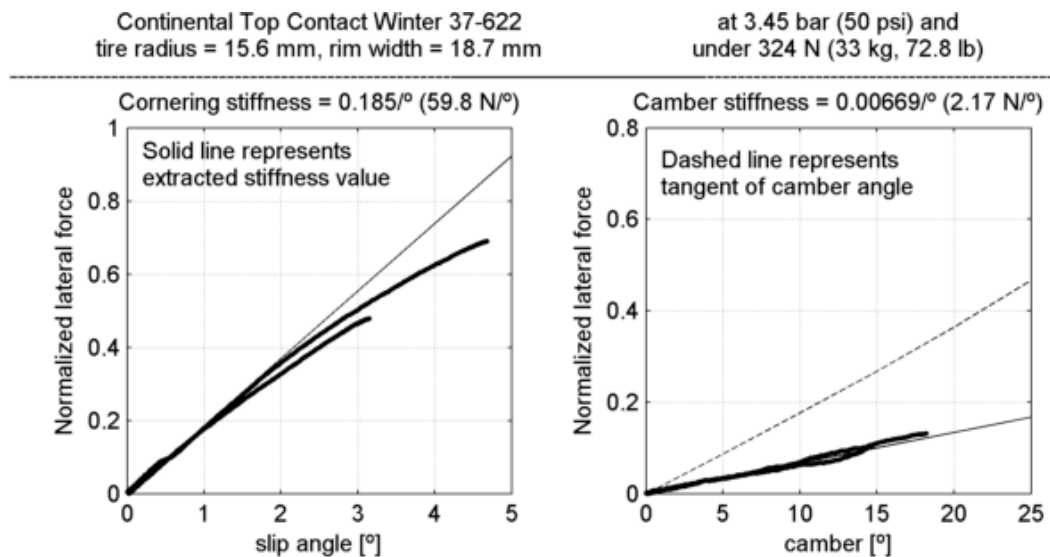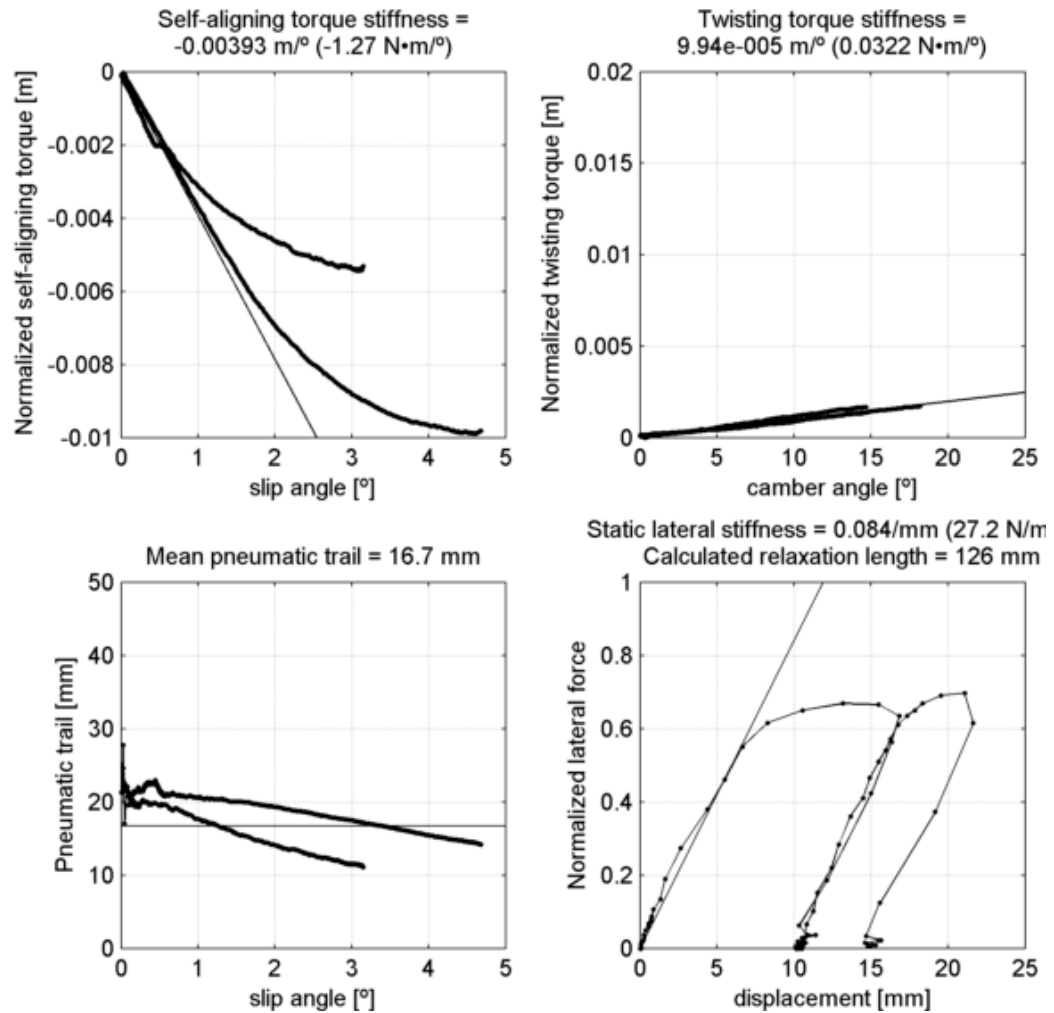
Figure 3-24: Formatted data from test device number 3.

3.10.2   Detection and subtraction of periodic noise in force data

With test device number 1, besides issues with the horizontal and vertical tracks, several

other sources of noise in the data were identified. A fast Fourier transform of the data,

shown below in **Figure 3-25**, reveals periodic signals with periods that closely match the

circumferences of the wheels that run against the vertical track and the support wheel that

runs against the braking surface of the rim. The original inexpensive casters were

replaced with neoprene wheels on ball bearings: off-the-shelf in-line skate and roller shoe

wheels and bearings.
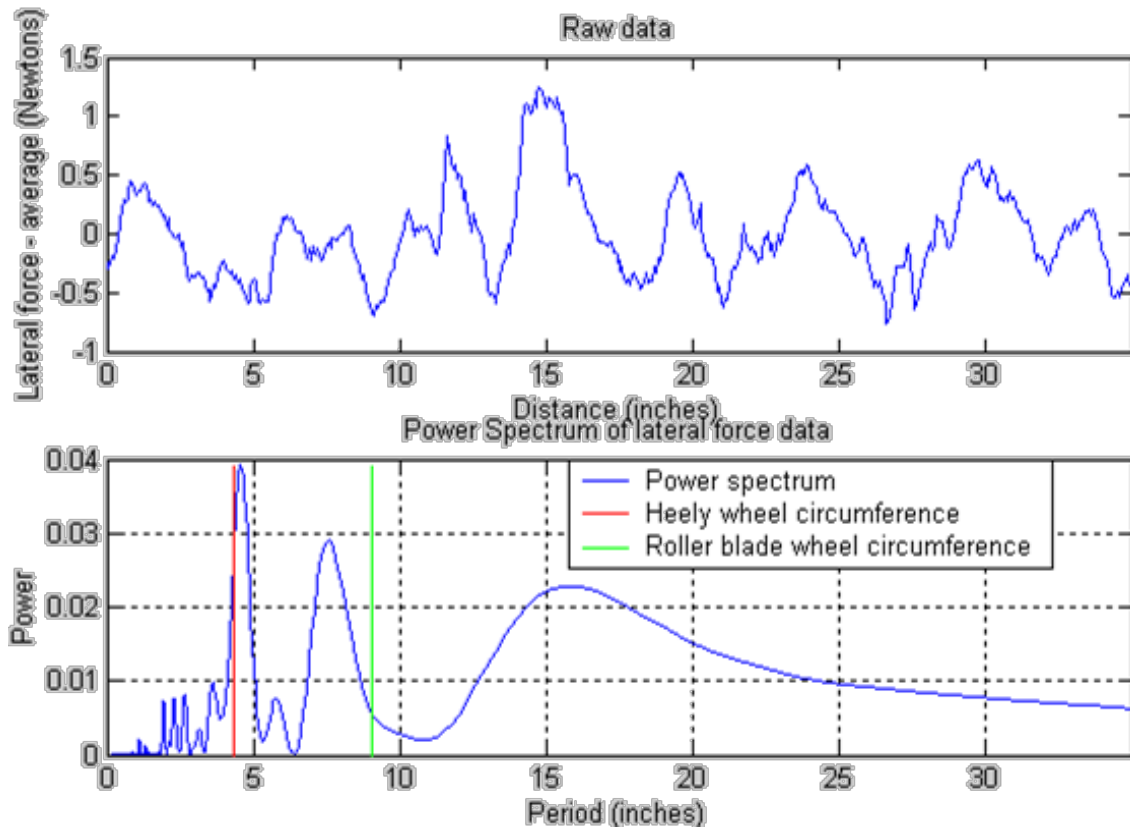
Figure 3-25: Fourier analysis of raw lateral force data showing signal from irregularity

in the Roller shoe wheels and the Inline skate wheel.

For test device number 2, because it was able to collect data over multiple rotations of the

bicycle wheel, the same analysis finds the largest signal has a period equal to the bicycle

wheel circumference, as shown below in **Figure 3-26**.
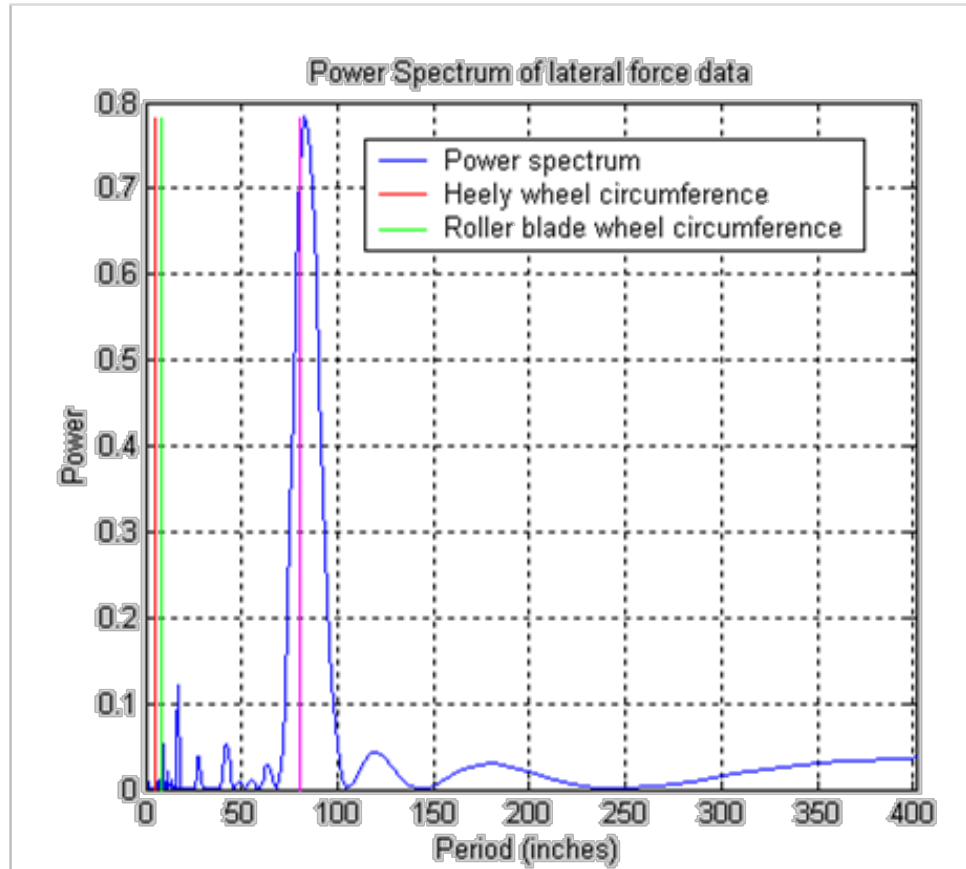
Figure 3-26: Power spectrum of force data collected from test device number 2. The vertical blue line at 83 inches is the circumference of the bicycle wheel with tire.

Once identified, this sinusoidal signal could be subtracted from the force data to smooth it out dramatically, as shown below in **Figure 3-27**.
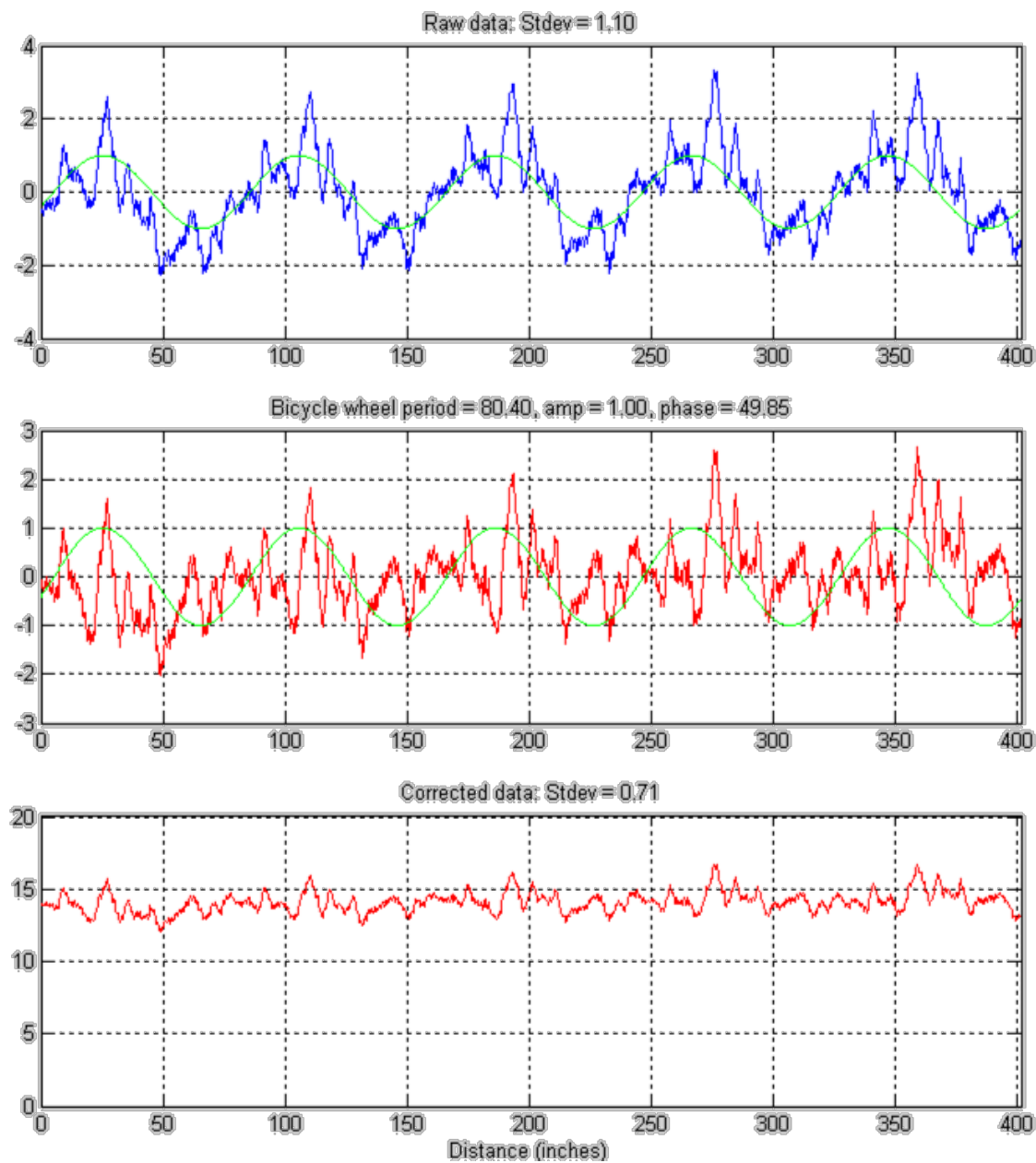
Figure 3-27: Raw data and data corrected by subtracting out the signal with period

equal to the circumference of the bicycle wheel.

3.10.3   Wheel lateral stiffness

A displacement sensor was mounted rigidly to the fork to measure wheel rim deflection relative to the fork.

Published values for wheel stiffness include the maximum, minimum, and average presented below in **Table 3-6**.[40]

Table 3-6: Published wheel lateral stiffnesses

maximum	78 N/mm	2006 Shimano WH-7801 Carbon 50
minimum	27 N/mm	2007 Xentris Mark 1 TT
average	52.7 N/mm	2005/2008 Campagnolo Bora G3 2008 Mavic Cosmic Carbone SL Premium 2007 Mavic Ksyrium Equipe

The measured values are approximately 50 N/mm, close to the published average, which translates to about 0.3685 degrees per 100 N. Thus, a maximum variation in camber angle from the values measured at the fork of less than 1 degree can be expected for the range of forces encountered during testing, up to 250 N. That, in turn, represents only a 0.01 variation in normalized lateral force, based on an approximate average normalized camber stiffness of 0.01 1/°1.

3.10.4   <u>Lateral force sensor misalignment</u>

The original design called for the fixed end of the lateral force sensor to also move on a curved track along with the forward pivot point to maintain a constant geometry during slip angle sweeps, but this was eliminated to simplify and speed up the construction.

As it turns out, the slip angle was usually limited to less than ±3°, so as not to exceed the limitations of the lateral force sensor, and so the distortion of the recorded lateral force is minimized. Even at 5°, the force value recorded is overstated by less than 0.5%

$$error = \frac{1}{\cos(5°)} - 1 = 0.00382 < 0.5\% \qquad \textbf{Eq. 3-5}$$

3.11   Other testing

3.11.1   <u>Instron compression stiffness</u>

Rate-independent hysteresis was observed in bicycle tires similar to that described by Foale in motorcycle tires.[41] The force required to create a given deflection in a tire depends on the history of force and deflection. If deflection is increasing, the required force is higher than if the deflection is decreasing, as shown below in **Figure 3-28**.
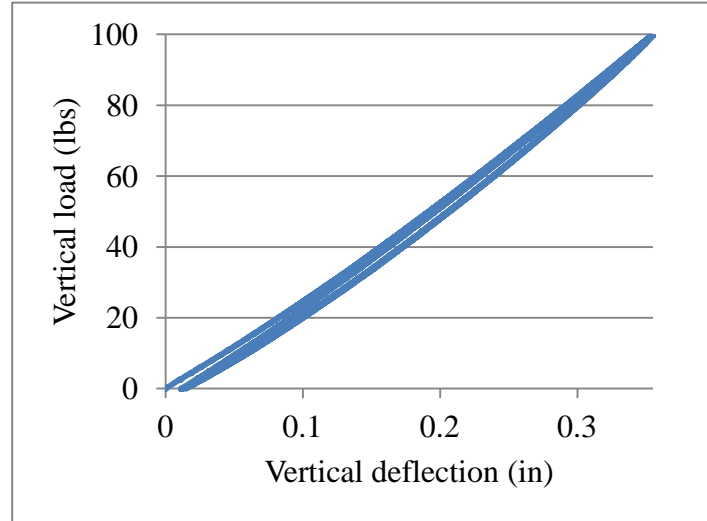
Figure 3-28: Vertical deflection vs. vertical load showing rate-independent hysteresis.

This property of the tires will affect the lateral stiffness measured by simply displacing a tire laterally and measuring the force required to do so.

3.11.2   Rate-dependent lateral stiffness

The lateral stiffness value was found to depend on the rate at which the tire is laterally displaced: rate dependent hysteresis. **Figure 3-29** shows the different stiffness obtained by applying a lateral displacement at varying rates, as shown in **Figure 3-30**.
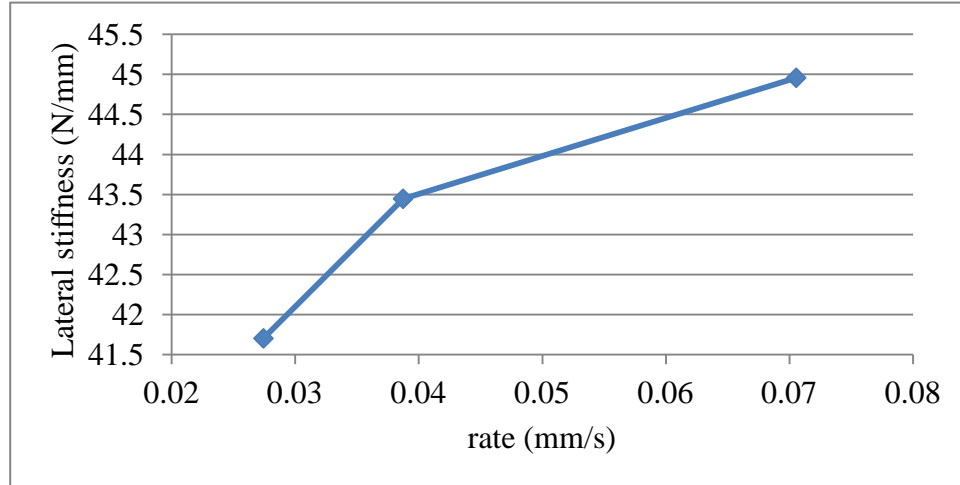
Figure 3-29: Lateral stiffness dependence on lateral displacement rate.

3.11.3 Stress relaxation

Another viscoelastic behavior observed was stress relaxation: stiffness decreases with time.
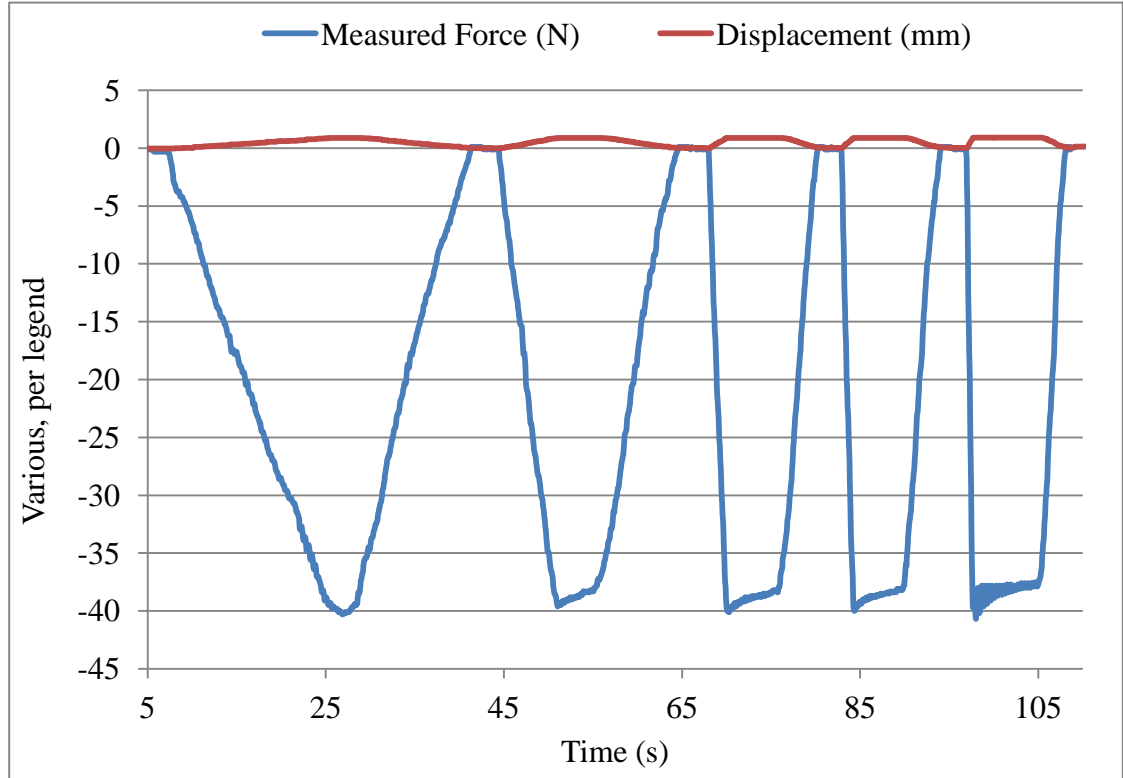
Figure 3-30: Force and displacement vs. time demonstrating stress relaxation.

This behavior, along with the rate dependent hysteresis graphed in **Figure 3-29**, have implications for stiffness testing. Applying a displacement too quickly or slowly will result in a value different from the value that the tire would exhibit when used on a bicycle.

3.11.4   Decay length

The displacement of a tire subjected to a point lateral load was measured to better understand how a tire transitions from being laterally displaced in the contact patch to being undisplaced far from the contact patch.

As shown below in **Figure 3-31**, deflection of 1/20" (half of white square at right, calibrated to scale in middle) appears to be completely gone after 3 inches (white dot just under edge of ruler at about 2.4 on the scale towards the left), so the decay length is 3(1-0.36788) ≈ 1.9 inch.
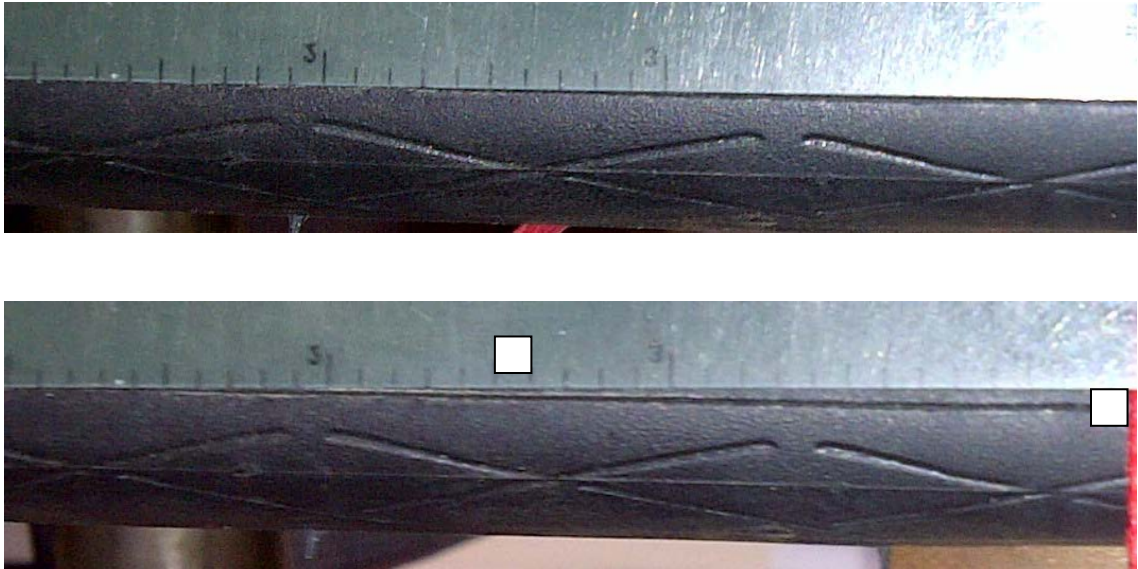
Figure 3-31: Schwalbe Durango 23-622 at 80 psi. Undeflected above, and deflected by a 25 lb localized lateral load (by red cord at far right) below.

As shown below in **Figure 3-32**, deflection of 1/10" (white square at right, calibrated to scale in middle) appears to have reduced to 0.036788 (white square at left) by 3.9 inches.
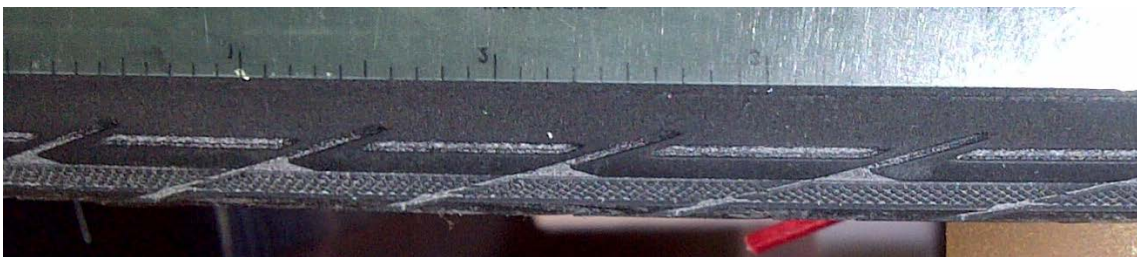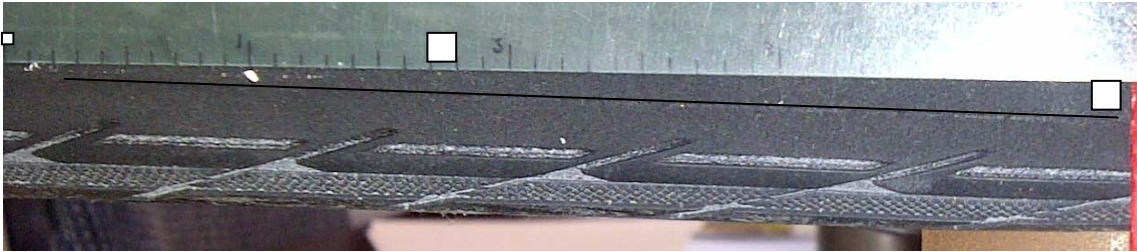
Figure 3-32: Bontrager All Weather 28-622 at 50 psi. Undeflected above, and deflected by a 25 lb localized lateral load (by red cord at far right) below.

3.11.5 Confirming slip angle during camber

Attempts were made to confirm the slip angle that occurs during pure camber, in order to validate or invalidate the "tangent rule", described above in section 2.4. A video recording device was mounted at the rear axle of a bicycle and aimed at the ground. Then the bicycle was rolled straight forward while kept vertical on pavement with a lateral slope. The idea was to measure the motion of the pavement, relative to the rear wheel, by tracking the trajectory of identifiable points on the pavement, in order to measure the side slip angle, as shown below in **Figure 3-33**.

Figure 3-33: Extracting slip angle from sequential video frames.

Unfortunately, uncertainty in attempts to extract the relative velocity of the pavement

from the resulting video was too large to confirm or deny the presence of small slip

angles, on the order of 1°. A better approach might be to use an existing device, such as

the Correvit Non-Contact Optical Sensor, pictured below in **Figure 3-34**, but corrections

would have to be made for camber angle. Such a device was beyond the budget of this

project, however, and attempts to borrow one from the manufacturer were

unsuccessful.[42]

Figure 3-34: Contact-less optical sensor for measuring sideslip angle.

3.12 Test protocols

3.12.1 Instrument warm up and calibration

Instruments were powered on and allowed to warm up for about a half hour before calibration or testing began. Calibration was performed by applying several known, independently measured inputs, forces or displacements, and recorded the reported voltages. These value pairs were then fitted to a first order polynomial with the polyfit() function in MATLAB®, and the resulting two constants, a slope and an offset, are used in MATLAB® to interpret the recorded values.

3.12.2 Tire break in and warm up

Tires were run on the drum at varying orientations for several minutes after being mounted on the rim to ensure that they were seated on the rim fully and to scuff off any die-release agent left over from the manufacturing process.

3.12.3   Forward rolling rate

Since testing has shown that the horizontal force generated in the contact patch decreases with time, see section 3.11.3, it is necessary to roll the wheel at a forward rate sufficient to minimize this effect.

3.12.4   Test device angle actuation rate

Since horizontal force due to side slip takes a finite time to develop,[16] it is necessary, in the case of continuous force and angle measurement, to modify the slip angle slowly enough, relative to the forward rolling rate, so that an accurate measure of the horizontal force can be taken.

3.12.5   Operating temperature

All tests were performed at "room temperature".

3.12.6   Steps for test device number 3

1   Inflate tire to desired pressure.

2   Attach desired vertical load to test frame.

3   Lift frame and place wheel on ink pad.

4   Lift frame and place inked tire on sheet of paper on top of the drum.

5   Lift frame and place wheel on ink pad.

6   Lift frame and place inked tire on sheet of paper on top of a flat plate.

7   Lift frame and place wheel on strip of non-skid tape on drum.

8   Detach lateral force sensor from fixed mounting point.

9   Start recording sensor data with LabVIEW application.

10 Pull on far end of lateral force sensor horizontally to create lateral deflection in tire.

11 Stop recording data with LabVIEW application.

12 Save data recorded by LabVIEW into an *.xls file whose name encodes the tire name, the inflation pressure, the vertical load, and drum rotation rate of zero.

13 Reattach lateral force sensor to fixed mounting point.

14 Start drum rotation.

15 Start recording sensor data with LabVIEW application.

16 Sweep orientation angles, first slip and then camber.

17 Stop recording data with LabVIEW application.

18 Stop drum rotation.

19 Save data recorded by LabVIEW into an *.xls file whose name encodes the tire name, the inflation pressure, the vertical load, and drum rotation rate.

20 Return to step one and repeat with next configuration.

CHAPTER 4

EXPERIMENTAL RESULTS

4.1   Physical measurements

4.1.1   <u>Ink prints of contact patch</u>

The contact patch images were scanned into PDF documents, cut and pasted into JPEG format images, and analyzed in MATLAB® to extract their length, width, and area. First, the image is converted from grey-scale to black-and-white. Then groups of pixels are identified and a convex hull is constructed around them. Finally, an ellipse is fitted to the convex hull, as shown below in **Figure 4-1** and **Figure 4-2**.
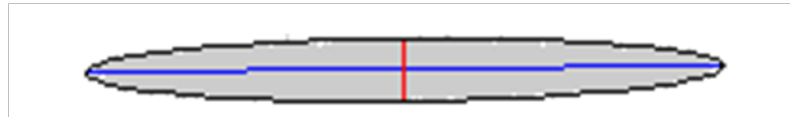
Figure 4-1: Contact patch on the plate analyzed in MATLAB®.

Contact patch on the plate image area = 15479 square pixels (461.7 mm^2), which, when assuming a uniform pressure, yields a pressure of 0.6587 N/mm (95.5% of inflation pressure). A fitted ellipse has a length = 453 pixels (78.2 mm) and width = 43.9 pixels (7.57 mm) for an ellipse area of 15588 pixels (464.9 mm^2), which yields a pressure of 0.6541 N/mm$_2$ (94.9% of inflation pressure).
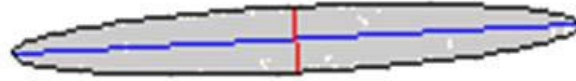
Figure 4-2: Contact patch on the drum analyzed in MATLAB®.

Contact patch on the drum analyzed in MATLAB®: image area = 14252 square pixels (425.1 mm^2), which yields a pressure of 0.7155 N/mm (104% of inflation pressure). A fitted ellipse has a length = 404 pixels (68.8 mm) and width = 45.6 pixels (7.87 mm) for an ellipse area of 14463 pixels (431.3 mm^2), which yields a pressure of 0.705 N/mm^2 (102% of inflation pressure).
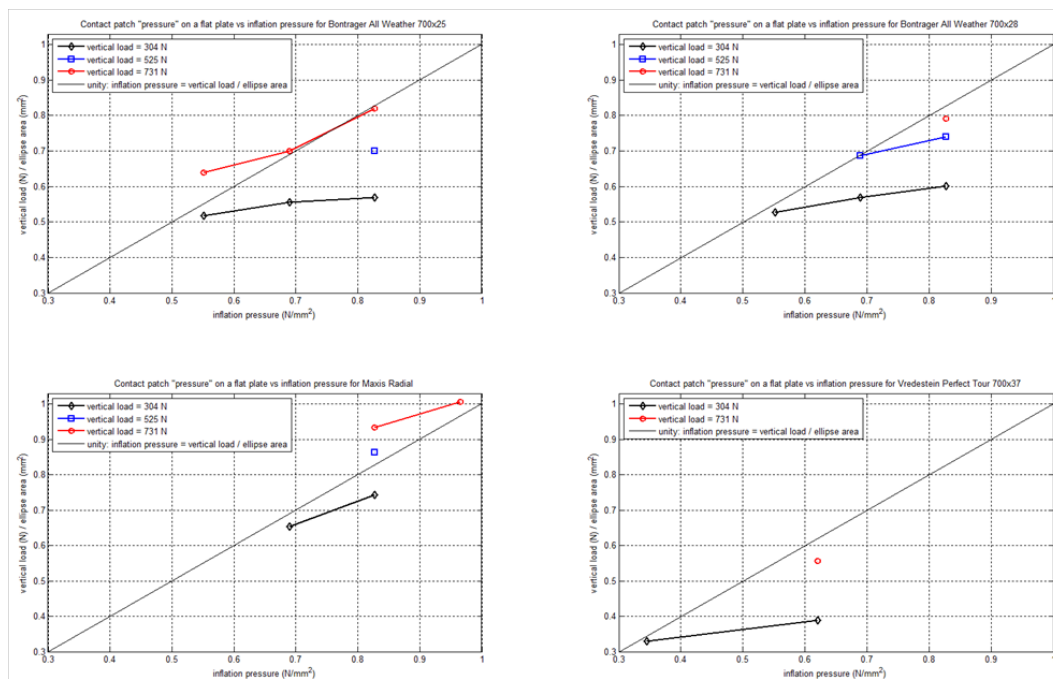
Figure 4-3: Comparison of vertical load divided by measured contact patch area vs. inflation pressure. The assumption that they are equal is represented by the black diagonal line.

4.1.1.1   *Drum vs. Plate*

As expected, the contact patches taken on the drum are slightly shorter and wider than those taken on the plate. The data is summarized below in **Table 4-1**.

Table 4-1: Summary of contact patch length, width, and area for plate and drum.

	Max	Min	Mean	Stdev
drum/plate length	105%	81%	92%	3.1%
drum/plate width	132%	83%	112%	8.5%
drum/plate area	119%	68%	104%	8.2%

4.1.2   Forces and moments

Despite the finite curvature radius of the drum, and the measurable effect it has on contact patch length and width, it is expected to have a negligible effect on forces and moments measured. A Calspan report from 1975 on testing bicycle tires on a 4-foot diameter (1.2192 meter, a little less than half the diameter of the 2.5 meter drum used at TU Delft) explicitly states "Since the bicycle tire contact patch is small, the drum closely approximates a flat surface."[20] This is confirmed by the good agreement between the measurements from test device number 2 at UWM and test device number 3 at TU Delft, described below in section 4.4.2, and by the Rotta model, described below in section 5.3.1.

4.1.3   Lateral stiffness
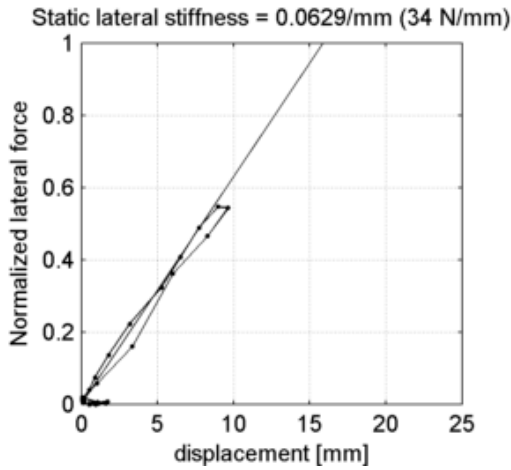
Figure 4-4: Lateral stiffness for Schwalbe Big Apple 55-622 on rim of width 18.7 mm

at 3 bar (44 psi) and under 450 N (55 kg, 121 lb) vertical load.

4.1.4   Cornering stiffness
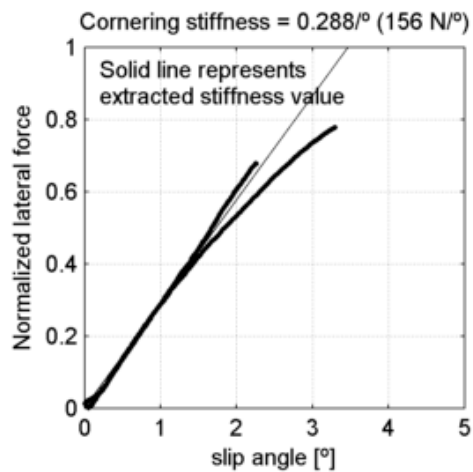
Figure 4-5: Cornering stiffness for Schwalbe Big Apple 55-622 on rim of width 18.7

mm at 3 bar (44 psi) and under 450 N (55 kg, 121 lb) vertical load.

4.1.5   Camber stiffness
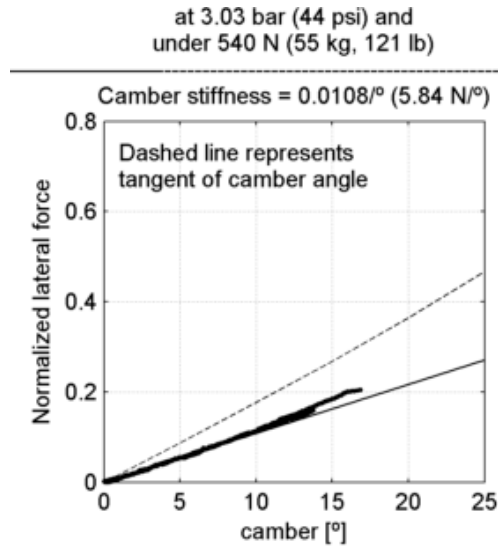
Figure 4-6: Camber stiffness for Schwalbe Big Apple 55-622 on rim of width 18.7 mm

at 3 bar (44 psi) and under 450 N (55 kg, 121 lb) vertical load.

4.1.6   Self-aligning torque
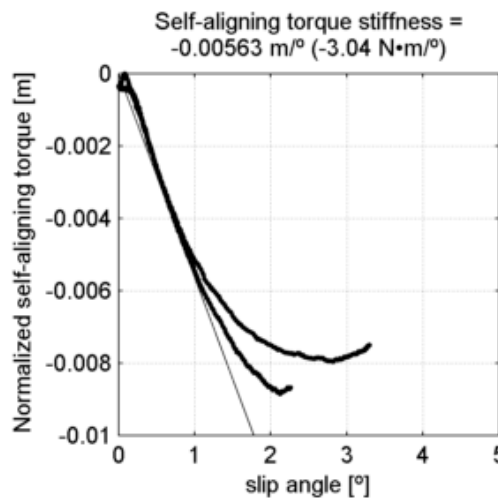
Figure 4-7: Self-aligning torque stiffness for Schwalbe Big Apple 55-622 on rim of

width 18.7 mm at 3 bar (44 psi) and under 450 N (55 kg, 121 lb) vertical load.

4.1.7   Twisting torque
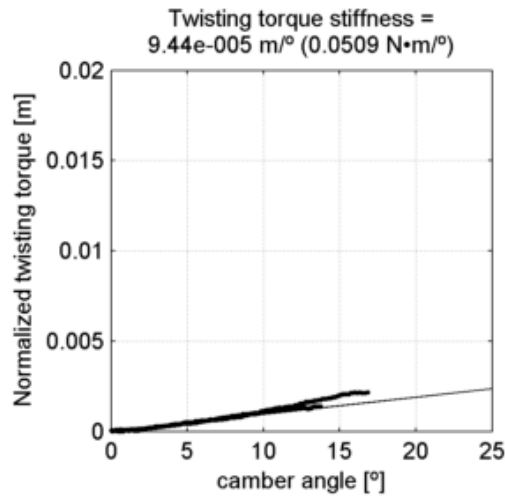
Figure 4-8: Twisting torque stiffness for Schwalbe Big Apple 55-622 on rim of width

18.7 mm at 3 bar (44 psi) and under 450 N (55 kg, 121 lb) vertical load.

4.1.8   Pneumatic trail

Calculate pneumatic trail from experimentally measured cornering force and self-aligning

torque via Pacejka's formula 1-87:[2]

$$t(\alpha) = \frac{M_z(\alpha)}{F_y(\alpha)}$$ **Eq. 4-1**

The calculated value varies with slip angle due to the variation in cornering force and

self-aligning torque, and so an arithmetic mean is calculated and reported.
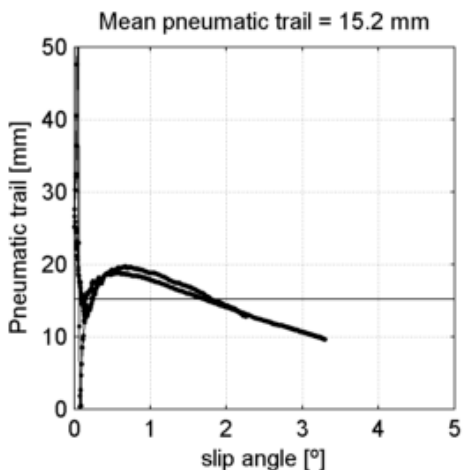
Figure 4-9: Pneumatic trail for Schwalbe Big Apple 55-622 on rim of width 18.7 mm

at 3 bar (44 psi) and under 450 N (55 kg, 121 lb) vertical load.

The 15.2 mm mean pneumatic trail reported in **Figure 4-9** is 14.76% of the 103 mm

contact patch length on the drum. The average ratio of mean pneumatic trail over contact

patch length for all tires and configurations tested was 12.76%.
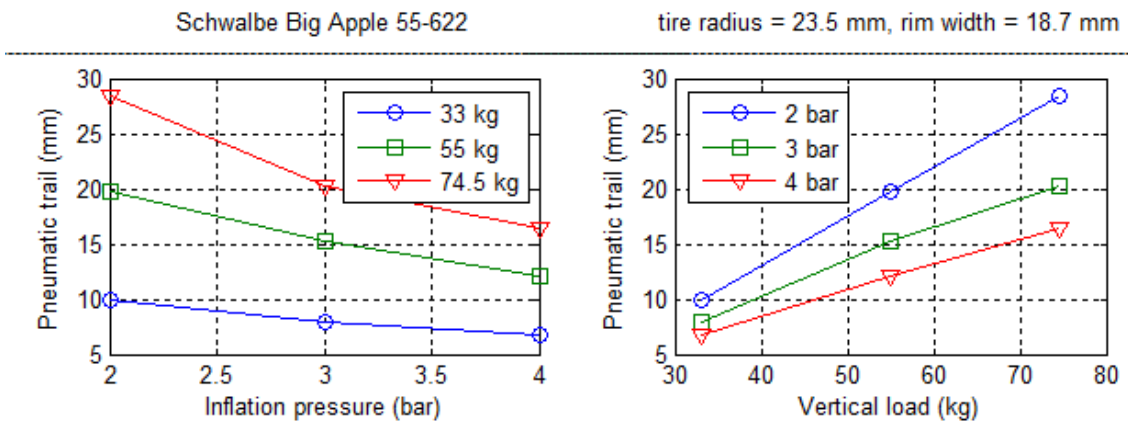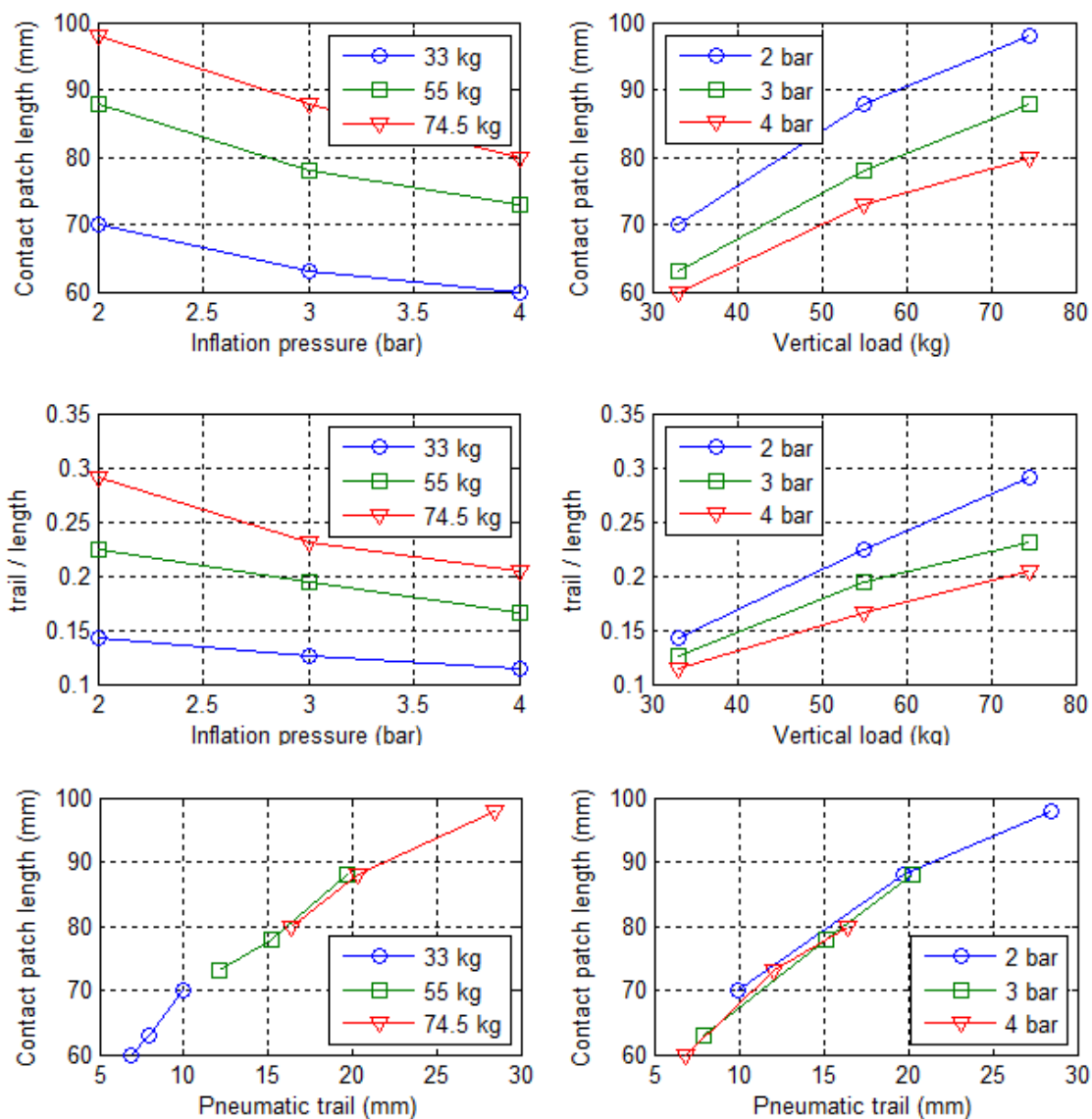
Figure 4-10: Summary of pneumatic trail for all Schwalbe Big Apple tests.

4.1.9 Relaxation length, decay length, and relaxation distance

Pacejka defines "relaxation length" as "an important parameter that controls the lag of the response of the side force to the input slip angle." He also uses the term to indicate the length of laterally deflected tire beyond the end of the contact patch: "The length σ,

designated as the relaxation length." For a "non-steady-state out-of-plane string-based" tire model, he calculates it in his equation 5.5 as [2]

$$\sigma = \sqrt{\frac{S}{c_c}}$$

Eq. 4-2

where c_c is the lateral carcass stiffness per unit length, and S is the circumferential component of the total tension force.

He also provides another expression (7.6) for calculating it in "single contact point transient" tire models. [2]

$$\sigma_\alpha = \frac{C_{F\alpha}}{C_{Fy}}$$

Eq. 4-3

The values calculated from the collected data agree with the rules of thumb Pacejka provides: "of the order of magnitude of the wheel radius" and "approximately equal to half the contact length of the tyre."[2]

It is not clear, however, that these two different parameters, the distance a tire rolls after a step change in slip angle before the side force reaches 63% of its steady-state, and the length along the tire beyond the end of the contact patch that is deflected laterally, are equivalent.

Therefore, in this document, "decay length", ℓ, will be used exclusively to mean the along the tire beyond the end of the contact patch that is deflected laterally, and "relaxation distance", κ, will be used exclusively to mean the distance a tire rolls after a step change in slip angle before the side force reaches 63% of its steady-state.

4.1.9.1   *Decay length*

Pacejka develops a decay length, given in **Eq. 4-2** above, for the string model of a tire.

The Rotta model for the stiffness of a single cross section has been combined with the model of a beam on an elastic foundation to describe the deflection of the centerline of the tire beyond the ends of the contact patch.

The fourth-order differential equation of a beam on an elastic foundation has a characteristic length $\dfrac{1}{\ell} = \sqrt[4]{\dfrac{k}{4EI}}$, where k is the stiffness of the foundation, E is the modulus of elasticity of the beam material, and I is the area moment of inertia of the beam cross section.

As can be seen in **Figure 4-11**below, the lateral deflection looks as might be expected from a non-zero slip angle, but is oddly S-shaped when the tire is not generating a side force.
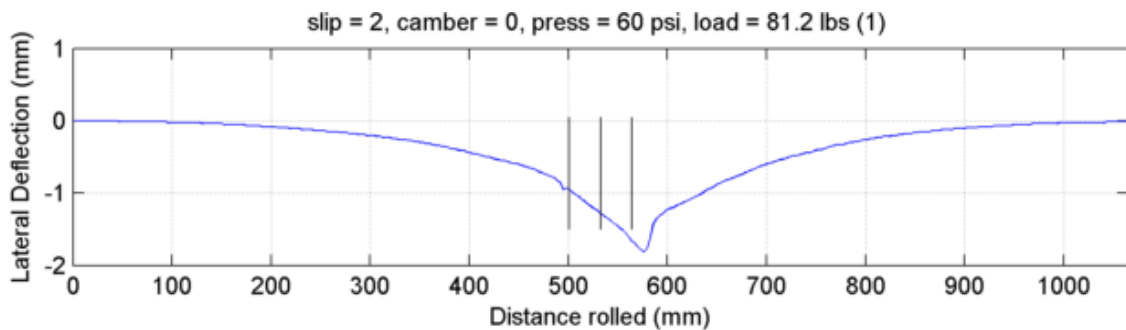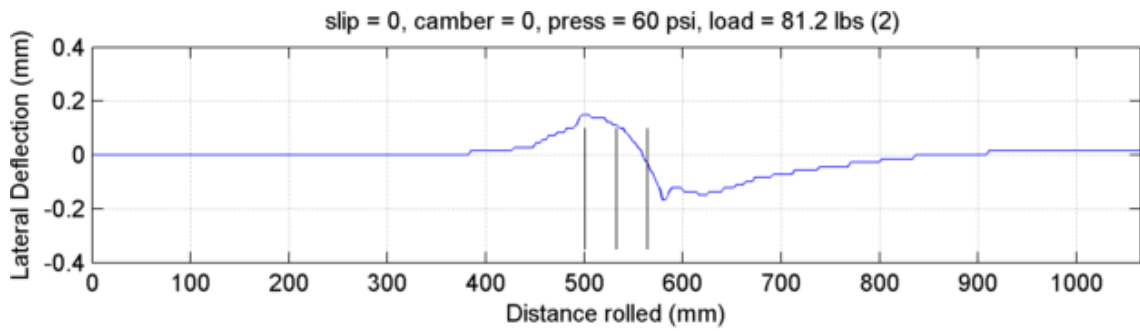
Figure 4-11: Recorded tire centerline lateral deflection non-zero and zero slip angles.

A wire-spoked wheel, however, under a vertical load experiences a vertical deflection of the rim over the contact patch, it will also deflect laterally "away from the spoke where the load is applied."[39] To confirm that this effect is contributing to the tire centerline lateral deflection data, the same tests were performed with the same tire mounted on a carbon tri-spoke wheel by HED Cycling, as pictured in **Figure 3-15**.
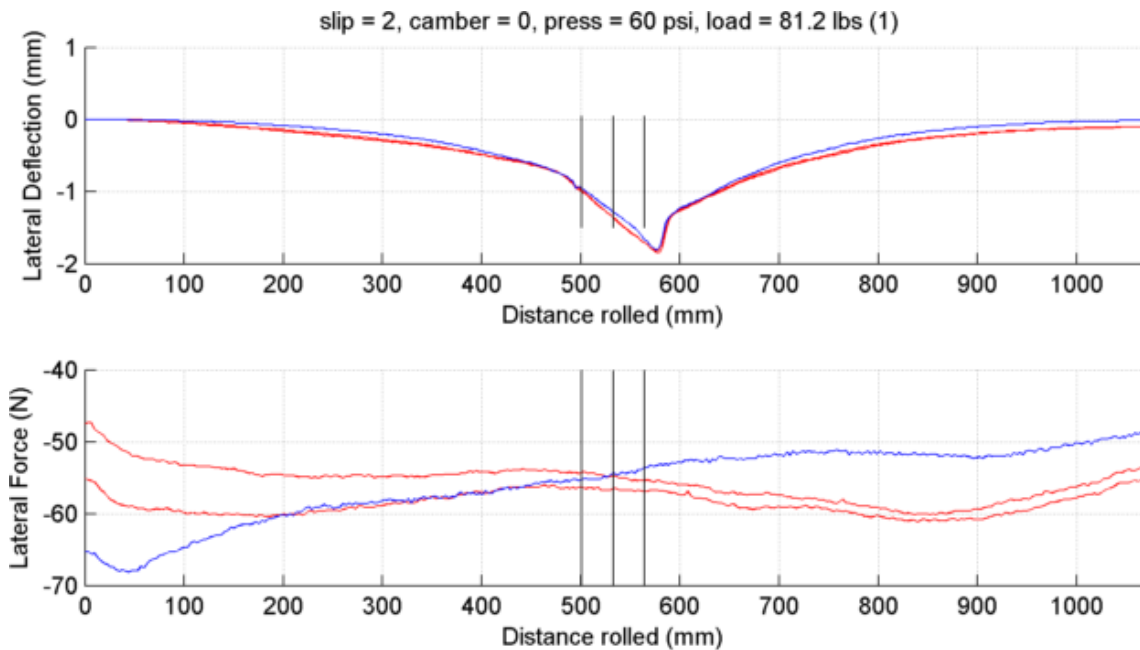
Figure 4-12: Recorded tire centerline lateral deflection for non-zero slip angle.

It appears, in **Figure 4-12** above, that the spoke arrangement to support the rim has negligible influence on tire centerline displacement during non-zero slip angles.
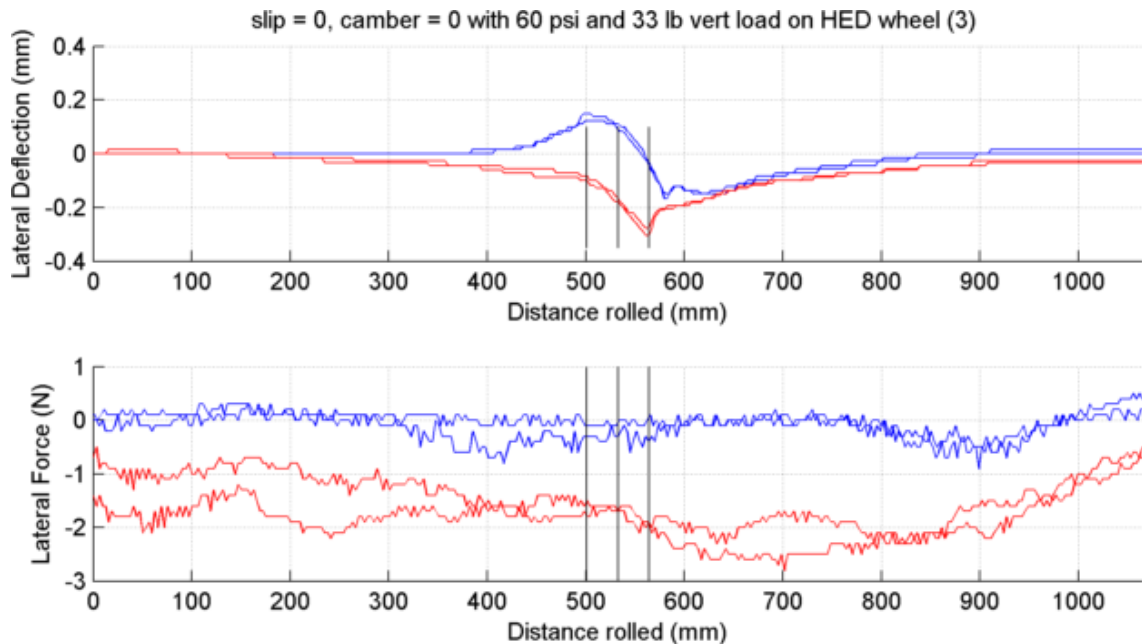
Figure 4-13: Recorded tire centerline lateral deflection for zero camber and slip angles.

In **Figure 4-13** above, the crude S-shape seen in the deflection of the wire-spoked rim (in blue) is completely absent in the deflection of the tri-spoked wheel (in red), and the remaining small deflection (0.3 mm ≈ 0.012 in) is likely due to some other deformation of the now-unsupported rim or due to the widening tire pressing the sensor string onto the non-skid tape.

The data presented below in **Figure 4-14** confirm that the small, ~0.1 mm, lateral displacement of the centerline actually occurs, is asymmetric, and is not just some artifact of the measuring technique. Instead, it is likely due to slightly non-zero camber or slip angles, even tire or rim non-uniformity, or some combination of these conditions.
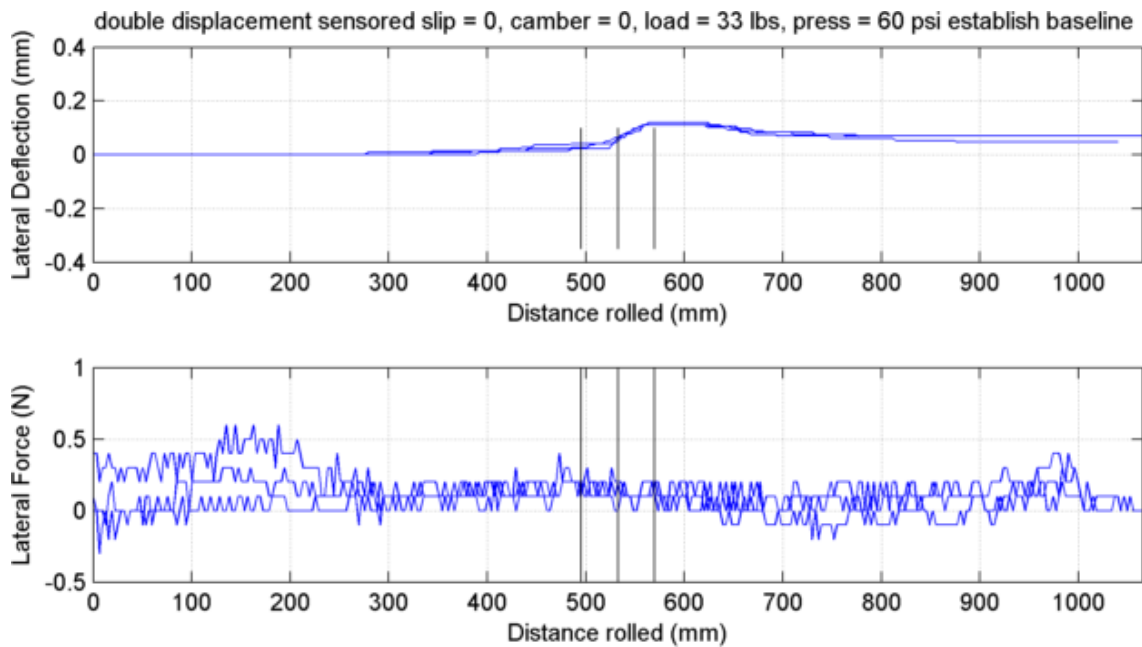
Figure 4-14: Tire centerline lateral displacement with zero slip and zero camber angles.

The sensors mounted on the wheel to measure lateral displacement of the tire centerline prevent the use of the pair of sensors mounted to the track and that measure the displacement of the rim, which are used to measure slip angle. Instead, zero slip angle was set for these tests by adjusting the track orientation until the measured lateral force generated by the tire was zero. It is possible that modifications to the test apparatus fork assembly, which were necessary for it to accept the wider hub of the rear tri-spoke wheel, could have caused the wheel to be slightly out of vertical alignment with the fork, and zero camber angle is set with a level mounted to one of the fork blades. Thus, even though the measured lateral force was zero, the camber angle might have been slightly non-zero, and that was countered by a slightly non-zero slip angle in the other direction.

Finally, the fact that slip angle has a larger effect on centerline lateral displacement could explain the small lateral displacement measured when the measured lateral force was zero.

Figure 4-15: Tire centerline lateral displacement with non-zero slip angle.

The long decay length, ~3 times the length of the contact patch, can clearly be seen before and after the nearly straight track through the contact patch in **Figure 4-15** above. Three separate test runs are superposed to show excellent repeatability of experiment.
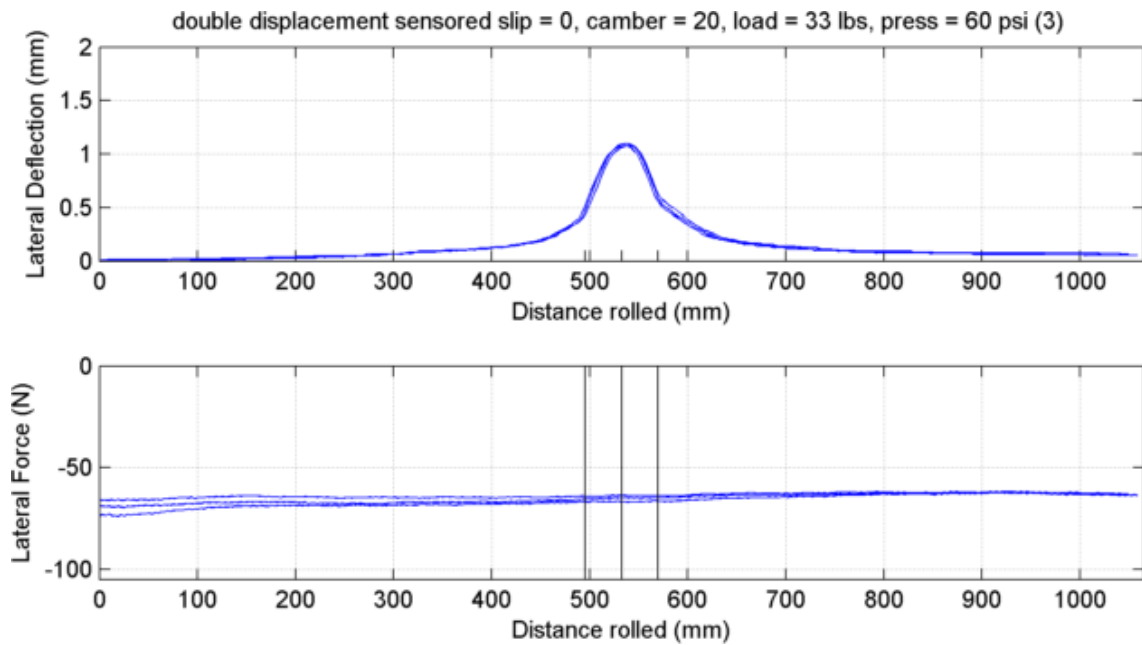
Figure 4-16: Tire centerline lateral displacement with non-zero camber angle.

The curved trajectory of the tire centerline through the contact patch of a cambered tire can clearly be seen in **Figure 4-16** above. The smaller lateral displacement required by the tire in camber to produce a larger lateral force than the tire in slip can be seen by comparing **Figure 4-16** with **Figure 4-15**.

Finally, the assumption that force and displacement due individually to non-zero camber and non-zero slip angles can be superposed to yield a good approximation of the combined effects of both is confirmed in **Figure 4-17** below. The slight misalignment of the peak lateral displacement is most likely due to experimental error: not starting each test run with identical wheel orientations.
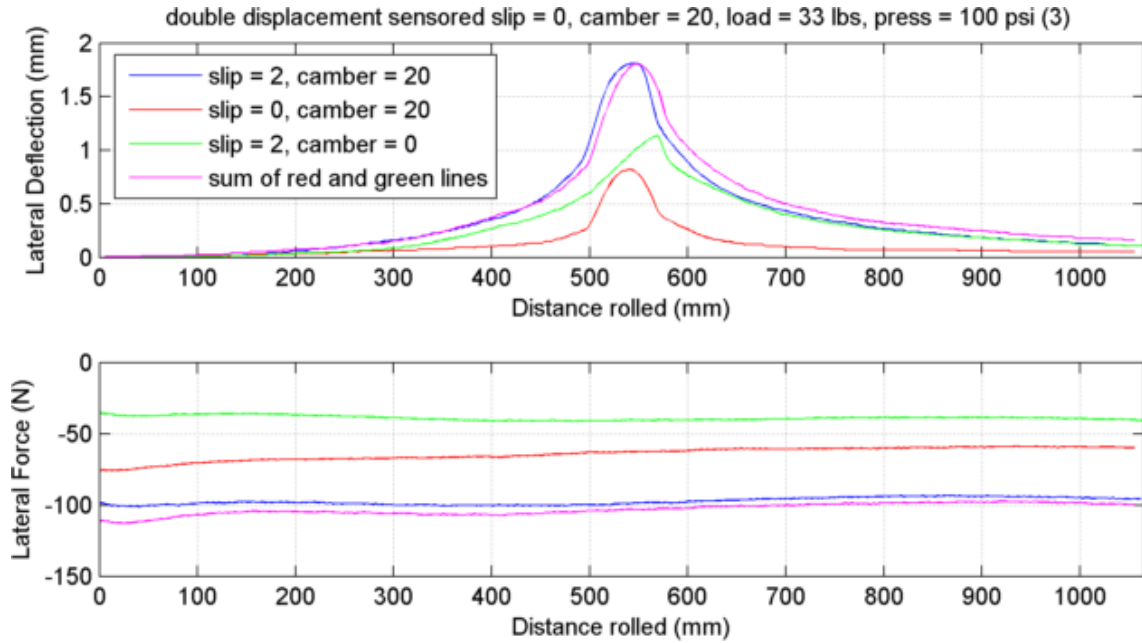
Figure 4-17: Tire centerline lateral displacement with non-zero slip and camber angles.

4.1.9.2 *Relaxation distance*

Pacejka's expression 7.6 for relaxation distance, **Eq. 4-3** above, can be derived from

developing a differential equation that describes the lateral displacement of a tire in terms

of lateral forces and stiffness constants. This derivation will use u for longitudinal

displacement, in the x direction, and v for lateral displacement, in the y direction, after

Pacejka, for example, equation 7.2.

Start with the definition of pure lateral displacement $F_y = vC_{Fy}$ (Pacejka's equation 7.5).

Rearrange slightly and take time derivative of both sides.

$$\frac{dv}{dt} = \frac{1}{C_{Fy}} \frac{dF_y}{dt}$$

Eq. 4-4

Next, combine the definition for pure slip displacement $F_y = \alpha C_{F\alpha}$ (Pacejka's equation

7.3) or $\alpha = \dfrac{F_y}{C_{F\alpha}}$, and the definition of slip angle α, $\tan \alpha = -\dfrac{V_y}{V_x}$ (Pacejka equation 1.3).

For small angles α, $\tan \alpha = \alpha$ so $\alpha = -\dfrac{V_y}{V_x} \Rightarrow \alpha V_x = -V_y$, which can be written as

$\dfrac{du}{dt}\alpha = -\dfrac{dv}{dt}$. Combine this last expression with the expression for pure slip to eliminate

α: $\dfrac{du}{dt}\dfrac{F_y}{C_{F\alpha}} = -\dfrac{dv}{dt}$.

$$\frac{dv}{dt} = -\frac{du}{dt}\frac{F_y}{C_{F\alpha}} \qquad \textbf{Eq. 4-5}$$

Then, combine lateral displacement, **Eq. 4-4**, with slip displacement, **Eq. 4-5**, to

eliminate $\dfrac{dv}{dt}$ and time altogether:

$$\frac{1}{C_{Fy}}\frac{dF_y}{dt} = -\frac{du}{dt}\frac{F_y}{C_{F\alpha}} \Rightarrow \frac{dt}{du}\frac{dF_y}{dt} = -\frac{C_{Fy}}{C_{F\alpha}}F_y \Rightarrow \frac{dF_y}{du} = -\frac{C_{Fy}}{C_{F\alpha}}F_y \qquad \textbf{Eq. 4-6}$$

Rewrite **Eq. 4-6** as a homogeneous first-order, ordinary differential equation, which

describes how lateral force varies with distance rolled.

$$\frac{dF_y}{du} + \frac{C_{Fy}}{C_{F\alpha}}F_y = 0 \qquad \textbf{Eq. 4-7}$$

Add a forcing function to **Eq. 4-7**, such as $A \cdot H(u)$, a constant lateral force A applied to the tire by the rim, for example, by a change in slip angle such that $A = \alpha C_{F\alpha}$, multiplied by the Heaviside step function $H(u)$, and let $\kappa = \dfrac{C_{F\alpha}}{C_{Fy}}$:

$$\frac{dF_y}{du} + \frac{1}{\kappa} F_y = \frac{A}{\kappa} \cdot H(u) \qquad \text{Eq. 4-8}$$

Eq. 4-8 has the form of a first order linear time-invariant system differential equation, but is in terms of forward distance rolled, instead of time. It has solutions, by Laplace transform, for example, of the form:[43]

$$F_y(u) = F_y(0)^{-u/\kappa} + A\left(1 - e^{-u/\kappa}\right) \qquad \text{Eq. 4-9}$$

If the initial force value is allowed to go to zero, $F_y(0) = 0$, then at the forward distance

$$u = \kappa = \frac{C_{F\alpha}}{C_{Fy}} \qquad \text{Eq. 4-10}$$

the lateral force generated by the tire against the pavement $F_y(\kappa) = A\left(1 - e^{-1}\right) = 0.632A$, or about 63% of its steady-state value, the input lateral force applied to the tire by the rim. The constant in **Eq. 4-10** has units of length, and is called the *relaxation distance*.

4.1.10   Summary

In summary, notable results from the physical experimentation include:

1. *Cornering and camber stiffness* depend on lateral displacement of tire centerline both in and beyond the ends of contact patch. This is predicted by the Rotta model when extended into 3D and confirmed by physical testing.

2. *Normalized lateral stiffness* increases as rim width increases for the same tire at the same inflation pressure and under the same vertical load. A 24% increase in rim width was observed to produce a 23% increase in stiffness. Tires marked as the same size vary in arc length by as much as 20%. To correctly characterize the size of a tire, it is necessary to incorporate the rim width and the arc length or carcass circumference.

3. *Normalized lateral stiffness* decreases as vertical load increases and increases as inflation pressure increases for any given tire on a given rim. It decreases as vertical load divided by inflation pressure, which approximately equals the contact patch area, decreases, for all tires, independent of size.

4. *Normalized cornering stiffness* decreases as vertical load increases and appears to either increase or decrease as inflation pressure increases, depending on the tire and the vertical load.

5. *Normalized camber stiffness increases* as vertical load increases and decreases as inflation pressure increases, but less clearly than either lateral stiffness or cornering stiffness. The values measured tend to be insufficient to produce a net ground reaction force that is in the plane of the wheel and agree with the tangent

rule, but not always. They are very seldom excessive and then only to a small degree.

6. *Correlations between stiffnesses*: As summarized in **Table 4-2**, there is a strong correlation between lateral stiffness and cornering stiffness, that remains even when all tires are considered together. There is a weaker anticorrelation between camber stiffness and both cornering stiffness and lateral stiffness, but this disappears when all tires are considered together.

Table 4-2: Correlations between stiffnesses.

Stiffnesses compared	Cornering and camber	Cornering and lateral	Camber and lateral
average of tire by tire	-0.711	0.922	-0.848
all tires combined	0.052	0.633	0.129

7. *Pneumatic trail* increases as vertical load increases and decreases as inflation pressure increases. Its ratio with contact patch length for a given tire is nearly a constant as vertical load and inflation pressure vary.

8. *Contact patch* area tends to be less than the value of vertical load divided by inflation pressure. The length is 6.7 times the width, on average, with a standard deviation of 1.2, a max of 9.8 times and a minimum of 4.7 times.

9. *Decay length*, the distance beyond the end of the contact patch over which a lateral displacement decreases to $1/e$ or 36.8% of its original value, exists and contributes to the total lateral stiffness, cornering stiffness, and possibly camber stiffness of the tire.

10. *Hysteresis*, both rate-independent and rate-dependent occurs, in both vertical and lateral deflection. Measured force at any given deflection depends both on previous deflection, rate at which current deflection was induced, and time spent at that current deflection.

11. *True tire size*, the actual diameter of the tire when mounted on the rim, correlates well with lateral stiffness, as can be seen in the last row of **Table 4-3** and in the graph in **Figure 3-4**.

Table 4-3: True size and normalized lateral stiffnesses for four different 37-622 tires mounted on the same 18.7 mm wide rim.

ID	Size (mm)	Arc length (mm)	Radius (mm)	"True size" (mm)	Size diff. (%)	max norm lat stiffness	min norm lat stiffness	average norm lat stiffness
5	37	78.0	15.6	31.2	-15.6	0.0979	0.0445	0.0664
10	37	73.5	14.9	29.8	-19.4	0.107	0.0417	0.0681
11	37	90.0	17.5	34.9	-5.6	0.149	0.0517	0.0888
12	37	85.7	16.8	33.6	-9.2	0.149	0.0492	0.0679
						0.896	0.9999	0.736
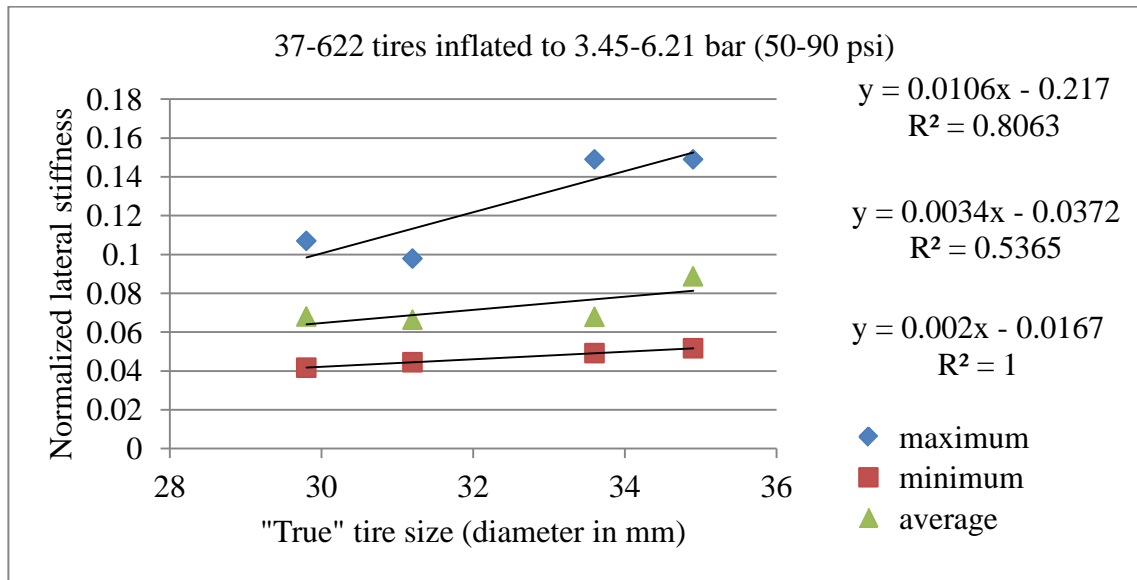
Figure 4-18: Graph of data from Table 4-3.

4.2 Repeatability

Test device number 3 provides good repeatability, except for the twisting torque, a very

small value. For example, the Bontrager All Weather 23-622 measured twice at 5.52 bar

(80 psi) and with a 304 N (31 kg, 68.3 lb) vertical load yields the results shown below in

Table 4-4.

Table 4-4: Example of repeatability of results from test device number 3.

Property	Test #1	Test #2	% Difference
Normalized cornering stiffness [1/°]	0.259	0.260	-0.39
Normalized camber stiffness [1/°]	0.0120	0.0119	0.84
Normalized self-aligning torque [m/°]	-0.00309	-0.00312	-0.97
Normalized twisting torque [m/°]	3.77E-05	8.91E-05	-81.07
Pneumatic trail [mm]	5.33	5.11	4.21

4.3 Tire described as "radial" does not stand out

The Maxxis prototype, which is described as "radial" and does appear to have cords aligned at about 22.5º, instead of the 45º usually associated with bias-ply, does not have stiffnesses noticeably different from other tires of similar size.

4.4 Physical measurements compared between three test devices

To a certain extent, the three different test devices represent an evolution in testing technique over two years, especially in the rate at which testing can be performed. Therefore, there is little overlap in the data collected. Never-the-less, in the cases where overlap exists, good agreement can be found.

4.4.1 TU Delft drum vs. UWM cart

Because of how testing protocols evolved from test device number 1 to test device number 3, no tire ended up being tested under the same configuration on both devices. Two tires, however, were tested under similar conditions so that a comparison can be made.

Table 4-5: Comparing results from test device 1 at UWM[37] and test device 3 at TU Delft.

Tire	Bontrager RL All Weather 23-622			Maxxis Radial Prototype 22-622		
Test device	1	3	3	1	3	3
Load (N)	402.6	303.8	524.9	378.1	303.8	524.9
Inflation pressure (bar)	6.895	6.895	8.274	6.895	6.895	9.653

| Normalized cornering stiffness | 0.198 | 0.252 | 0.193 | 0.212 | 0.217 | 0.174 |
| Normalized camber stiffness (1/°) | 0.0138 | 0.0117 | 0.0135 | 0.0168 | 0.0158 | 0.0168 |

As can be seen in **Table 4-5** above, both devices yield similar stiffnesses for the two tires. Notably, camber stiffness approaches but does not exceed the tangent rule value of 0.0172 1/°.

4.4.2 TU Delft drum vs. UWM flat track

As can be seen in the plots below in **Figure 4-19**, the few data points collected for the Maxxis 22-622 radial tire at UWM fit well with the data points collect at TU Delft for the same tire. The sample size is small, but it was all that time permitted. Unfortunately, the UWM data was collected on different rims and before the dependence on rim width was discovered, so that parameter was not recorded, although all the rims are still available and can still be measured.
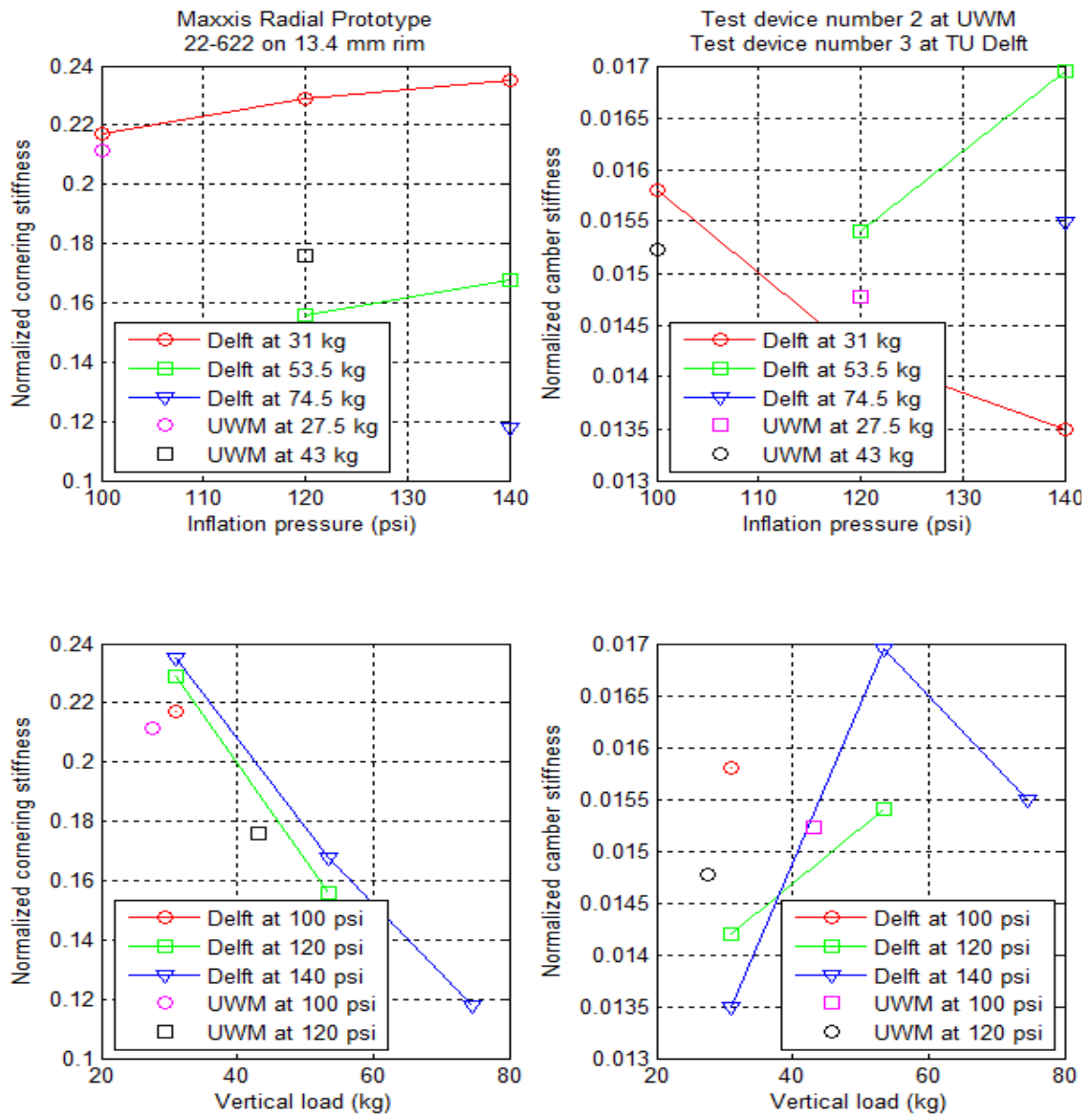
Figure 4-19: Comparing results from test device 2 at UWM and test device 3 at TU Delft.

4.5 Physical measurements compared to previous results

For cornering stiffness, there is reasonable agreement between the several sources, as can be seen in **Figure 4-20**.
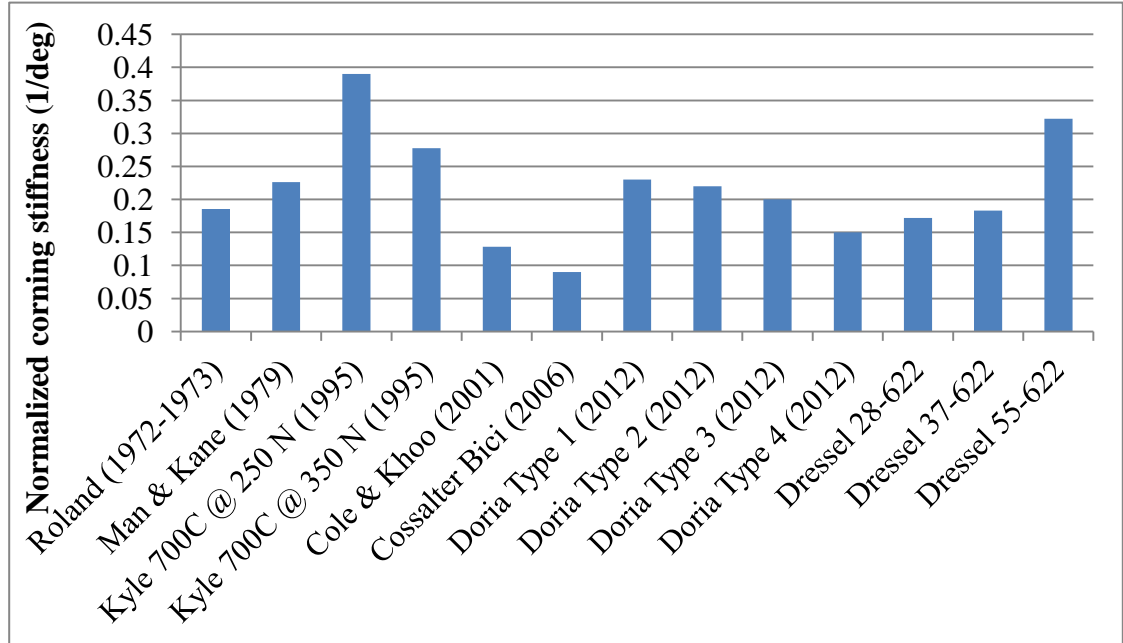
Figure 4-20: Comparison of all known bicycle tire normalized cornering stiffnesses at 1 degree slip angle.

For camber stiffness, there is much greater disagreement between the several sources, as can be seen in **Figure 3-4**. In particular, the values from Kyle stand out for far exceeding a friction coefficient of 1. Roland and Man and Kane fall far below the tangent rule, and Doria *et al.* exceed the tangent rule by as much as 45%.
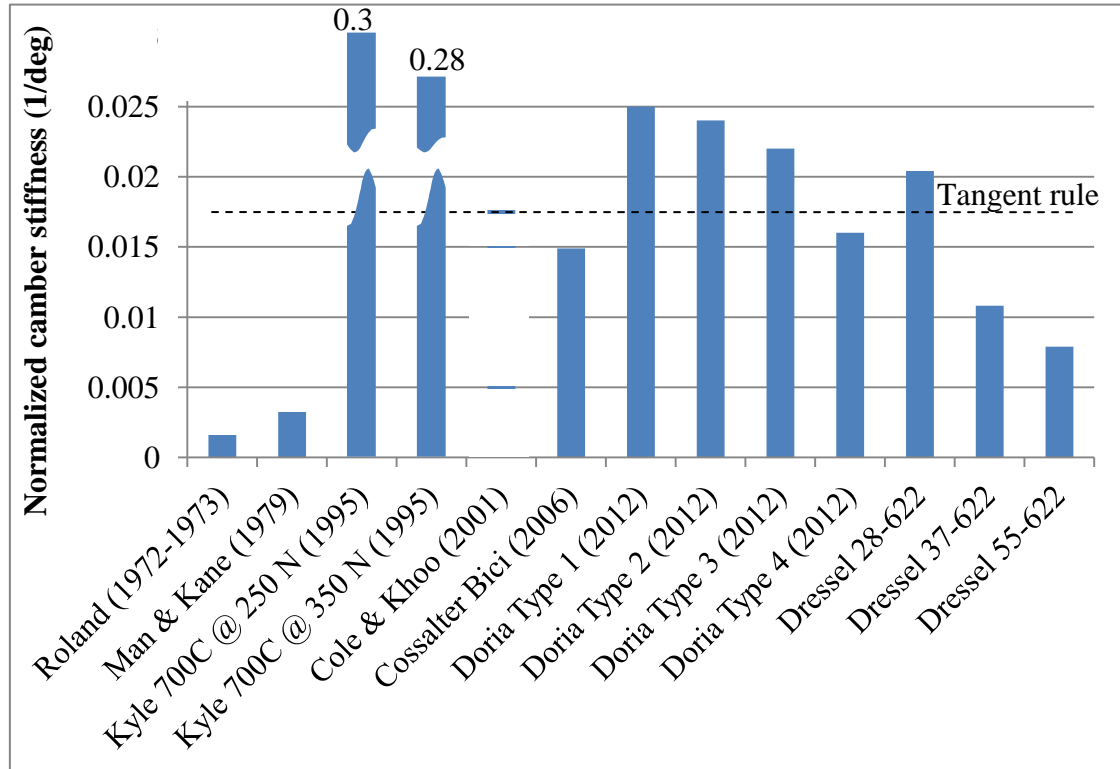
Figure 4-21: Comparison of all known bicycle tire normalized camber stiffnesses at 10 degree camber angle.

4.5.1 Doria *et al.*, 2012

Because the results reported recently by Doria *et al.* have so many details, [28] a more-detailed comparison can be made, as seen below in **Table 4-6**. In general, for similar tires, there is good agreement between the cornering stiffness values, but the camber stiffnesses measured at Padua are much higher than those found at TU Delft. A more-detail discussion of the differences is presented below in section 7.2.

Table 4-6: Comparison of stiffness found by Doria *et al.* at Padua with tires tested on device number 3 at TU Delft.

	Tire number	Tire name	Tire "size"	Vertical load [kg]	Inflation pressure [psi]	Normalized Cornering Stiffness [1/°] at 1°	Difference between Padua and Delft values	Normalized Camber Stiffness [1/°] at 10°	Difference between values and tangent rule
at Padua	1	unknown	37	41	58	0.247		0.0242	37%
	2	Vredestein Perfect Tour	37	41	58	0.220	-18%	0.0232	31%
	3	Schwalbe Marathon Plus	35	41	58	0.200	11%	0.0211	19%
	4	Continental Top Contact Winter	37	41	58	0.147	-11%	0.0153	-13%
		Normalized Camber stiffness of tangent rule						0.0176	
at TU Delft	5	Continental Top Contact Winter	37	33	58	0.183		0.0067	-62%
				44	58	0.165	11%	0.0077	-56%
				55	58	0.146		0.0087	-51%
	9	Schwalbe Kojak	35	33	58	0.315		0.0085	-52%
				44	58	0.270		0.0088	-50%
				55	58	0.224		0.0091	-48%
	10	Schwalbe Marathon Plus	37	33	58	0.197		0.0056	-68%
				44	58	0.179	-11%	0.0064	-64%
				55	58	0.161		0.0072	-59%
	11	Vittoria Randonneur Hyper	37	33	58	0.350		0.0105	-40%
				44	58	0.305		0.0114	-35%
				55	58	0.260		0.0123	-30%
	14	Vredestein Perfect Tour	37	31	60	0.292		0.0079	-55%
				42	60	0.265	18%	0.0087	-51%
				54	60	0.237		0.0094	-47%

The inflation pressure used in Padua is nearly halfway between two of the inflation pressures used in Delft, so a simple average is calculated for comparison in **Table 4-6**.

CHAPTER 5

MODELING

Collecting force and moment values for all interesting combinations of tire, size, inflation pressure, vertical load, and orientation is a time- and labor-intensive operation that requires specialized testing equipment. A model than can predict these values from a few simple static measurements is desirable.

A variety of predictive models have been developed for pneumatic tires in general, and some have been specific to motorcycle tires. Wanatabe describes a motorcycle tire specific finite element model for investigating camber thrust, but it predicts a net ground reaction force orientation of only about a half of his measured data, which in turn is only about 60% of the value predicted by the tangent rule.[44]

The simplest existing tire model available, the original 'brush' model of Fromm and of Julien,[2] was chosen as a place to start, and it was enhanced only as necessary to predict the values found from physical experiments. The Rotta model was used to find the bristle stiffness from just inflation pressure, vertical load, rim width, and tire radius.[45] Finally, the relaxation rate of the tire was accounted for with a model of a taut beam on an elastic foundation. The tension can be found as a function of inflation pressure and cross section area; the area moment of inertia can be found from the cross section radius and casing thickness; the modulus of elasticity can be estimated from published data; and the foundation stiffness can be calculated with Rotta's model.

5.1   Rotta's model

Bicycle tires, by virtue of their relatively thin carcass and relatively high inflation pressure, are expected to behave differently than automobile and even motorcycle tires. They assume a toroidal shape, as do motorcycle tires, but the stiffness of their much-thinner carcass plays a reduced role in their total stiffness.

There is a model developed by Rotta for toroidal tires, specifically aircraft tires, that predicts the forces generated in the contact patch and that might be applicable to bicycles.[45]

Rotta's premise is simply that, in the plane of a cross section that is perpendicular to the midplane of the wheel and collinear with a radius, the two side bulges of a tire that is pressed against a flat surface form constant-radius circular arcs on either side of the contact patch and that those arcs are tangent to the ground plane.

Thus, the total arc length of the carcass, from bead to bead, is divided between a straight line representing the contact patch, and two circular arcs between the rim and the contact patch, which are tangent to the ground at the contact patch. For a given position and angle of the rim relative to the ground contact of the tire centerline, there is no closed-form solution, but the radii of the arcs can be calculated by a simple iteration.

Forces generated in each cross section can be calculated from the air pressure and resulting geometry. Vertical force is just contact patch width times distance to next cross section times inflation pressure. Horizontal force is the difference between the radii of the circular arcs tangent to the ground, times inflation pressure.

Vertical displacement of the wheel may be specified directly or found by iteration to support a given vertical load at a given inflation pressure.

The calculation of the size of these arcs and the corresponding width of the contact patch, can be started with the known constants:

- the width of the rim, w;

- the arc length of the tire from bead to bead, s_0, and

- the distance that the particular cross section is pressed into the surface. If the tire is vertical, the cross section is vertical, and the surface is horizontal, this is the vertical displacement of the wheel, δ_w.

Note that, in this discussion, dimensions for the entire tire have a subscript of 0 and dimensions for the left and right sides have subscripts of 1 and 2, respectively. Also note that Rotta uses a different coordinate system for the cross section of a tire, shown below in **Figure 5-1**, than that for an entire tire described above in **Figure 2-1**. [45]
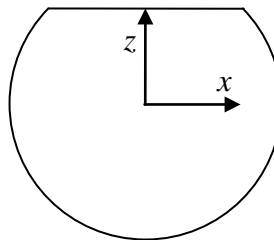
Figure 5-1: Rotta model coordinate system. The y-axis is into the page.

First, a quick detour to calculate the necessary arc length of the undeformed tire from bead to bead, s_0, from the rim width, w, and the undeformed tire radius, r_0:
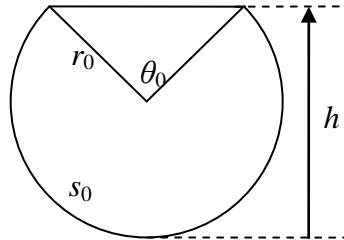
Figure 5-2: Geometric parameters of tire cross section.

$$w = 2r_0 \sin\left(\tfrac{1}{2}\theta_0\right)$$

Eq. 5-1

$$\theta_0 = 2\sin^{-1}\left(\frac{w}{2r_0}\right)$$

Eq. 5-2

One more value that will be needed below is rim height, h, which can be calculated directly:

$$h = r_0 + r_0 \cos\left(\tfrac{1}{2}\theta_0\right) = r_0\left(1 + \cos\left(\tfrac{1}{2}\theta_0\right)\right)$$

Eq. 5-3

For example: a tire with a "size" of 28 mm tire on a 15 mm wide rim

   (assuming that tire size = 2 * tire radius).

Table 5-1: Sample calculations for tire cross section geometry (part 1).

rim_width	given	15 mm

tire_size	given	28 mm
tire_radius	tire_size / 2	14 mm
rim_angle	2 * asin(rim_width / (2 * tire_radius))	1.1307 rad
arc_length	tire_radius * (2*pi -rim_angle)	72.1347 mm
rim_height	tire_radius *(1 + cos(rim_angle / 2))	25.8216 mm

If, instead of starting with the specified tire size and assuming that it equals the tire diameter, which is rarely the case, an actual measurement of the carcass arc length is used:

$$w = 2r_0 \sin\left(\tfrac{1}{2}\theta_0\right) \text{ and } s_0 = r_0\left(2\pi - \theta_0\right) \qquad \textbf{Eq. 5-4}$$

$$r_0 = \frac{w}{2\sin\left(\tfrac{1}{2}\theta_0\right)} \text{ and } r_0 = \frac{s_0}{2\pi - \theta_0} \qquad \textbf{Eq. 5-5}$$

$$\frac{w}{2\sin\left(\tfrac{1}{2}\theta_0\right)} = \frac{s_0}{2\pi - \theta_0} \text{ so } \frac{w}{s_0} = \frac{2\sin\left(\tfrac{1}{2}\theta_0\right)}{2\pi - \theta_0} \qquad \textbf{Eq. 5-6}$$

It appears that there is no closed-form solution, but it can be found by iteration, with the fzero() function in MATLAB®, for example:

```
    rim_angle = fzero(@(theta) (rim_width/arc_length -
        (2*sin(0.5*theta))/(2*pi - theta)), pi/4)
```

Table 5-2: Sample calculations for tire cross section geometry (part 2).

rim_width		15 mm
arc_length		72.1347 mm

rim_angle	`fzero(@(theta)`
	
	`(rim_width/arc_length -`
	
	`(2*sin(0.5*theta))/(2*pi - theta)),`
	
	`pi/4)`
		1.1307 rad

tire_radius	`rim_width / (2*sin(0.5*rim_angle))`	14 mm
rim_height	`tire_radius *(1 + cos(rim_angle / 2))`	25.8216 mm

There may also be a known horizontal displacement relative to the rim, δ_h, of the part of a cross section in the contact patch. This displacement may simply be imposed on the contact patch, but in the case of a camber angle, σ, this displacement is based on the premise that once a section of tire touches the pavement, it remains stationary as the wheel rolls forward over it, until it is lifted back off the pavement.
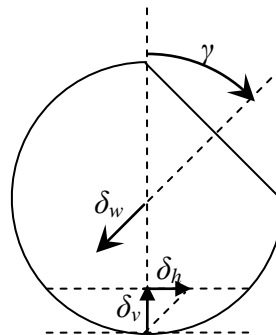
Figure 5-3: Geometric parameters of cambered and displaced tire cross section.

As the cambered tire rolls forward, the rim moves both horizontally and vertically with respect to the lowest point of a section of tire once it is in the contact patch and therefore stationary. Since the rim remains centered about the midplane of the wheel, by definition, the parts of the tire in the contact patch move relative to the rim in a line parallel to the midplane of the wheel, and the horizontal displacement of any point in the contact patch, δ_v, can be calculated from the vertical displacement of that point, δ_v, simply as:

$$\delta_h = \delta_v \tan(\sigma)$$

Eq. 5-7

The vertical displacement of the lowest cross section, δ_v, the one perpendicular to the horizontal ground, in turn can be calculated from the displacement of the entire wheel in the plane of the wheel, δ_w by:

$$\delta_v = \delta_w \cos(\sigma)$$

Eq. 5-8

Together, of course, they sum to the total displacement of the wheel in the plane of the wheel:

$$\delta_w = \sqrt{\delta_h^2 + \delta_v^2}$$

Eq. 5-9

Thus, continuing the numerical example from above in **Table 5-3** below.

Table 5-3: Sample calculations for tire cross section geometry (part 3).

wheel_vert_d	given (in the plane of the wheel)	5 mm
wheel_horz_d	Given	0 mm

camber_angle	45 * pi / 180	0.7854 rad
slice_vert_d	wheel_vert_d * cos(camber_angle)	3.5355 mm
slice_horz_d	slice_vert_d * tan(camber_angle)	3.5355 mm
wheel_vert_d	sqrt(slice_horz_d^2 + slice_vert_d^2)	5 mm
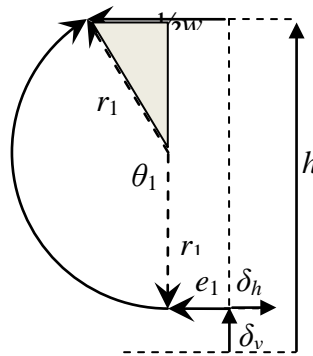
Figure 5-4: Geometric parameters of one side of cambered and displaced tire cross section.

The arc length of the left side bulge s_1 can be related to the total arc length s_0, horizontal displacement δ_h, and the horizontal distance to the edge of the contact patch from the center of the undeformed tire, e_1:

$$s_1 = \tfrac{1}{2}s_0 - e_1 - \delta_h \text{ and } s_2 = \tfrac{1}{2}s_0 - e_2 + \delta_h \qquad \textbf{Eq. 5-10}$$

For a tire of original rim height h and vertical displacement δ_v, the shaded right triangle can be analyzed with Pythagoras's theorem to find the radius of the left side bulge r_1:

$$r_1^2 = \left(\tfrac{1}{2}w - e_1\right)^2 + \left(h - \delta_v - r_1\right)^2$$

$$r_1^2 = \left(\tfrac{1}{2}w - e_1\right)^2 + \left(h - \delta_v\right)^2 - 2\left(h - \delta_v\right)r_1 + r_1^2 \qquad \textbf{Eq. 5-11}$$

$$r_1 = \frac{\left(\tfrac{1}{2}w - e_1\right)^2 + \left(h - \delta_v\right)^2}{2\left(h - \delta_v\right)}$$

The same shaded triangle can be analyzed to find the angle swept by the left side bulge

θ_1:

$$\tfrac{1}{2}w - e_1 = r_1 \sin\left(\pi - \theta_1\right)$$

$$\theta_1 = \pi - \sin^{-1}\left(\frac{\tfrac{1}{2}w - e_1}{r_1}\right) \qquad \textbf{Eq. 5-12}$$

Since the width of the contact patch, the radius of the side bulge, and the angle that it

sweeps out are all unknown, and there is no closed-form solution for them, they also must

be found by iteration. For example: increment e_1, calculate s_1, r_1, and θ_1, and then check

to see if $\mathrm{abs}\left(r_1\theta_1 - s_1\right) >$ tolerance .

This is slow, however, and can be sped up with a zero finding algorithm that finds the

zeros of $\left(r_1\theta_1 - s_1\right)$, where $s_1 = \tfrac{1}{2}s_0 - e_1 - \delta_h$, $r_1 = \dfrac{\left(\tfrac{1}{2}w - e_1\right)^2 + \left(h - \delta_v\right)^2}{2\left(h - \delta_v\right)}$, and

$\theta_1 = \pi - \sin^{-1}\left(\dfrac{\tfrac{1}{2}w - e_1}{r_1}\right)$. Combine into a single expression to eliminate r_1 and θ_1, leaving

just e_1.

$$\frac{\left(\frac{1}{2}w-e_1\right)^2+\left(h-\delta_v\right)^2}{2\left(h-\delta_v\right)}\left[\pi-\sin^{-1}\left(\frac{\frac{1}{2}w-e_1}{\frac{\left(\frac{1}{2}w-e_1\right)^2+\left(h-\delta_v\right)^2}{2\left(h-\delta_v\right)}}\right)\right]-\left(\frac{1}{2}s_0-e_1-\delta_h\right)\qquad \textbf{Eq. 5-13}$$

Again, the fzero() function in MATLAB® works well.

One detail remains, however, and it is that the horizontal displacement δ_h, due to camber angle, used in the above expression needs to be adjusted by a delta for how much additional length of tire cross section is pulled towards this side by the cambering. This delta is approximately the tire radius times the tangent or the sine of the camber angle, but really depends on the radius of the side bulge which has not yet been calculated. All attempts to find this delta value correctly with a closed-form expression have failed, and a two step iteration has been developed that works well:

1. Estimate the delta value with tire_radius * sin(camber_angle).

2. Calculate the geometry of the deformed tire in the cross section, as explained above, and add to this delta value the difference between the found contact patch center and the originally-calculated horizontal displacement.

The delta value needs to be calculated only once, for the vertical cross section, at the center of the contact patch, and then can be used on all the cross sections in the contact patch.

Lastly, it is useful to calculate where in the contact patch the lowest point of the cross section has ended up. When plotted in 3D, this provided visual confirmation that all the

above calculations do not contain some gross error, if this point from all the cross sections in the contact patch forms a straight line from one tip of the contact patch to the other. Also, the value must be the same when calculated from the left side and from the right.

Finally, continuing the example of a 28 mm tire on a 15 mm wide rim with 5 mm vertical displacement at 45º further, in **Table 5-4** below:

Table 5-4: Sample calculations for tire cross section geometry (part 4).

rim_width		15 mm
tire_radius		14 mm
rim_height		25.8216 mm
arc_length		72.1347 mm
camber_angle		0.7854 rad
slice_vert_d		3.5355 mm
slice_horz_d		3.5355 mm
delta	tire_radius * sin(camber_angle)	9.8995
delta	delta + (cp(1,2) - slice_horz_ds(slice))	10.9956
cp_edges(1)	fzero(@(cp_edge) ((((rim_widths(1) - cp_edge)^2 + (rim_heights(1) + tire_radius - slice_vert_d)^2) / (2 * (rim_heights(1) + tire_radius - slice_vert_d))) * (pi - asin((rim_widths(side) - cp_edge) /	-0.7780 mm

	`(((rim_widths(1) - cp_edge)^2 +` `(rim_heights(1) + tire_radius -` `slice_vert_d)^2) / (2 * (rim_heights(1) +` `tire_radius - slice_vert_d ))))) -` `(arc_length/2 + cp_edge - (slice_horz_d -` `delta))), x0)`	
`cp_edges(2)`	`fzero(@(cp_edge) ((((rim_widths(2) -` `cp_edge)^2 + (rim_heights(2) +` `tire_radius - slice_vert_d)^2) /` `(2 * (rim_heights(2) + tire_radius -` `slice_vert_d))) * (pi -` `asin((rim_widths(side) - cp_edge) /` `(((rim_widths(2) - cp_edge)^2 +` `(rim_heights(2) + tire_radius -` `slice_vert_d)^2) / (2 * (rim_heights(2) +` `tire_radius - slice_vert_d ))))) -` `(arc_length/2 - cp_edge + (slice_horz_d -` `delta))), x0)`	9.7218 mm
`cp_mids(1)`	`cp_edges(1) - (r(1)*a(1) - tire_radius *` `(pi - 0.5*rim_angle + camber_angle)`	3.5355 mm
`cp_mids(2)`	`cp_edges(2) + (r(2)*a(2) - tire_radius *` `(pi - 0.5*rim_angle 1 camber_angle)`	3.5355 mm

The resulting geometry is depicted below in **Figure 5-5**.
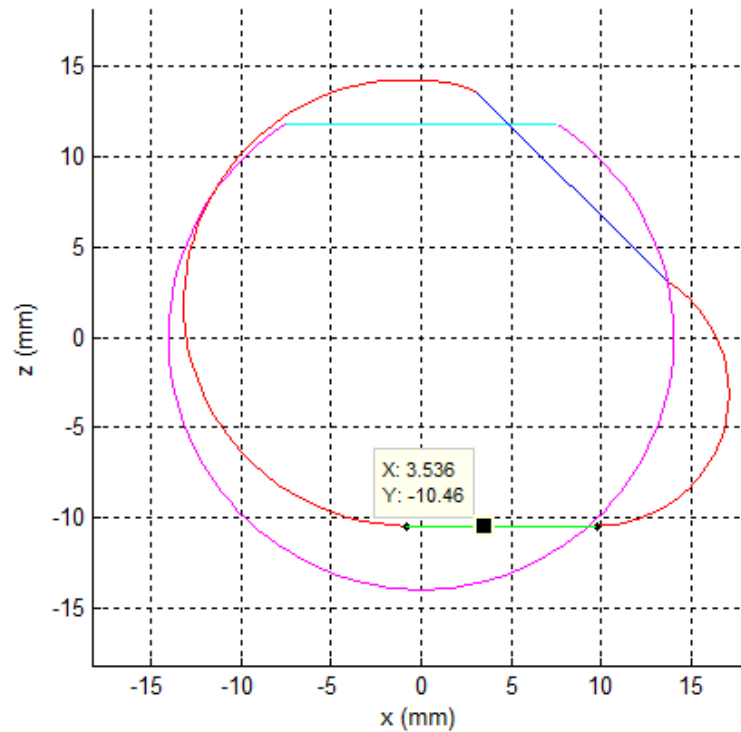
Figure 5-5: Plot of sample tire cross section calculated using the Rotta model.

So far, all calculations have been limited to a single cross section, the one that is vertical, at the lowest point on the rim, and at the center of the contact patch, if it is symmetrical.

5.2   Expanding Rotta's model in 3D

In order to calculate the forces and moments generated over the entire contact patch, it is necessary to consider multiple cross sections. Each tire carcass cross section is oriented perpendicular to the midplane of the wheel and parallel to wheel radii.

For a tire with radius r_0, finite sinkage δ_w, and non-zero camber angle γ, the lowest point of the tire as it touches the ground, at the leading and trailing edges of the contact patch, is at a distance around the cross section of $s = r_0 * \gamma$ from the centerline of the tire.
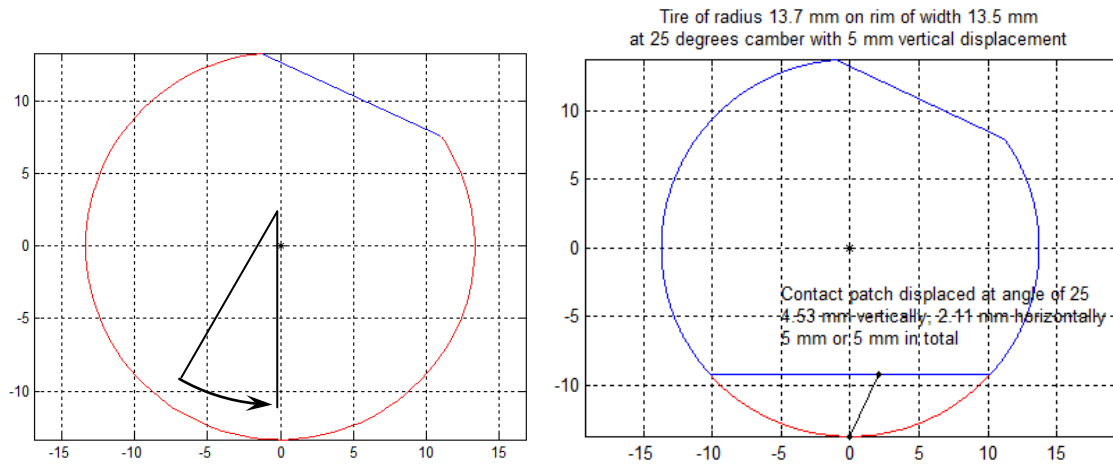
Figure 5-6: Location of lowest point on tire, first point to touch ground, on cambered tire.

Then, as the wheel rolls forward, this point on the cross section remains stationary with respect to the pavement and so moves relative to the rim both vertically, towards the rim, and horizontally in the direction of camber. This occurs independent of the bulging modeled by Rotta, and is depicted on the right in **Figure 5-6** above, for the sake of simplicity, as though the tire were a solid material with Poisson's ratio of zero, cork for example, and thus compresses in one direction without a corresponding bulge in the perpendicular directions.
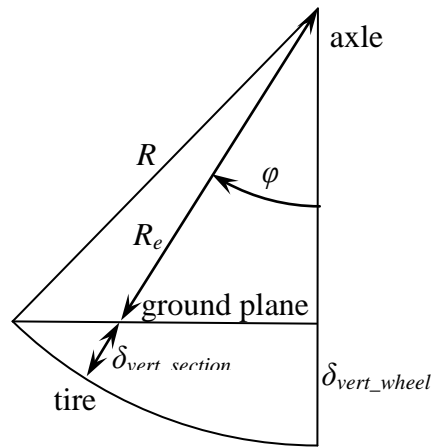
Figure 5-7: Side view of wheel dimensions as the tire is pressed into the ground.

In the plane of each radial cross section, the plane in which the Rotta geometry is determined, the vertical component of this displacement can be calculated as:

$$\delta_{vert\_section} = R - R_e$$

$$R_e \cos(\varphi) = R - \delta_{vert\_wheel}$$

$$\delta_{vert\_section} = \left(R - \frac{\left(R - \delta_{vert\_wheel} \right)}{\cos(\varphi)} \right)$$

Eq. 5-14

Where **Figure 5-7** above shows dimensions on the side view of a portion of the entire wheel, R, is the height of the axle, also the radius of the wheel including the tire, and the distance from the axle to the farthest point on the tire; and $\theta_{section}$ is the angle of the cross section in question as measured from the bottom-dead-center of the wheel. The distance from the axle to the ground plane is R_e, and the displacement of the tire in the radial plane is $\delta_{vert\_section}$.

Finally, to adjust for non-zero camber angle, multiply by cosine of the camber angle:

$$\delta_{vert\_section} = \left(R - \frac{\left(R - \delta_{vert\_wheel} \right)}{\cos(\varphi)} \right) \cos(\gamma)$$
<div align="right">**Eq. 5-15**</div>

When this vertical component of the displacement is applied to each radial cross section

of the tire, and then each cross section is oriented correctly in 3D, their bottom edges

form a plane, as shown below in **Figure 5-8**.
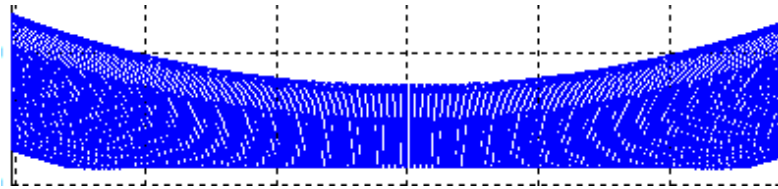
Figure 5-8: Side view of tire calculated with 3D Rotta model showing flat contact

patch.

It is not, however, perfectly flat for non-zero camber angles. A tire with 5 mm vertical

displacement on a 622 mm diameter wheel is modeled to have the small variations shown

in **Figure 5-9** below, for two different camber angles.
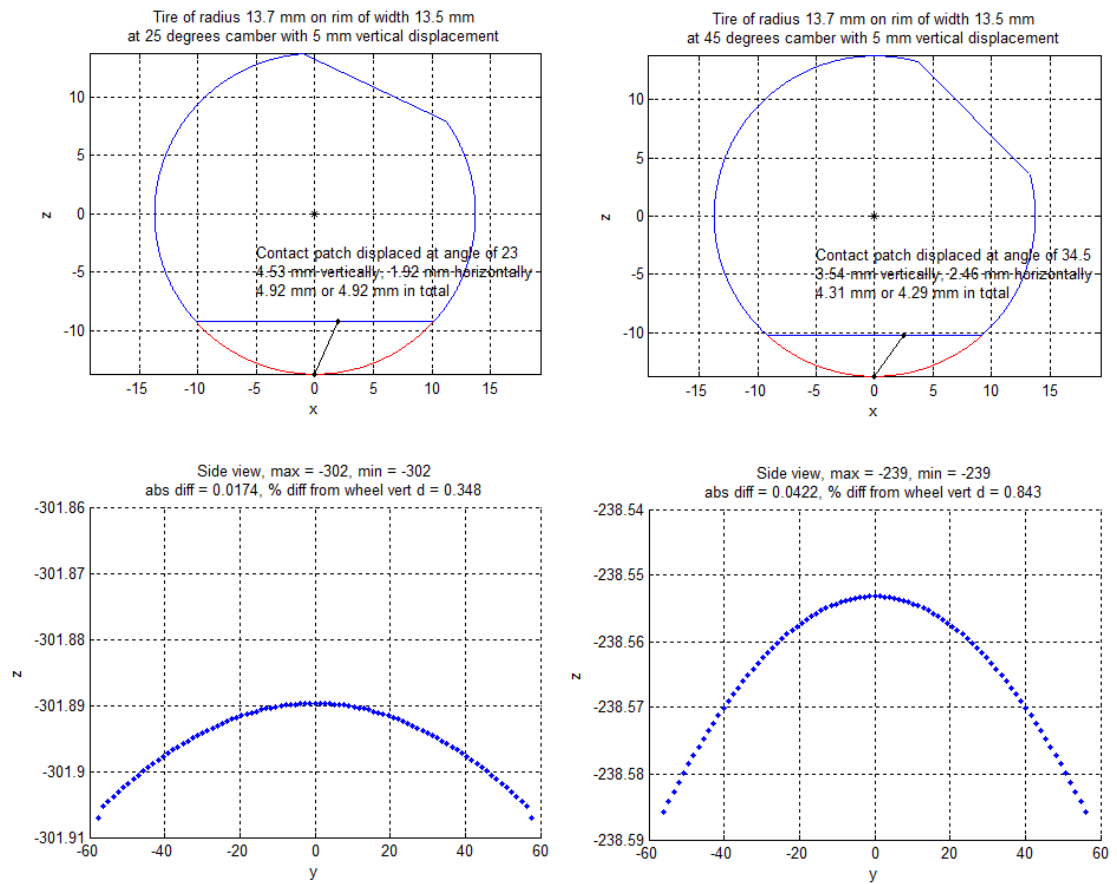
Figure 5-9: Flatness of contact patch developed by the 3D Rotta model.

The two lower plots in **Figure 5-9** above show an exaggerated side view of the calculated vertical location of the bottom, flattened part of each cross section, as plotted along the length of the contact patch.

The corresponding horizontal component of the displacement can be calculated, also in the plane of the radial cross section, by an expression that is actually more assembled through trial and error than derived:

$$\delta_{horz\_section} = r_0\gamma + r_0\sin(\gamma) + ...$$

$$\left(\begin{array}{c} -\left(R\left(1 - \dfrac{\delta_{vert\_wheel}}{R} \right)\cos(\gamma) \right) + ... \\ R\cos(\varphi)\cos(\gamma) \end{array} \right)\tan(\gamma) \qquad \textbf{Eq. 5-16}$$

This too is not exactly correct and a tire with 5 mm vertical displacement on a 622 mm

diameter wheel is modeled to have the variations shown below:
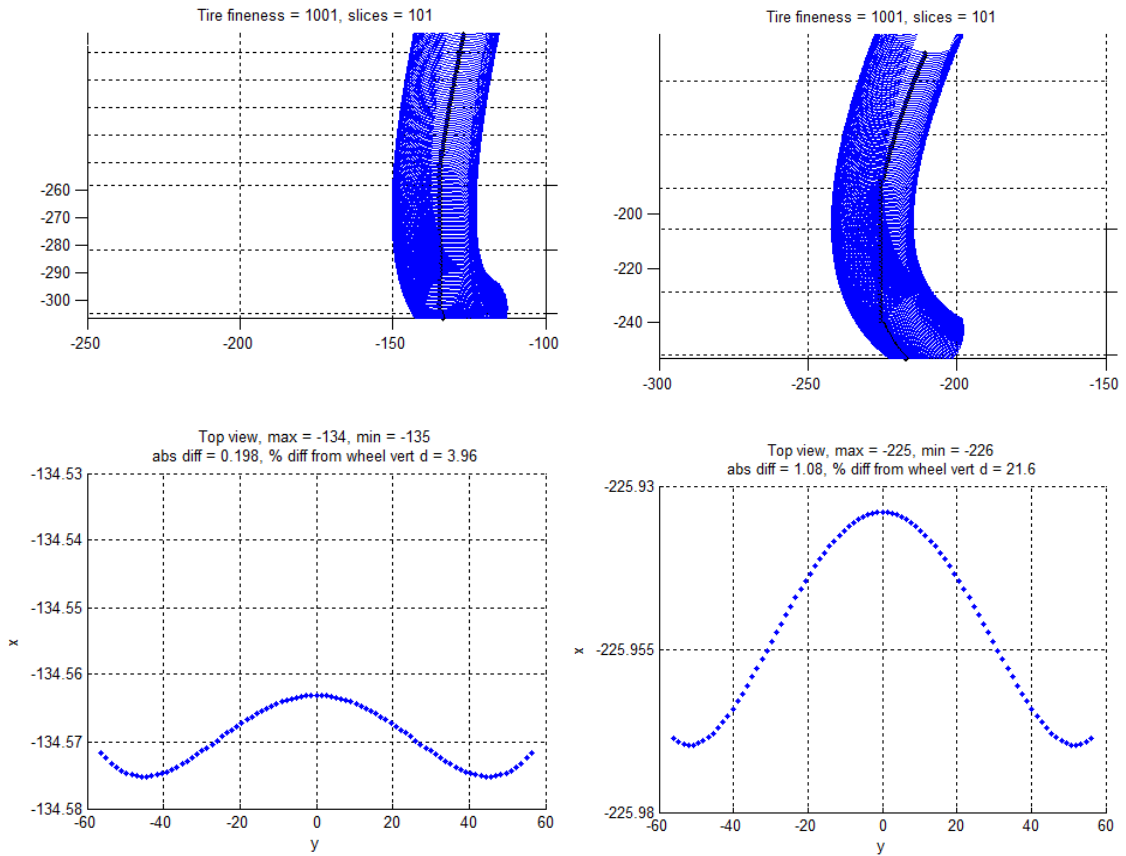
Figure 5-10: Straightness of contact patch developed by the 3D Rotta model (part 1).

The lower two plots in **Figure 5-10** above show an exaggerated top view of the calculated horizontal location of the point on each cross section that first touched the ground, as plotted along the length of the contact patch.

This variation in the horizontal displacement of each cross section, with respect to a straight line, can be improved by recognizing the desired relationship between the horizontal and vertical displacements, and so calculating the horizontal displacement directly from the vertical displacement:

$$\delta_{horz\_section} = \delta_{vert\_section} \tan(\gamma)$$

Eq. 5-17
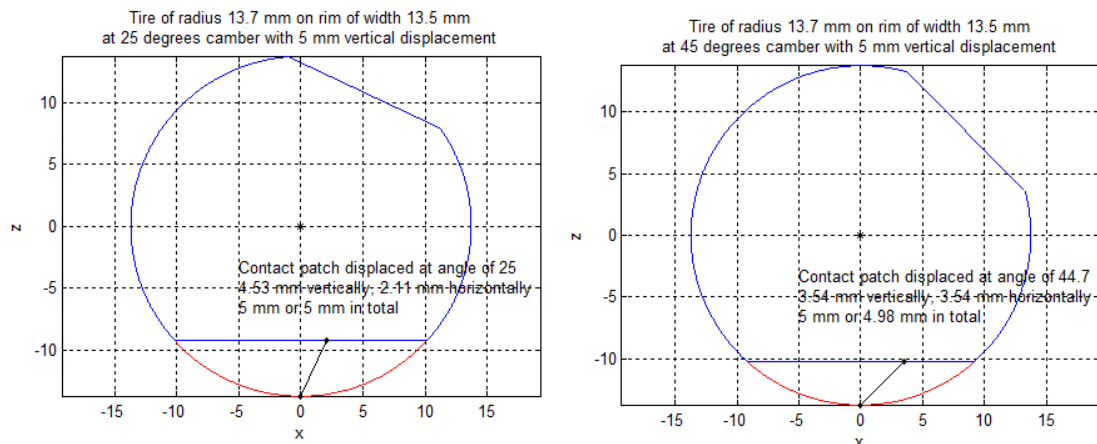
Figure 5-11: Plots showing horizontal and vertical displacement of contact patch.
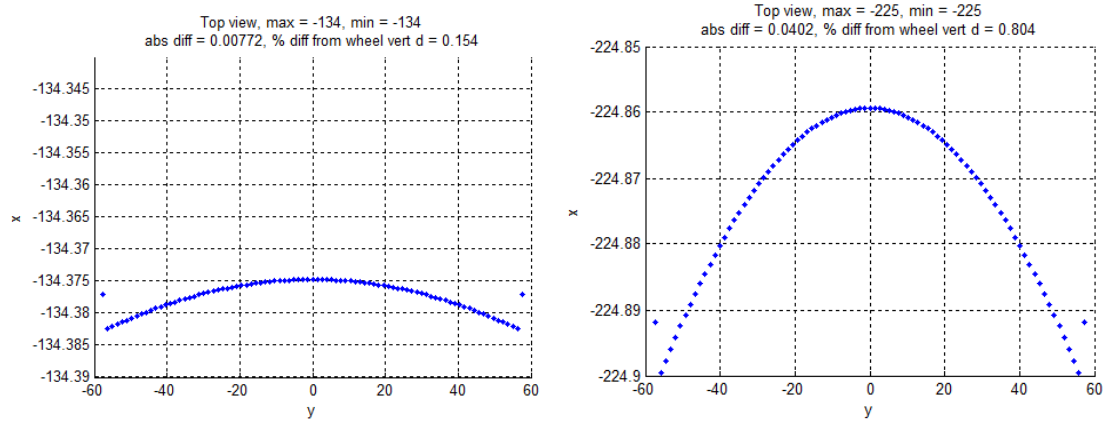
Figure 5-12: Straightness of contact patch developed by the 3D Rotta model (part 2).

This simplification shows a nice improvement, as can be seen in the difference between the lower two plots in **Figure 5-12** immediately above vs. the lower two plots in **Figure 5-10** on the previous page.

With this final version, the cross sections flattened in the contact patch, and modeled to bulge out on the sides with Rotta's method, now blend seamlessly with the undeformed cross sections beyond the ends of the contact patch, as shown in **Figure 5-13** on the next page and as expected in the case of vertical deflection, also called sinkage, and camber angle without additional horizontal deflection.

Vertical force = 696.9 N (71.04 kg, 156.7 lb)
Horizontal force = 370 N (37.72 kg, 83.18 lb), which normalizes to 0.5309
Resultant force orientation = 27.97° from vertical, (62.15% of camber angle)
Relaxation length = 0 mm
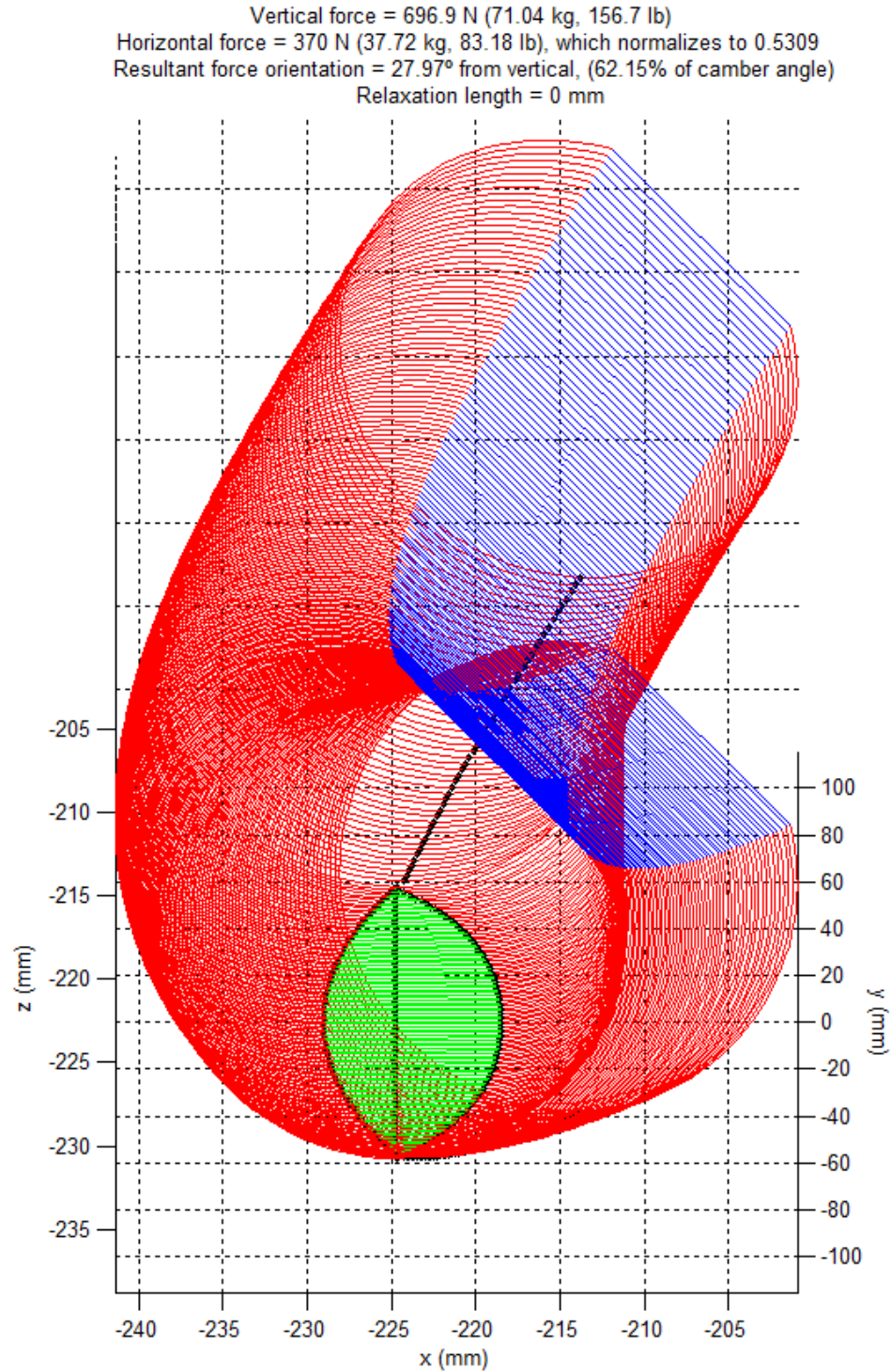
Figure 5-13: 3D representation of cambered tire modeled by Rotta.

5.3 Drums and disks

The above calculations can be adjusted to account for the curvature of various testing surfaces: such as the 2.5 meter drum at TU Delft and the 3 meter disk at Padua.

5.3.1 Drums

If the vertical displacement of the wheel, δ_{vert_wheel}, is taken to be with respect to the top of the drum, then the new radial deflection of each tire cross section due to the drum can be calculated as a decrease in the deflection already calculated above for the case of flat pavement.

$$\delta_{vert\_section} = R - R_e - \delta_{drum\_section} \qquad \textbf{Eq. 5-18}$$

This change, $\delta_{drum\_section}$, indicated below in **Figure 5-14**, can be calculated as in **Eq. 5-19**.
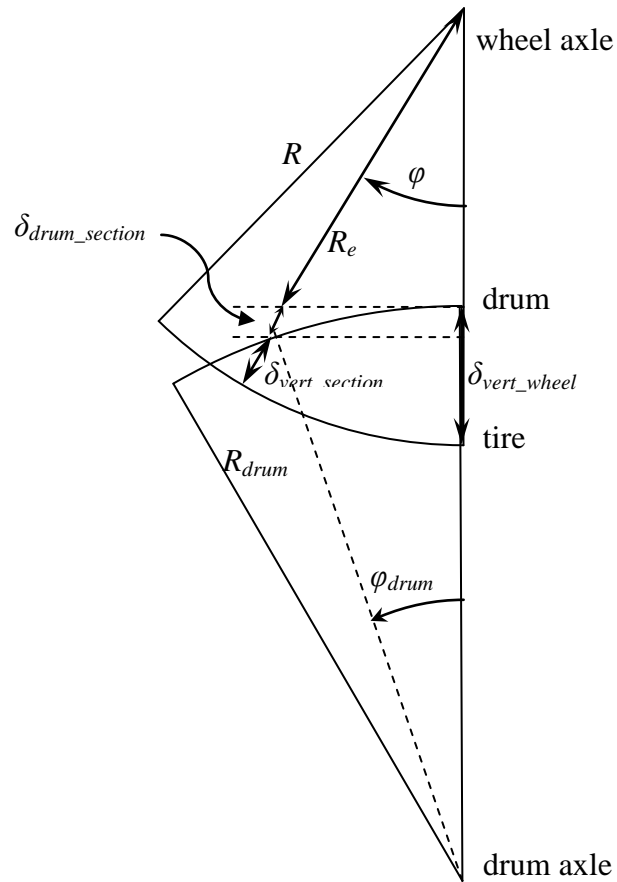
Figure 5-14: Side view schematic of wheel and drum.

$$\delta_{drum\_section} = \frac{R_{drum}\left(1-\cos\left(\varphi_{drum}\right)\right)}{\cos\left(\varphi\right)}$$

Eq. 5-19

Finally, assuming that the wheel radius and the drum radius are both much larger than the wheel vertical displacement, which is the case for narrow bicycle tires, then the two angles can be approximately related for any radial cross section:

$$R_{drum}\sin\left(\varphi_{drum}\right) = R\sin\left(\varphi\right)$$

Eq. 5-20

$$\varphi_{drum} = \sin^{-1}\left(\frac{R}{R_{drum}}\sin\left(\varphi\right)\right)$$

All together:

$$\delta_{vert\_section} = R - R_e - \frac{R_{drum}\left(1 - \cos\left(\sin^{-1}\left(\frac{R}{R_{drum}}\sin\left(\varphi\right)\right)\right)\right)}{\cos\left(\varphi\right)}$$

Eq. 5-21

This implementation of the Rotta model predicts only a 2% difference in camber force between flat pavement and the 2.5 meter drum at TU Delft. A 37 mm tire at 4 bar and under 400 N generates a 210 N horizontal force when cambered 45° on a straight track. If the track is a 2.5-meter diameter drum that curves away from the tire, then it generates 206 N.

5.3.2   Disks

In the case of a disk, the situation is nearly the same except that the curvature of the disk needs to be accounted for in the horizontal displacement of each cross section of the tire, $\delta_{horz\_section}$.

$$\delta_{disk\_section} = \frac{R_{disk}\left(1 - \cos\left(\varphi_{disk}\right)\right)}{\cos\left(\varphi\right)}$$

Eq. 5-22

Finally, assuming that the wheel radius and the disk radius are both much larger than the wheel vertical displacement, which is the case for narrow bicycle tires, then the two angles can be approximately related for any radial cross section:

$$R_{disk} \sin\left(\varphi_{disk}\right) = R \sin\left(\varphi\right)$$

$$\varphi_{disk} = \sin^{-1}\left(\frac{R}{R_{disk}} \sin\left(\varphi\right)\right)$$

Eq. 5-23

All together:

$$\delta_{disk\_section} = \frac{R_{disk}\left(1 - \cos\left(\sin^{-1}\left(\frac{R}{R_{disk}} \sin\left(\varphi\right)\right)\right)\right)}{\cos\left(\varphi\right)}$$

Eq. 5-24

Lastly, the entire contact patch needs to be offset by the difference between $\delta_{disk\_section}$ for the center section and $\delta_{horz\_section}$ of the end section so that the ends of the contact patches are located where they would be if there were not a disk curvature.

This cannot be calculated in closed-form before hand, and simply needs to be calculated from the resulting contact patch and then used to create the contact patch again from scratch.

Modeling a tire, such as those tested by Doria *et al.*, with the 3 meter disk they describe,[28] confirms their assumption that averaging the lateral force from a tire first cambered one way and then cambered the same amount in the opposite direction does indeed yield the lateral force the tire generates on a flat surface.

For example, a size 37 mm tire at 4 bar and under 400 N generates a 210 N horizontal force when cambered 45º on a straight track. If the track is a 3-meter diameter disk that curves away from the camber, then it generates 206 N, and if the track curves toward the camber, then it generates 214 N. The average of 214 and 206, of course is 210, which matches the expected value.

5.4   Contact patch ellipse

The contact patch predicted by extending the Rotta model in 3D has pointed ends which are obviously different from the contact patches recorded in ink. One way to generate a more-realistic contact patch is to fit an ellipse to it, at least in the case of vertical displacement only, without slip or camber angles, when the contact patch is symmetrical. This tends to shorten the contact patch and reduce the area, as shown below in **Figure 5-15**, both of which better match the experimental results.
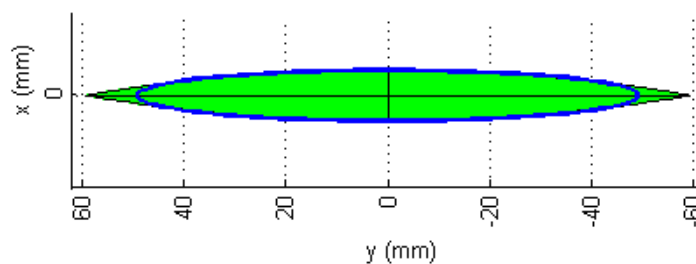
Figure 5-15: Rotta model contact patch fitted with an ellipse.

5.5   Calculating forces generated

In all cases, the lateral force generated, dS, in a cross section of depth dy is calculated from just the inflation pressure, p, and the radii of the two circular arcs, r_1 and r_2, as shown below in **Eq. 5-25**. The lateral force generated by the entire tire is simply the sum of all the dSs.

$$dS = p(r_1 - r_2)dy \qquad \text{Eq. 5-25}$$

Following the assumption that the vertical force between a piece of tire and the pavement is equal to the inflation pressure multiplied by the area of the piece of tire in question, the vertical force borne by each cross section can be calculated. The vertical force, dN, in a cross section of depth dy is simply the width of the contact patch in that cross section multiplied by its depth, the area dA, multiplied by the inflation pressure, p, as shown below in **Eq. 5-26**.

$$dN = p(e_2 - e_1)dy \qquad \text{Eq. 5-26}$$

5.6   Friction

Dividing into the lateral force from **Eq. 5-25** by the vertical force from **Eq. 5-26** yields the minimum coefficient of friction necessary to prevent sliding, as shown in **Eq. 5-27**.

$$\mu_{min} = \frac{dS}{dN} \qquad \text{Eq. 5-27}$$

One solution to the problem of excessive coefficients of friction, for example $\mu > 1$, is to allow a cross-section to slide in the direction of the lateral force until the lateral force is

reduced enough to be withstood by the available friction, and the sliding can be

implemented by simple iteration. In practice, however, because the cross-sectional slices

are completely decoupled, it was found that cross-sectional slices near the ends of the

contact patch would slide nearly all the way back to their undeflected location before

friction became sufficient, and this causes an unlikely kink in the centerline of the tire.

Instead, it was found that limiting the sliding of any cross section to some fraction of its

immediate neighbor produces much more realistic results. More details about how this

fraction is decided and implemented are given below in section 5.8.

Another issue that arises once deflection of the tire beyond the ends of the contact patch

is considered is the additional lateral force generated there that must be borne by friction

in the contact patch. Since this cannot be known until those cross-sectional slices are

considered, a simple iteration was implemented to calculate the lateral force generated by

the entire tire by at first ignoring the additional force that must be applied to the contact

patch, and then recalculating each cross-sectional slice and its possible sliding due to this

additional force. This process usually converges to a stable total lateral force in just three

or four iterations.

5.7 Extending Rotta's model beyond the ends of the contact patch

If a cross section does not intersect the contact patch, then its contact patch width can

simply be set to zero, the radii of the two sides are equal, and it forms a single, constant-

radius circular arc from bead to beam. If there is no lateral displacement of the tire

centerline at this point, then the two zones blend together seamlessly. If there is a lateral

displacement of the tire centerline at the end of the contact patch, however, simply reverting to completely undeflected cross sections will result in a discontinuity.

The published Rotta analysis does mention displacement of the tire beyond the contact patch, but it does not predict that behavior, and it has been observed that a lateral displacement of even a point contact patch causes lateral displacement over a significant length of the tire. Here, the Rotta model is used to calculate a foundation stiffness for the model of a beam on an elastic foundation.

If the assertion that tire cross sections are displaced laterally beyond the ends of the contact patch is correct, and logical analysis and physical testing presented in section 3.8.6.3 suggests that it is, then it begs the question, what shape does it take? Rotta does not say, so additional models must be considered.

Choices are:

- Even displaced, it is a perfectly circular arc from bead to bead.

- The radius of curvature varies continuously in some way.

- The radius of curvature is constant for parts and there is a discontinuity, either in radius or the rate at which it changes.

The first choice, constant radius for the entire cross section, can be rejected by geometric analysis, as shown below in **Figure 5-16**. If the radius is constant from bead to bead, and the tire forms a perfect constant-radius circular arc, then it cannot be displaced laterally.

There is no way to draw a straight rim from one point on a circle to some other point on a circle so that it is not bisected by a radius:
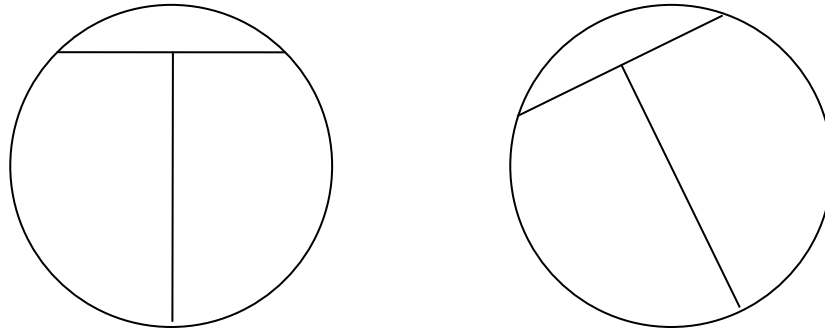
Figure 5-16: Geometric argument that displaced tire cannot have constant radius.

So that leaves a continuously varying radius or multiple distinct radii. Of the two, the former is more likely to match reality because it does not require something to create the discontinuity, as the contact patch does in the case of the Rotta model. Examples are shown in **Figure 5-17** below in green.
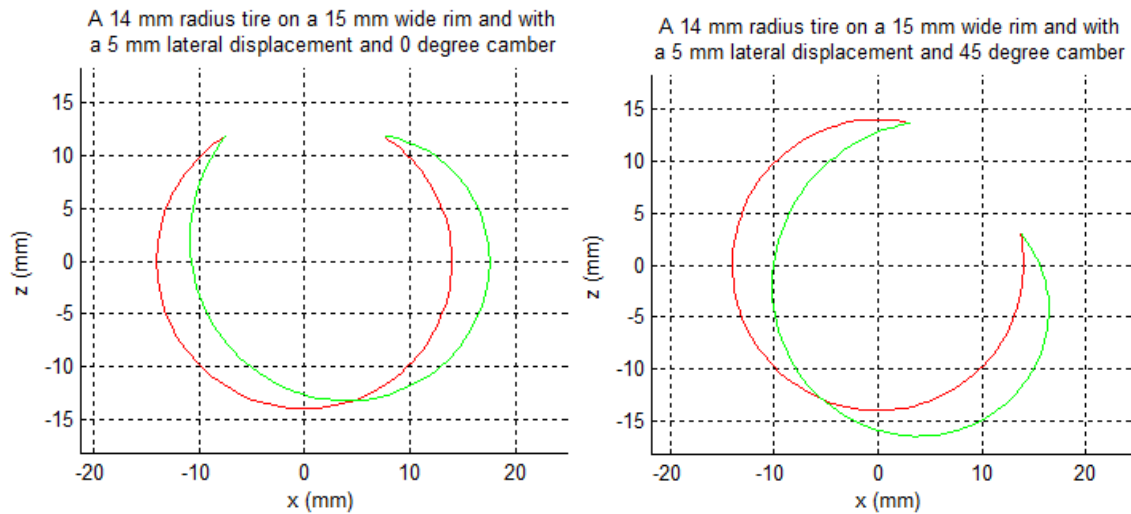
Figure 5-17: Continuously varying radius in laterally displaced tire cross sections.

This then begs the next question, however, of what two radii should be used to calculate the lateral force applied to the rim with Rotta's *dS* equation, **Eq. 5-25**? It can be argued that the difference between the angles at which the tire departs the two sides of the rim should be used instead of radii, but there is also another issue. There is not a closed-form solution to either configuration, and the variable radius configuration does not lend itself to speedy solution with the fzero() function in MATLAB®.

Instead, consider treating the problem as simply a zero-width contact patch and use the existing technology, which is already greatly speeded along with the fzero() function in MATLAB®, to calculate the radii of the two, distinct side bulges. As can be seen in **Figure 5-18** below, the results (in blue) are very similar to the continuously varying radius results (in green).
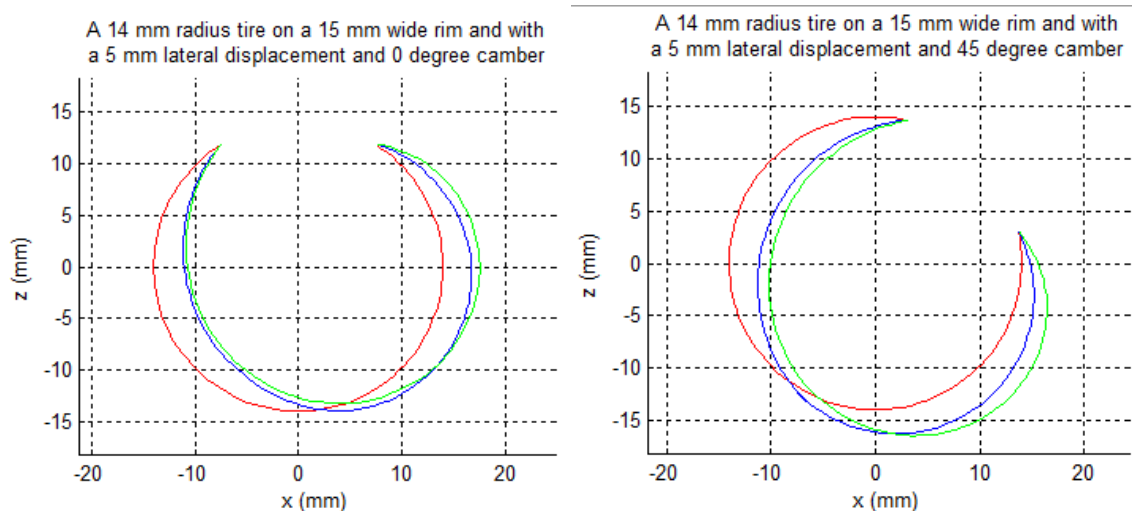
Figure 5-18: Comparison of constantly varying radius vs. two discrete radii.

5.8   Decay rate and length

The first method implemented to calculate the horizontal displacement of each cross section beyond the end of the contact patch is simply exponential decay. Each subsequent horizontal displacement is calculated by multiplying the previous horizontal displacement by a relaxation rate, i, in the range of 0.9 to 0.99.

$$\lambda = 1 - i$$

Eq. 5-28

This relaxation rate, i, can be converted to an exponential decay rate, λ, with **Eq. 5-28** above and the exponential decay rate, λ, can be converted to a decay length, ℓ, with **Eq. 5-29** below:

$$\ell = \frac{1}{\lambda}$$

Eq. 5-29

Vertical force = 490.3 N (49.98 kg, 110.2 lb)
Horizontal force = 57.52 N (5.864 kg, 12.93 lb), which normalizes to 0.1173
Resultant force orientation = 6.692° from vertical, (Inf% of camber angle)
Relaxation length = 131 mm, Static lateral stiffness = 28.76 N/mm (0.05866 1/mm)
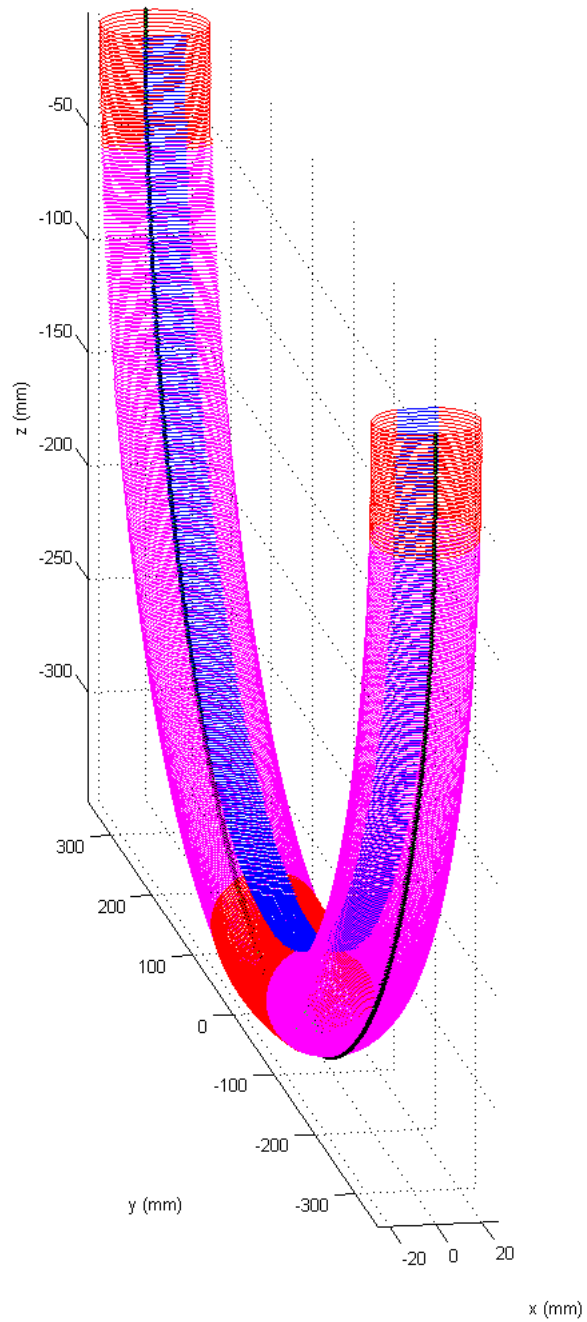
Figure 5-19: Rotta model of tire with 2 mm lateral displacement blended smoothly back to undisplaced over nearly half the wheel circumference with a 0.993 relaxation rate.

This produces reasonable looking results, as shown in **Figure 5-19** and **Figure 5-20**, but leaves the question of what rate to use unanswered. One method of determining the rate would be to measure the relaxation rate of an actual tire, but this is more easily said than done and goes against the motivation for the model in the first place.

Horizontal (dS) and vertical (dN) force per unit lenght (N/mm)
actual horizontal deflections vary from 2 to 0.09948 mm

Pneumatic trail = 0 mm

y, distance along long axis of tire and contact patch (mm)

Horizontal displacement (blue in mm), dS (black) and dN (red) (both in N)
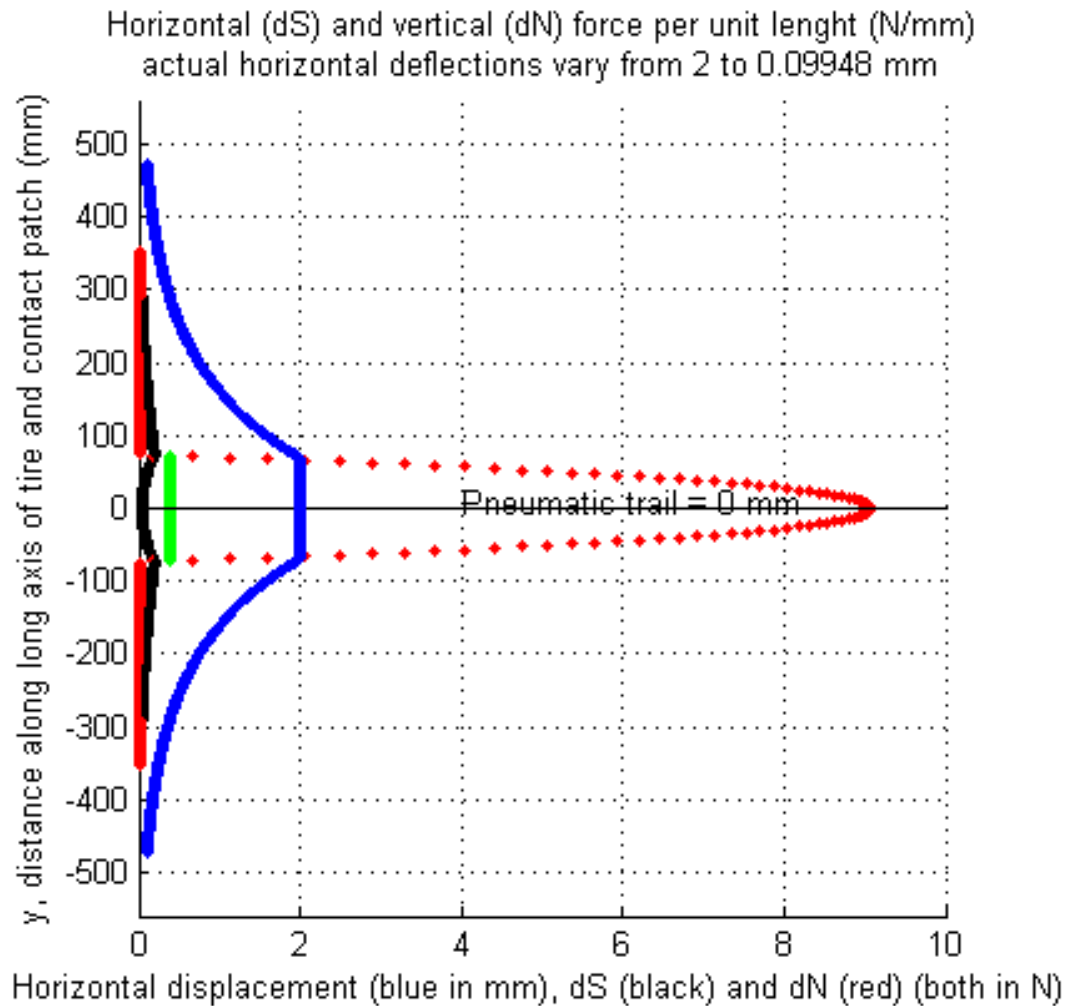
Figure 5-20: Graph showing lateral displacement of each cross section, the lateral and vertical forces generated in each cross section, and the average lateral force applied in the contact patch for a tire with 2 mm lateral displacement blended smoothly back to undisplaced over nearly half the wheel circumference with a 0.993 relaxation rate.
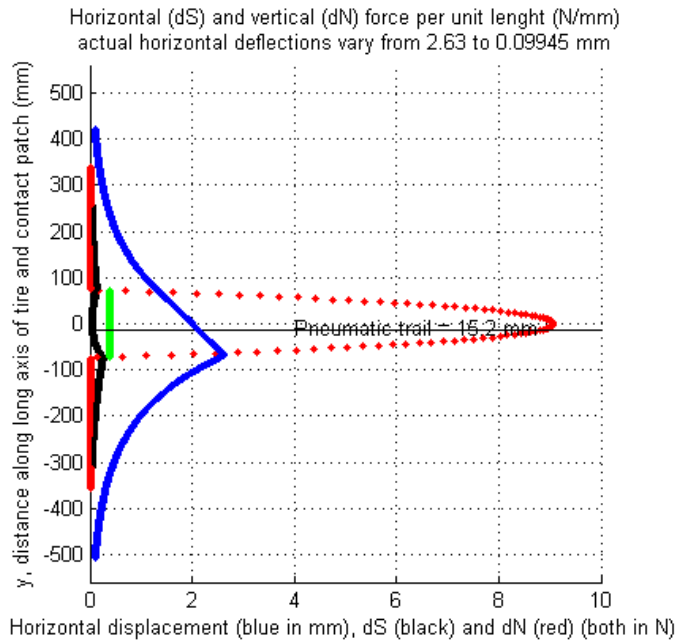
Figure 5-21: Graph showing same values and for same tire as **Figure 5-21**, but now with a 0.5º slip angle. Note similarity with measured lateral displacement shown in **Figure 4-15**.
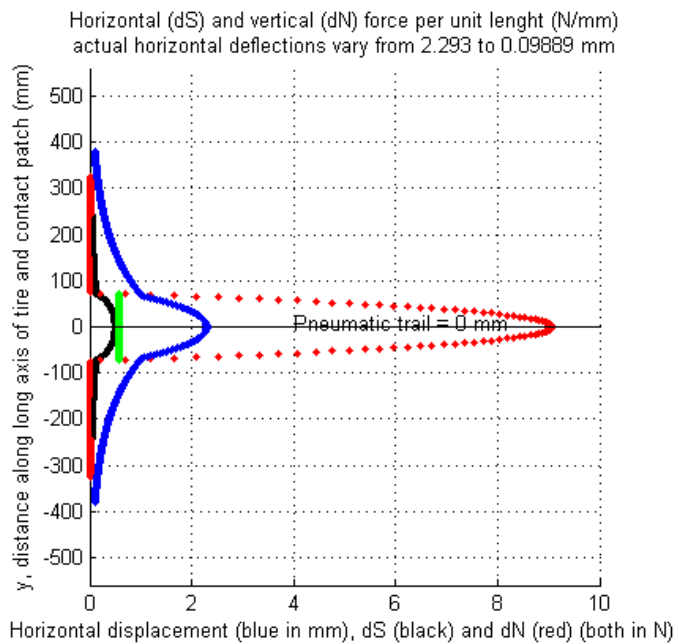
Figure 5-22: Graph showing same values and for same tire as **Figure 5-21**, but now with a 10º camber angle. Note similarity with measured lateral displacement shown in **Figure 4-16**.

5.9   Taut beam on an elastic foundation

In an effort to enhance the Rotta model so that it can provide a relaxation rate as output instead of require one as input, the model of a taut beam on an elastic foundation was considered. The resistance to lateral flexing of a tire is the beam lateral stiffness, the circumferential tension in the tire is the tension in the beam, and the lateral stiffness of each cross section according to the Rotta model is the elastic foundation.

5.9.1   Differential equation

The lateral displacement behavior of a taut beam on an elastic foundation can be modeled by a fourth-order ordinary differential equation, as shown in **Eq. 5-30**.

$$-\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 u}{\partial x^2}\right)+T\frac{\partial^2 u}{\partial x^2}-k\left(x\right)u\left(x\right)+f\left(x\right)=0 \qquad \textbf{Eq. 5-30}$$

5.9.2   Numerical implementation

The boundary value problem represented by this ordinary differential equation is solved by collocation with the bvp4c() function in MATLAB®,[46] as shown below in **Figure 5-23**. The length of the beam is some large fraction of the circumference of the wheel. Because the lateral stiffness of the tire causes any lateral displacement to eventually go to

zero beyond the ends of the contact patch, any length beyond the ends of this

displacement is sufficient.

Beam with I = 0.0169 in⁴, E = 5.48e+004 lb/in², A = 0.644 in², and L = 20.7 in
on a foundation with stiffness rate 4e+003 lb/in/in for a net stiffness of 8.26e+004 lb/in
and subject to T = 51.5191 lb and a load rate of 200 lb/in over a contact patch length 0.125 in for a net force of 25 lb
undergoes a maximum displacement of 0.05 in for a net stiffness of 500 lb/in (an increase of 0.01338%), and a relaxtion length of 3.02 in
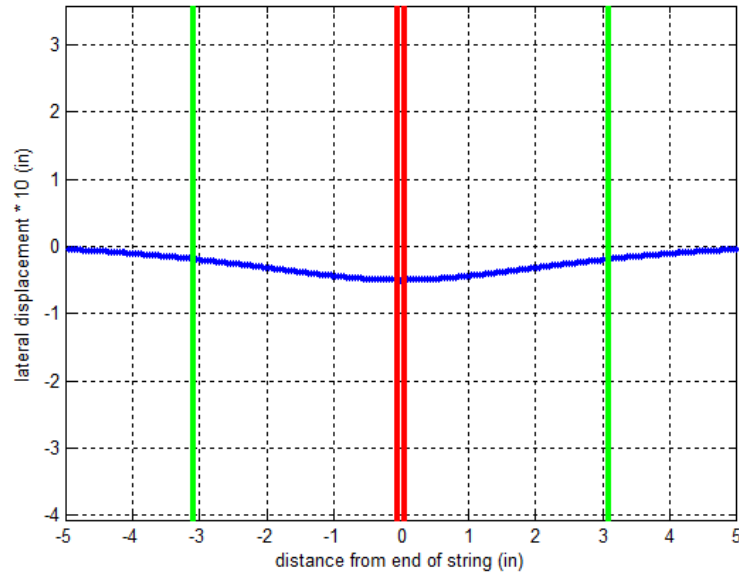
Figure 5-23: Lateral displacement, in blue, of a taut beam on an elastic foundation

subject to a concentrated load at the midspan.

5.9.3   Constants

The governing equation for a taut beam on an elastic foundation requires several

constants.

5.9.3.1   *Area Moment of Inertia*

An area moment of inertia can be calculated directly from the tire cross section geometry.

As the lateral deflection of the tire decreases as the tire nears the rim, only the area of the

half of the tire cross section furthest from the rim is considered to approximate this reality, as shown in equation **Eq. 5-31**.

$$I = \tfrac{\pi}{4}\left(r^4 - (r-t)^4\right)$$
<div align="right">**Eq. 5-31**</div>

5.9.3.2 *Modulus of Elasticity*

An estimate of the modulus of elasticity for the composite material of nylon cords embedded in a rubber matrix that makes up the carcass can be found in the literature.[47]

5.9.3.3 *Tension*

The tension in the tire, normal to the cross section, can be estimated by simply multiplying the inflation pressure by the internal cross sectional area, as shown in **Eq. 5-32**.

$$T = p\pi r^2$$
<div align="right">**Eq. 5-32**</div>

Another estimation is to consider the radial pressure acting on the tire carcass over the wheel circumference,[2] as shown in **Eq. 5-33**.

$$T = p(R + 2r)$$
<div align="right">**Eq. 5-33**</div>

For a 28-622 tire, **Eq. 5-32** predicts about twice the tension as does **Eq. 5-33**.

5.9.4 Integration with Rotta model

The integration of this model with the Rotta model begins by picking some small lateral displacement, scaled to the arc length of the tire, and calculating the contact patch length,

the stiffness rate, and the force rate with the Rotta model. In the case of zero

displacement beyond the contact patch, which would give an infinite stiffness, use instead

the maximum stiffness, found just inside the ends of the contact patch. Plug those values

into the taut beam on elastic foundation model to calculate displacements. Plug those

displacements into the Rotta model to calculate the contact patch length, the stiffness

rate, and the force rate. Iterate until the orientation angle of the ground reaction force

converges.
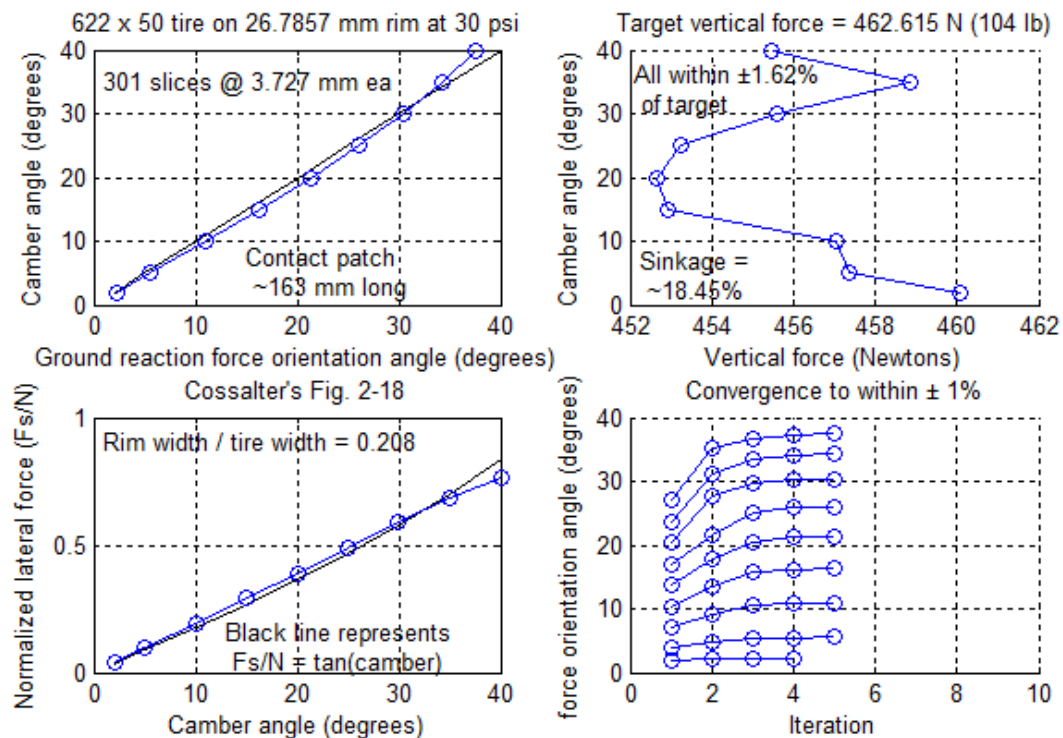
Figure 5-24: Camber force results of iteration between Rotta model and model of taut

beam on elastic foundation.

CHAPTER 6

MODELING RESULTS

6.1   Model predictions

6.1.1   Contact patch

That the contact patch area equals the vertical load divided by the inflation pressure is an assumption intrinsic to the Rotta model, but all measured contact patches have slightly less area than this.

6.1.2   Sinkage

Increasing vertical load without changing inflation pressure increases sinkage. The Rotta model predicts that increasing sinkage has small effect on lateral stiffness, but the increasing load decreases normalized lateral stiffness.
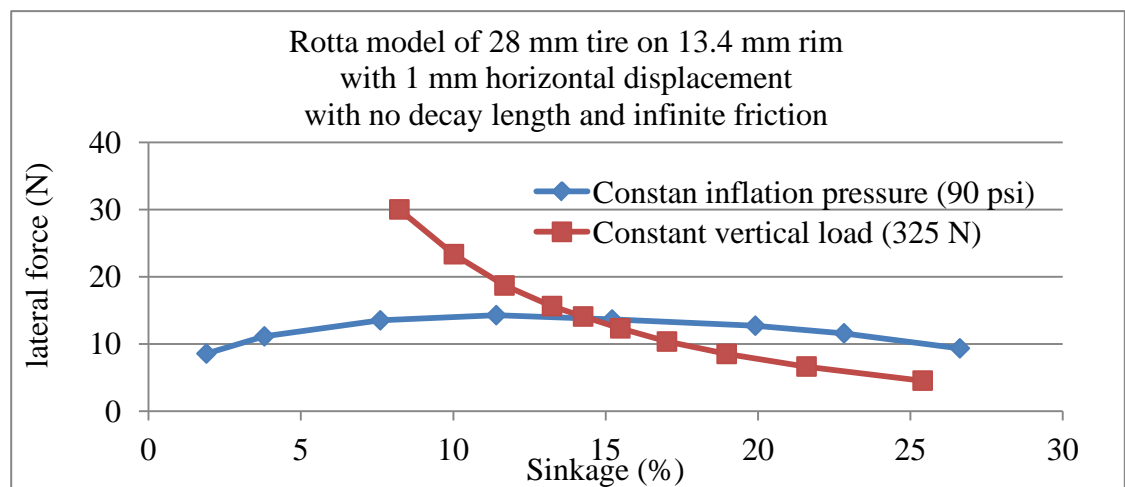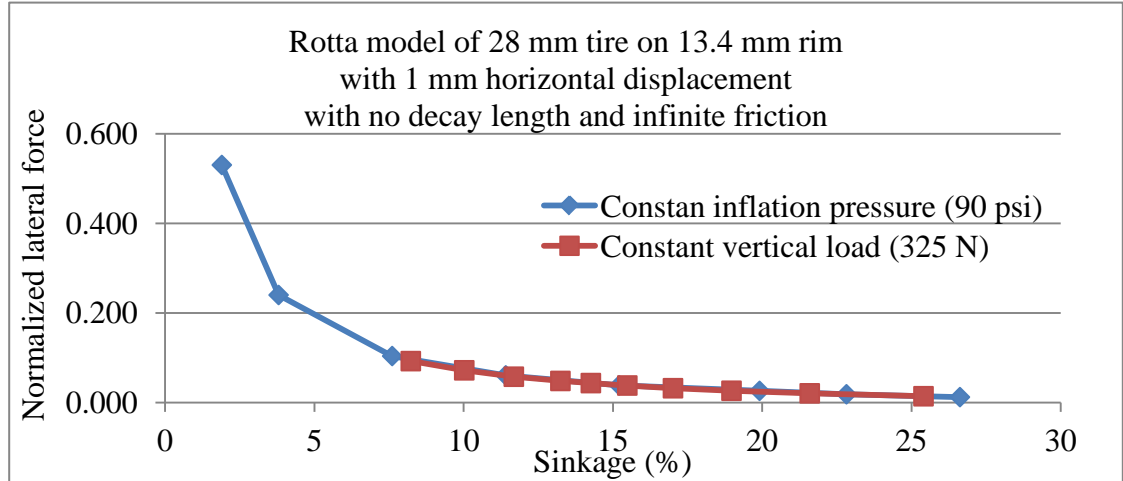
Figure 6-1: Lateral force vs. sinkage for constant inflation pressure and constant vertical load.

- Sinkage calculated by the Rotta model increases linearly ($R^2 = 0.999$) with vertical load, and decreases proportional to about the -0.7 power of inflation pressure ($R^2 = 1$).

- For values from the Rotta model, the ratio between sinkage and contact patch area raised to the 0.704 power is nearly constant (+0.2%, -0.6%) for sinkages from 9% - 25%.

6.1.3 Stiffness

Increasing inflation pressure without changing vertical load decreases sinkage, and the Rotta model predicts that decreasing sinkage has small effect on lateral stiffness, but the increasing inflation pressure increases lateral stiffness and thus normalized lateral stiffness.

The lateral force calculated by the Rotta model increases linearly with the lateral displacement of the centerline of the tire.

6.1.4 Tangent rule

The Rotta model casts doubt on the applicability of the tangent rule derived from the radial spoke model to bicycle tires.[17][7] The Rotta model predicts that camber thrust deficiency tends to decrease as sinkage decreases, but there is no guarantee that it will go to zero before sinkage does. Instead, the model suggests that camber thrust deficiency appears to be sensitive also to the ratio of tire arc length over rim width, and a deficiency can be decreased, even to the point of becoming a surplus, be widening the rim on which the tire is mounted, as shown in **Figure 6-4**.

Limited physical testing of this prediction to date has proven inconclusive.

Jim Papadopoulos writes, "The deformation is only unchanged if you have a brush or spoke model of the world. Rotta calculations show us that the force aligns with the wheel only for very tiny radial deformations, otherwise it deviates more and more as deformation increases. That is the first flaw of the brush model: that it takes a linear approximation of the radial and lateral tire stiffness, and assumes that this is unaffected by camber. The second flaw of course is the lack of a belt or string, but of course that is not your discovery. (Except for finding that the decay length is of order half the contact length, for typical loadings.)"
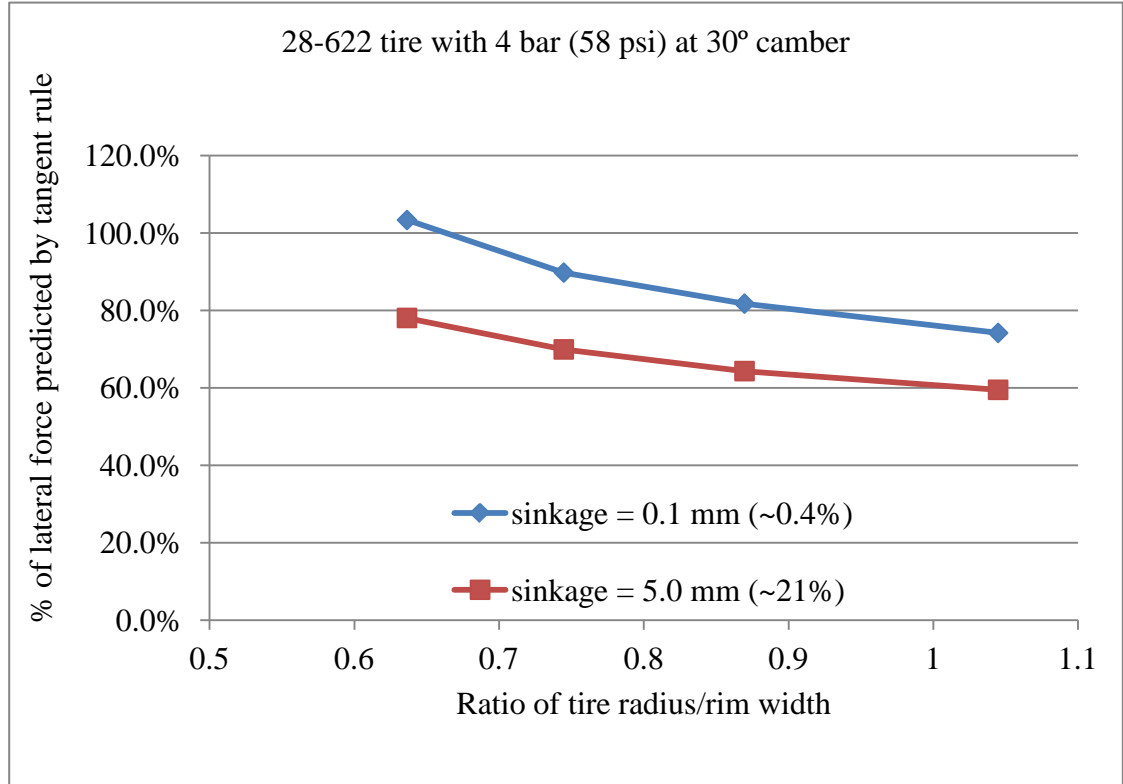
Figure 6-2: Plot of Rotta model prediction of % agreement with tangent vs. ratio of tire radius over rim width.

6.2   Physical measurements compared to model predictions

The length and width of the contact patch predicted by the Rotta model for a given contact patch area is compared against the lengths and widths of contact patches measured from physical tires under various vertical loads and at various inflation pressures. In general, as can be seen below in **Figure 6-3**, measured contact patch lengths are shorter and the widths are wider than those predicted by the Rotta model.

6.2.1 Contact patch size and shape

Theoretical tire of size = 623 x 23 (mm)

Theoretical tire of size = 623 x 23 (mm)
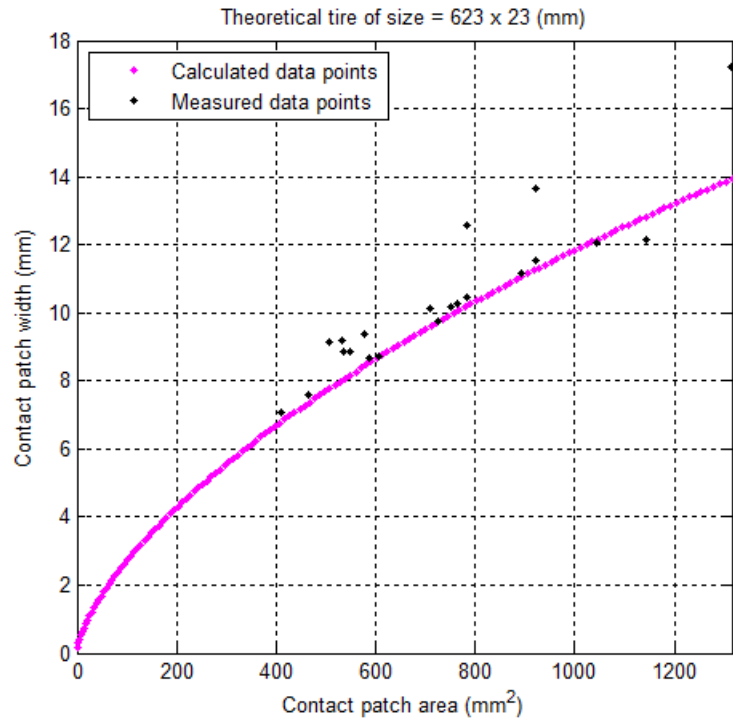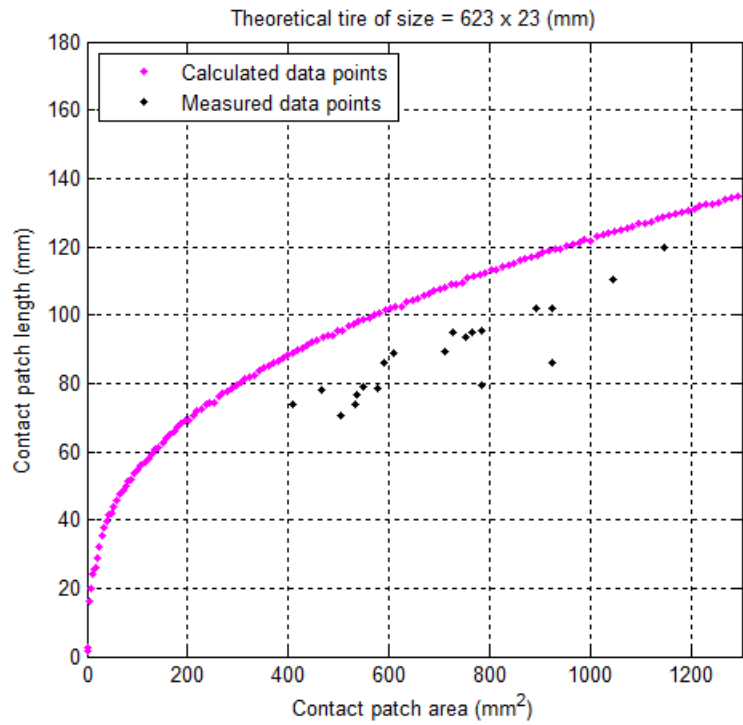
Figure 6-3: Comparison of contact patch area vs. length and width between Rotta model and measured physical tires.

6.2.2   Variation in stiffness with rim width

As can be seen in **Figure 6-4**, the lateral stiffness of a tire varies almost perfectly linearly with the width of the rim. The result is also seen in the Rotta model.
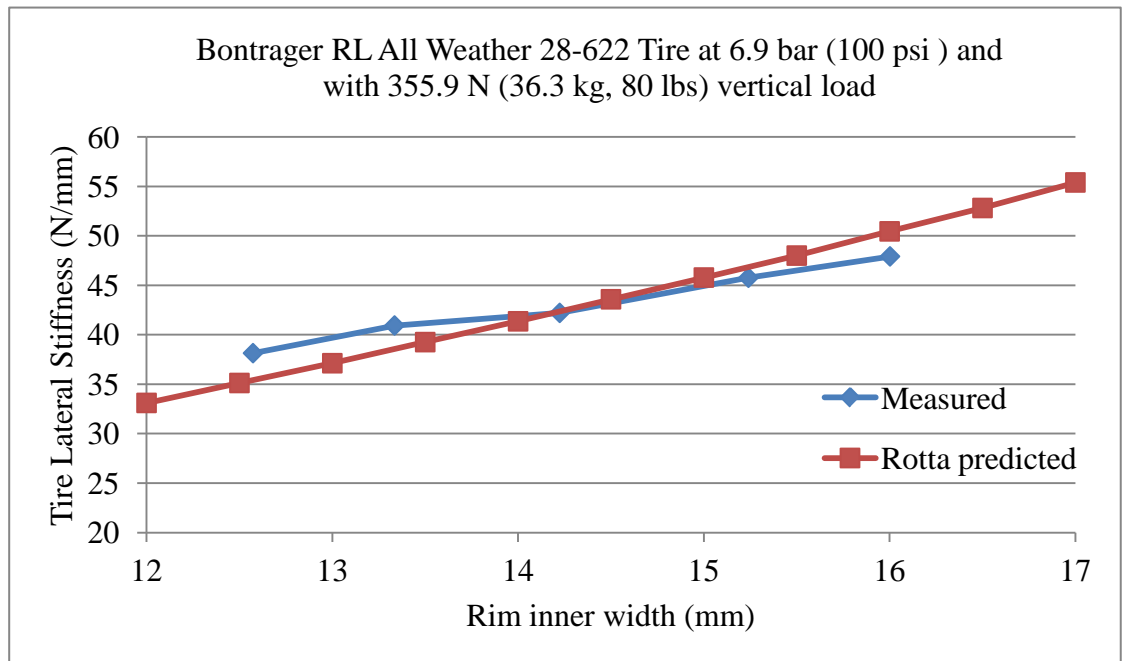
Figure 6-4: Variation in lateral stiffness with rim width, measure and modeled.
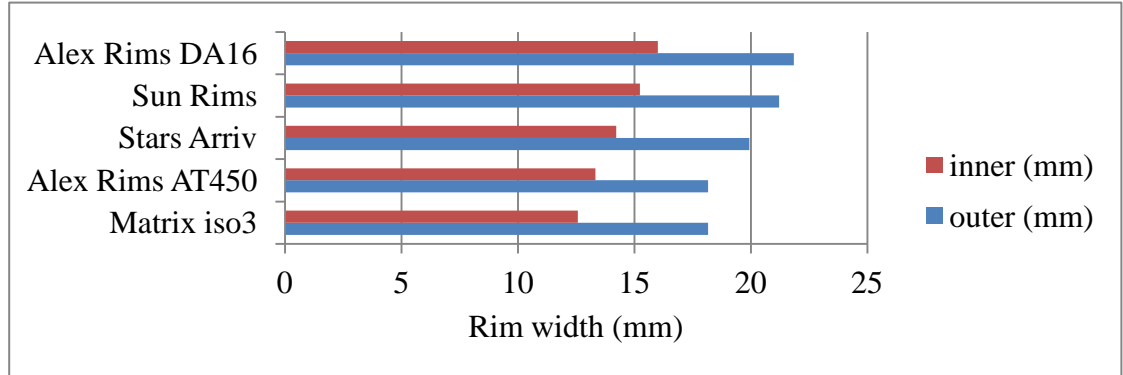
Figure 6-5: Rims and their widths used to generate data in **Figure 6-4**

6.2.3 Variation in stiffness with tire arc length

As shown above, in **Table 4-3** and **Figure 3-4**, four tires with the same 37-mm size designation were found to have actual carcass arc lengths that vary from 73.5 mm to 90 mm and stiffness tends to increase with carcass arc length. The Rotta model predicts the opposite, all else remaining the same. A wider tire, however, would have greater resistance to bending, all else remaining the same, simply due to having a greater area moment of inertia and greater circumferential tension, and thus could have a slower exponential decay of lateral displacement. This could cause more cross sections to be deflected and therefore generate more force. In the current implementation of a taut beam on an elastic foundation, however, as described above in section 0, this is not sufficient to overcome the decrease in stiffness of each cross section.

On the other hand, the trend seen in the measured data could just as well be due to other factors. The four tires in question are not known to be identical except for their carcass

arc length, and the observed increases stiffness with carcass arc length could be due to several other factors, such as increased carcass thickness or modulus of elasticity.

6.2.4   Other stiffnesses

1. Measured normalized lateral stiffness increases as inflation pressure increases in 11 of 11 cases and decreases as vertical load increases in 12 of 12 cases. The Rotta model predicts both of these trends.

2. Measured normalized cornering stiffness decreases as vertical load increases in 14 of 14 cases. The Rotta model predicts this trend for a fixed lateral displacement:

    a. decreases as inflation pressure increases in 8 of 14 cases, ranging in size from 23-37 mm,

    b. increases as inflation pressure increases in 2 of 14 cases, the Cheng Shin Classic Zeppelin at 50 mm and the Maxxis radial at 22 mm,

    c. increases and decreases as inflation pressure increases in 4 of 14 cases, ranging in size from 35 mm to 55 mm.

3. Measured normalized camber stiffness:

    a. decreases as inflation pressure increases and increases as vertical load increases in 13 of 14 cases, which is the opposite of what the Rotta model with no lateral displacement predicts,

b. increases as inflation pressure increases and remains constant as vertical load

increase in 1 of 14 cases.

CHAPTER 7

DISCUSSION

7.1   Difficulties in the physical testing

7.1.1   Difficulty detecting edges of contact patch in ink prints

Connecting isolated sections of tire treads was not too difficult, but determining which sections near the edge actually bore weight and which merely touched the paper was far more problematic.

Once a judgment was made on which isolated ink spots should be included, the convex hull algorithm worked well. The fitted ellipse, however, is slightly problematic because the true shape of the contact patch is not necessarily an ellipse.

7.1.2   Difficulty in setting one angle to zero while second angle is swept

Ideally, camber angle would be set to zero before sweeping slip angle in order to measure cornering stiffness. After sweeping slip angle, it should ideally be set to zero while camber angle is swept in order to measure camber stiffness. The remote actuators on test device number 3 at TU Delft were not designed to do this precisely. Instead, it was up to the operator to observe the reported slip and camber angles, and get them as close to zero as possible.

7.1.3   Flexibility

As with the bicycle wheels used, and mentioned in section 3.3, the frame of all three test devices are flexible, and the sensors used and their locations, especially the displacement sensors on the rim, were designed to accommodate this. The one measurement that could not easily be made, however, is relaxation distance (also Pacejka's relaxation length σ, and Cossalter's L), the distance the tire rolls after a step change in orientation before developing 1-1/e or about 63.2% of its steady-state force. Any measured value would be greatly exaggerated as the wheel and the frame flexed under the building lateral force.

One idea for overcoming this difficulty was to support the rim directly over the contact patch with a stiff mechanism that connected directly to a force sensor. Time did not permit implementing this, however.

7.1.4   Headset bearing in test device number 3

A poorly implemented headset in test device number 3 at TU Delft caused poorly recorded torques for thin tires. This was corrected before testing of the wide tires.

7.1.5   Resonance

Under some configurations, test device number 3 at TU Delft could be seen bouncing significantly. This, of course, was captured in the recorded force and displacement data.

7.2   The difference between measured camber stiffness presented here and values

from Padua published by Doria *et al.*

7.2.1   Reasons that the data presented here may be correct

At TU Delft, the same sensor is used for cornering force, which mostly agrees with Padua values, as for camber force. Errors in measurement of lateral force, tire inflation pressure, or vertical load are not likely to produce a cornering stiffness that agrees with Padua and also a camber stiffness that does not.

The inability to zero slip angle accurately on test device number 3 could introduce noise to the camber stiffness values from configuration to configuration and tire to tire, but the recorded results do not show this inconsistency. If anything, an undetected non-zero slip angle should increase the measured camber stiffness, not decrease it.

The camber force measurements from test device number 3 are symmetrical about zero camber angle, as can be seen in the plots in Appendix B.

As shown above, in section 4.4.2, there is good agreement between the measured values from TU Delft and similar measurements made on the flat-track tester at UWM: The Maxxis Radial Prototype was found to have normalized camber stiffness between 0.0135 and 0.017 at TU Delft and 0.015 at UWM. These are both just below the 0.0174 that represents the net ground reaction force aligning with the plane of the wheel.

7.2.2   Possible problems with the data presented here

There is no correction applied for drum curvature, as explained in section 4.1.2. The

implementation of the Rotta model with no decay length predicts only a 3% difference in

camber force between flat pavement and the 2.5 meter drum at TU Delft.

The Rotta model predicts that a 37 mm tire at 4 bar and under 400 N, which is the best

guess at what Padua used, generates a 214.3 N horizontal force when cambered 45° on a

straight track. If the track is a 2.5-meter diameter drum that curves away from the tire,

then it generates 208.5 N: a 2.7% decrease.

7.2.3   Possible problems with the data presented by Doria *et al.*

The finite radius and asymmetrical nature of the disk used is an obvious suspect, despite

the correction for disk curvature they describe. This implementation of the Rotta model,

however, completely agrees with them on this point, when not using any enhancements

for tire deflections beyond the ends of the contact patch.

The Rotta model predicts that a 37 mm tire at 4 bar and under 400 N, which is the best

guess at what Padua used, generates a 214.3 N horizontal force when cambered 45° on a

straight track. If the track is a 3-meter diameter disk that curves away from the camber,

then it generates 222.4 N, and if the track curves toward the camber, then it generates

208.1 N. The average is 215.25 N, just 0.44% high.

Another possibility is that Doria *et al.* used a much wider rim for their testing than the

18.7 mm wide rim used to test the 37-622 tires on device number 3. As described above

in section 6.1.4, the Rotta model predicts a sensitivity in camber stiffness to rim width.

7.3 Shortcomings of the model

7.3.1   Contact patch shape

The modeled contact patches have sharp points at each end instead of the rounded shape of real contact patches captured in the ink prints. The fitted ellipse matches reality better, but it can only be applied when the contact patch is symmetrical about the centerline.

7.3.2   Flatness and straightness of contact patch in cambered tire

The contact patch generated by the model for non-zero camber angles is not perfectly flat and straight.

7.3.3   The bending of a beam might not be the best analogy for the lateral deflection of an inflated bicycle tire

Other options include the torsion of a thin-walled tube, although that is not perfect either.

7.3.4   Difficulty with convergence

Although initially promising, the integration of the model of a taut beam on an elastic foundation with the Rotta model has proven to be problematic, because the stiffness of the foundation predicted by Rotta is almost perfectly linear. It is possible to reach equilibrium between the foundation stiffness and the force applied in the contact patch at nearly any reasonable lateral displacement. Thus, the model quickly converges to some lateral displacement near the initial guess.

CHAPTER 8

CONCLUSIONS AND FUTURE WORK

8.1 Conclusions

8.1.1 Contributions

The contributions of this project include:

1. Collecting and comparing previously measured bicycle tire data. While a handful of published papers have been found with some bicycle tire data, few refer to another, and none summarizes all the data.

2. Developing three different test devices specific to bicycle tires. When this project began, it was explained that the motorcycle tire test device in Padua was too heavy and therefore unsuitable for testing bicycle tires. There appeared to be no other option available, but to create test devices from scratch.

3. Measuring all relevant properties from a wide variety of tires and under a wide variety of configurations. In most cases, the existing published bicycle tire data includes only one or two variables, and is missing many of the parameters, such as inflation pressure, vertical load, or tire size, necessary to duplicate it.

4. Documenting lateral stiffness dependence on rim width.

5. Documenting rate-independent and rate-dependent hysteresis, which may contribute to rolling resistance.

6. Documenting existence of long decay length beyond the ends of the contact patch as laterally deflected tire centerline returns to equilibrium which needs to be considered for accurate modeling.

7. Documenting that the so-called tangent rule, based on the radial brush model, does not accurately predict bicycle tire camber stiffness.

8. Developing a numerical model, implemented in MATLAB®, which suggests the capability of predicting hard-to-measure properties from properties which are easier to measure. A working predictive model minimizes the need for future testing.

9. Additional observations from the collected data are enumerated above in section 4.1.10.

8.1.2 Tire properties should matter to bicycle stability and handling

The numerical simulation that showed how a change in tire stiffness can affect the wobble mode, shown above in **Figure 1-3**, was based on an assumption that a 10% difference in stiffness is possible. Much larger variations in stiffness than that can be seen in tire test results. Normalized cornering stiffness varies from below 0.15 to over 0.35, which is ±40% from the average, and normalized camber stiffness varies from below 0.0075 to over 0.015, which is ±33% from the average.

8.1.3   Tangent rule for camber stiffness

Tires approach the camber stiffness necessary for the net ground reaction force to be in the plane of the wheel, but most tires with most inflation pressures and under most loads presented here fall below that. Some possibilities for the difference between the results presented here and the results presented by Doria *et al.* are offered above in sections 7.2.2 and 7.2.3, but a conclusive reason has yet to be found.

8.1.4   Model fidelity

Although actual values generated by the model do not exactly match measured values, the trends in contact patch size and lateral stiffness values generated do correspond well with measured data as parameters vary, such as inflation pressure, vertical load, and rim width. Cornering stiffness is problematic because the measured value depends to a large extent on the lateral displacement of the tire centerline, and attempts to model this displacement have proven fruitless. Camber stiffness also depends in part on lateral displacement, but the displacement values are smaller. The trends in measured camber stiffness with vertical load and inflation pressure, however, are opposite those predicted by the Rotta model.

8.2   Future Work

8.2.1   Measuring

8.2.1.1   *Measure sinkage directly*

The current Rotta model implementation assumes that the vertical load divided by the

contact patch area must equal the inflation pressure, but experimental results show that is

is only approximately true. If sinkage was measured physically, and that was used in the

Rotta model, perhaps the contact patch dimensions predicted by the Rotta model would

better match reality. It would also enable comparison with Lyasko's model relating

sinkage, contact patch dimension, and tire dimensions.[48]

8.2.1.2   *Measure contact patch pressure distribution*

Pressure sensitive paper exists that that would enable measuring pressure throughout the

contact patch. This would enable better calculation of contact patch area, and possibly

enable enhancement of the model to better predict exceeding the limits of friction.

8.2.1.3   *Measuring slip angle directly during camber to confirm camber force*

Since an unexplained discrepancy remains between data collected for camber force at

UWM and TU Delft and that most recently published by Doria, *et al.*[28], an independent

method of confirming one or the other set of values is desired. One such method would

be to measure the slip angle of a cambered bicycle tire.

8.2.2   Modeling

8.2.2.1   *Extract analytical relationships from numerical model*

The modeling results are more likely to be useful to more people if they are expressed as analytical relationships linking contact patch behavior to tire dimensions.

8.2.2.2   *Find proper model for lateral displacement of tire centerline for non-zero slip and camber angles*

Since the Rotta model depends on this value so much, predictions of cornering and camber stiffness can not be made from simple geometric tire parameters until this value is also well predicted.

One possibility is to examine the effect mentioned by Pacejka and Sharp in which "a structural interaction involving a tendency of the tyre tread to deflect laterally if an overturning moment M_x is applied to the tread occurs."[17]

8.2.2.3   *Enhance numerical model of bicycle with force generating tires*

Before expending the time, money, and energy required for physical testing, as described below in the next point, the bike with the pair of tires that might reasonably be mounted on it should be modeled together to confirm that they will cause a detectable change in bicycle behavior.
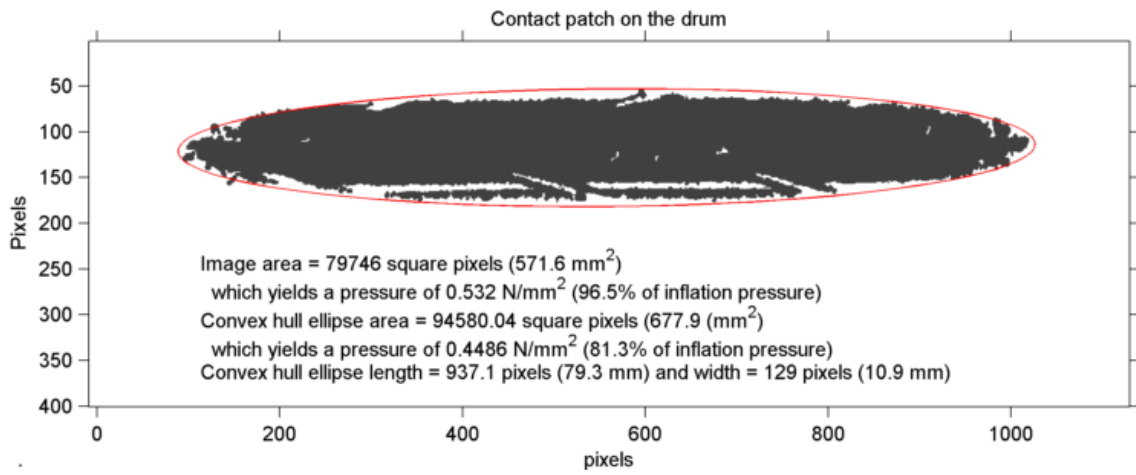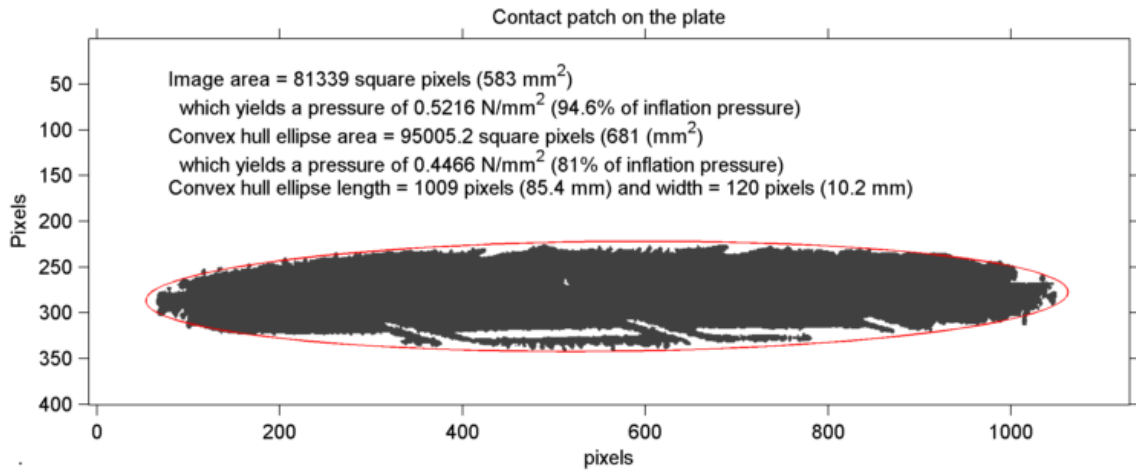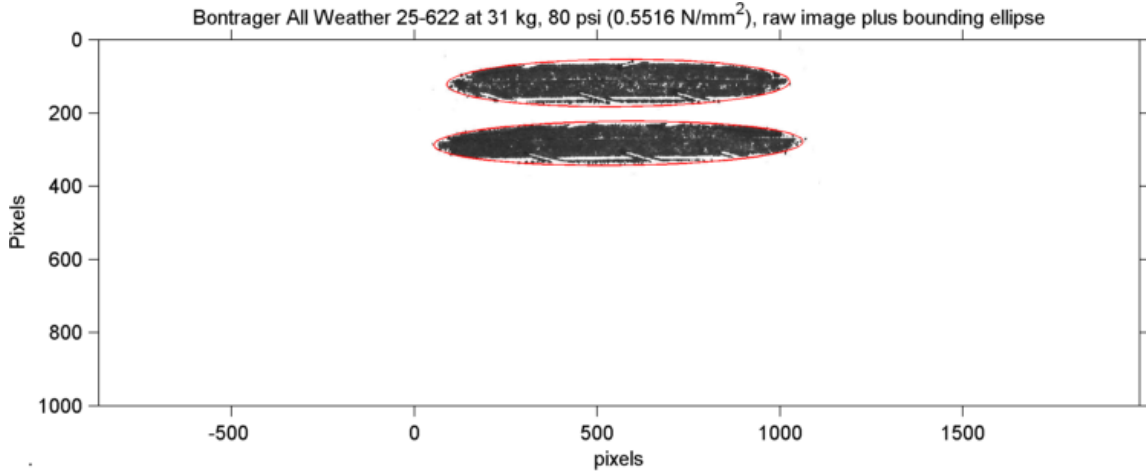
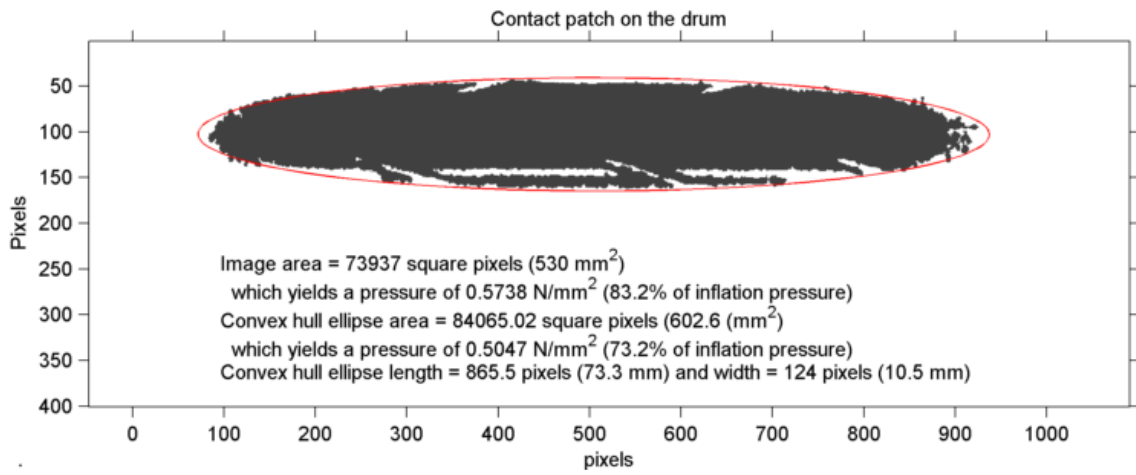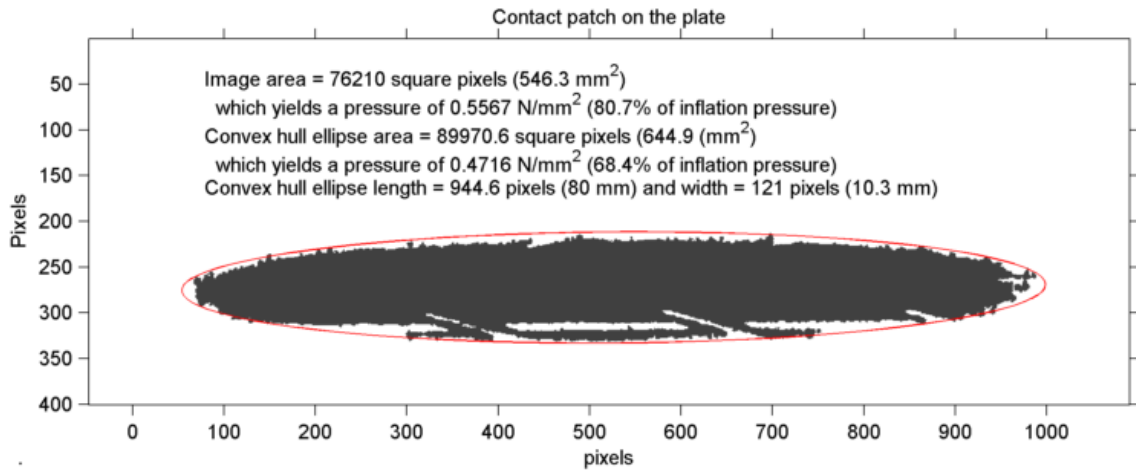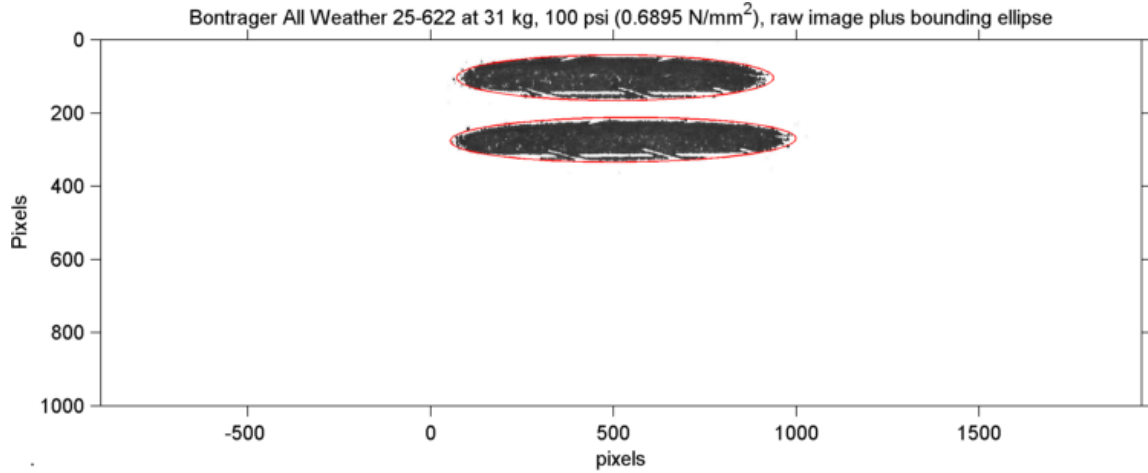### 8.2.3 Validation of tire properties by measuring bicycle behavior

Since one of the purposes of collecting this data in the first place is to model bicycle behavior more accurately, the ideal validation of this data would be to model the behavior of a bicycle with these tires and then measure that behavior physically in a way similar to Kooijman, *et al.*[10]
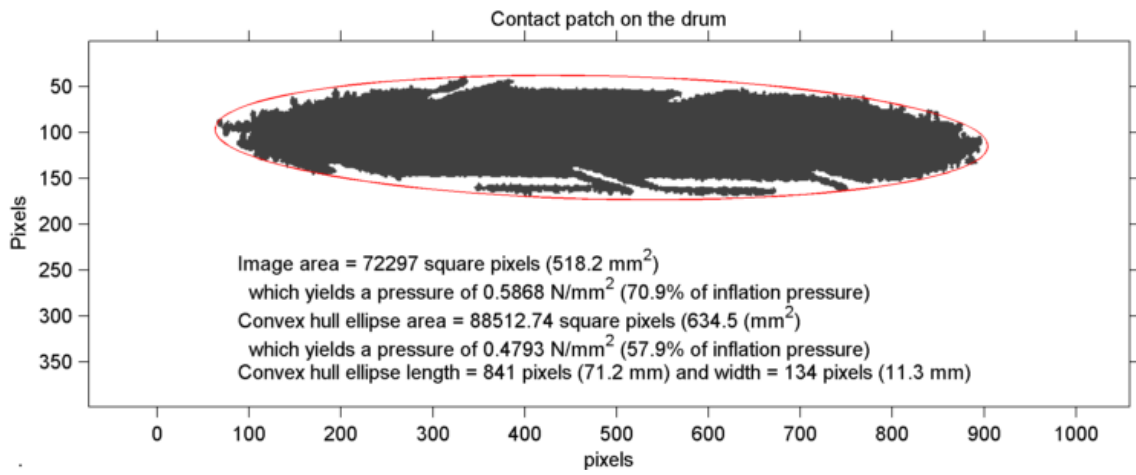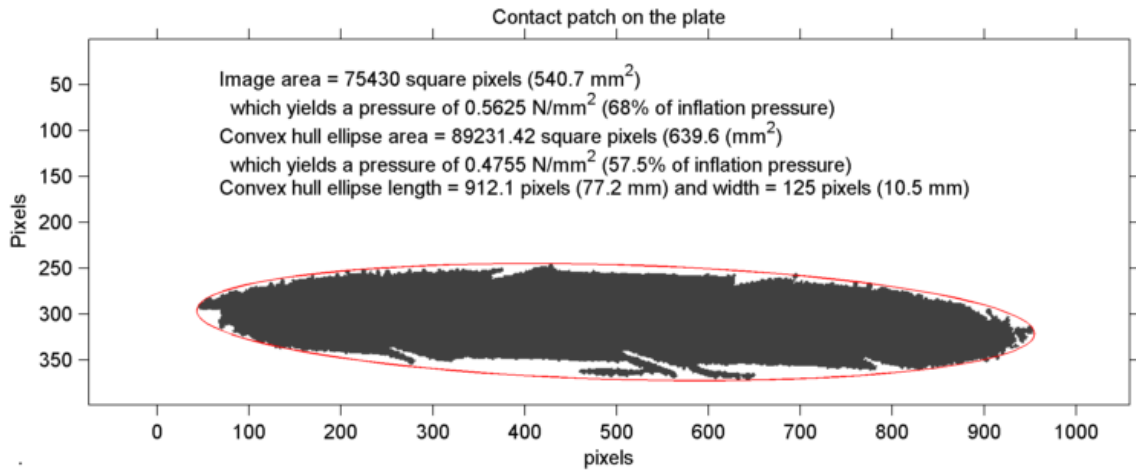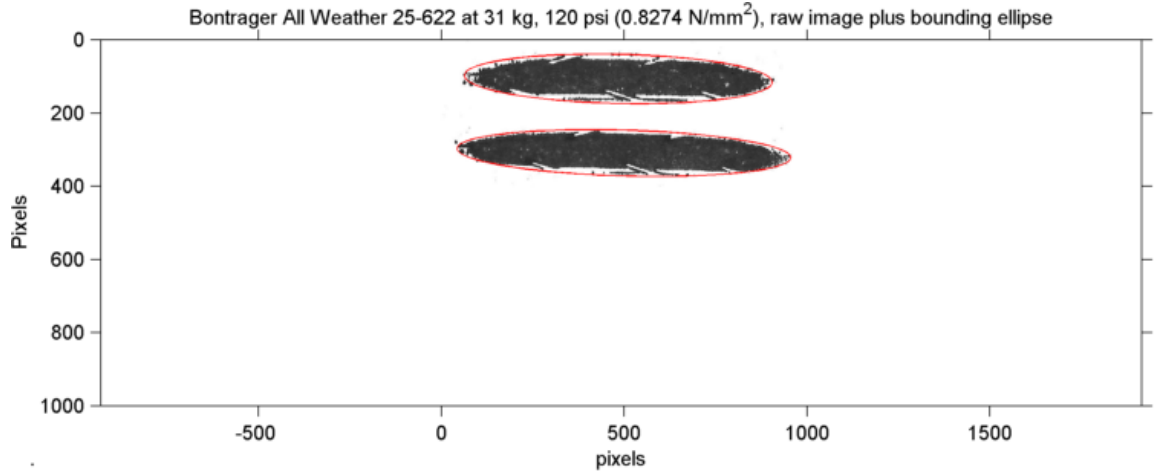
8.2.4 Design a better bicycle tire

While modern pneumatic bicycle tires perform admirably in many situations, they continue to suffer from loss of inflation pressure, both slowly because of the finite permeability of the inner tube, or catastrophically because of a puncture. Meanwhile, non-pneumatic tires suffer from poor rolling resistance and poor handling characteristics. Perhaps a solid understanding of what forces and moments a tire generates and how it does so will enable the design of a non-pneumatic tire that would provide the best of both worlds.

APPENDIX A

INDIVIDUAL TIRE CONTACT PATCHES

Bontrager All Weather 25-622 at 31 kg, 80 psi (0.5516 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 81339 square pixels (583 mm^2)
  which yields a pressure of 0.5216 N/mm^2 (94.6% of inflation pressure)
Convex hull ellipse area = 95005.2 square pixels (681 (mm^2))
  which yields a pressure of 0.4466 N/mm^2 (81% of inflation pressure)
Convex hull ellipse length = 1009 pixels (85.4 mm) and width = 120 pixels (10.2 mm)

Contact patch on the drum

Image area = 79746 square pixels (571.6 mm^2)
  which yields a pressure of 0.532 N/mm^2 (96.5% of inflation pressure)
Convex hull ellipse area = 94580.04 square pixels (677.9 (mm^2))
  which yields a pressure of 0.4486 N/mm^2 (81.3% of inflation pressure)
Convex hull ellipse length = 937.1 pixels (79.3 mm) and width = 129 pixels (10.9 mm)

Bontrager All Weather 25-622 at 31 kg, 100 psi (0.6895 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 76210 square pixels (546.3 mm^2)
  which yields a pressure of 0.5567 N/mm^2 (80.7% of inflation pressure)
Convex hull ellipse area = 89970.6 square pixels (644.9 (mm^2))
  which yields a pressure of 0.4716 N/mm^2 (68.4% of inflation pressure)
Convex hull ellipse length = 944.6 pixels (80 mm) and width = 121 pixels (10.3 mm)

Contact patch on the drum

Image area = 73937 square pixels (530 mm^2)
  which yields a pressure of 0.5738 N/mm^2 (83.2% of inflation pressure)
Convex hull ellipse area = 84065.02 square pixels (602.6 (mm^2))
  which yields a pressure of 0.5047 N/mm^2 (73.2% of inflation pressure)
Convex hull ellipse length = 865.5 pixels (73.3 mm) and width = 124 pixels (10.5 mm)

Bontrager All Weather 25-622 at 31 kg, 120 psi (0.8274 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 75430 square pixels (540.7 mm^2)
which yields a pressure of 0.5625 N/mm^2 (68% of inflation pressure)
Convex hull ellipse area = 89231.42 square pixels (639.6 (mm^2))
which yields a pressure of 0.4755 N/mm^2 (57.5% of inflation pressure)
Convex hull ellipse length = 912.1 pixels (77.2 mm) and width = 125 pixels (10.5 mm)

Contact patch on the drum

Image area = 72297 square pixels (518.2 mm^2)
which yields a pressure of 0.5868 N/mm^2 (70.9% of inflation pressure)
Convex hull ellipse area = 88512.74 square pixels (634.5 (mm^2))
which yields a pressure of 0.4793 N/mm^2 (57.9% of inflation pressure)
Convex hull ellipse length = 841 pixels (71.2 mm) and width = 134 pixels (11.3 mm)

Bontrager All Weather 25-622 at 53.5 kg, 120 psi (0.8274 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 102894 square pixels (737.5 mm^2)
 which yields a pressure of 0.7116 N/mm^2 (86% of inflation pressure)
Convex hull ellipse area = 123058.6 square pixels (882.1 (mm^2))
 which yields a pressure of 0.595 N/mm^2 (71.9% of inflation pressure)
Convex hull ellipse length = 1096 pixels (92.8 mm) and width = 143 pixels (12.1 mm)

Contact patch on the drum

Image area = 101408 square pixels (726.9 mm^2)
 which yields a pressure of 0.722 N/mm^2 (87.3% of inflation pressure)
Convex hull ellipse area = 121098.5 square pixels (868 (mm^2))
 which yields a pressure of 0.6046 N/mm^2 (73.1% of inflation pressure)
Convex hull ellipse length = 1034 pixels (87.6 mm) and width = 149 pixels (12.6 mm)

Bontrager All Weather 25-622 at 74.5 kg, 80 psi (0.5516 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 153194 square pixels (1098 mm^2)
 which yields a pressure of 0.6656 N/mm^2 (121% of inflation pressure)
Convex hull ellipse area = 184550.2 square pixels (1323 (mm^2))
 which yields a pressure of 0.5525 N/mm^2 (100% of inflation pressure)
Convex hull ellipse length = 1403 pixels (119 mm) and width = 167 pixels (14.2 mm)

Contact patch on the drum

Image area = 147454 square pixels (1057 mm^2)
 which yields a pressure of 0.6915 N/mm^2 (125% of inflation pressure)
Convex hull ellipse area = 180044 square pixels (1291 (mm^2))
 which yields a pressure of 0.5663 N/mm^2 (103% of inflation pressure)
Convex hull ellipse length = 1307 pixels (111 mm) and width = 175 pixels (14.9 mm)

Bontrager All Weather 25-622 at 74.5 kg, 100 psi (0.6895 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 140356 square pixels (1006 mm^2)
 which yields a pressure of 0.7264 N/mm^2 (105% of inflation pressure)
Convex hull ellipse area = 174553.3 square pixels (1251 (mm^2))
 which yields a pressure of 0.5841 N/mm^2 (84.7% of inflation pressure)
Convex hull ellipse length = 1293 pixels (109 mm) and width = 172 pixels (14.5 mm)

Contact patch on the drum

Image area = 134665 square pixels (965.3 mm^2)
 which yields a pressure of 0.7571 N/mm^2 (110% of inflation pressure)
Convex hull ellipse area = 170568.4 square pixels (1223 (mm^2))
 which yields a pressure of 0.5978 N/mm^2 (86.7% of inflation pressure)
Convex hull ellipse length = 1189 pixels (101 mm) and width = 183 pixels (15.5 mm)

Bontrager All Weather 25-622 at 74.5 kg, 120 psi (0.8274 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 122238 square pixels (876.2 mm^2)
 which yields a pressure of 0.8341 N/mm^2 (101% of inflation pressure)
Convex hull ellipse area = 147416.1 square pixels (1057 (mm^2))
 which yields a pressure of 0.6917 N/mm^2 (83.6% of inflation pressure)
Convex hull ellipse length = 1206 pixels (102 mm) and width = 156 pixels (13.2 mm)

Contact patch on the drum

Image area = 123946 square pixels (888.4 mm^2)
 which yields a pressure of 0.8226 N/mm^2 (99.4% of inflation pressure)
Convex hull ellipse area = 150335.8 square pixels (1078 (mm^2))
 which yields a pressure of 0.6782 N/mm^2 (82% of inflation pressure)
Convex hull ellipse length = 1165 pixels (98.6 mm) and width = 164 pixels (13.9 mm)

Bontrager All Weather 28-622 at 31 kg, 80 psi (0.5516 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 84699 square pixels (607.1 mm^2)
 which yields a pressure of 0.5009 N/mm^2 (90.8% of inflation pressure)
Convex hull ellipse area = 103430.3 square pixels (741.4 (mm^2))
 which yields a pressure of 0.4102 N/mm^2 (74.4% of inflation pressure)
Convex hull ellipse length = 991.8 pixels (84 mm) and width = 133 pixels (11.2 mm)

Contact patch on the drum

Image area = 80423 square pixels (576.5 mm^2)
 which yields a pressure of 0.5275 N/mm^2 (95.6% of inflation pressure)
Convex hull ellipse area = 97135.45 square pixels (696.3 (mm^2))
 which yields a pressure of 0.4368 N/mm^2 (79.2% of inflation pressure)
Convex hull ellipse length = 928.4 pixels (78.6 mm) and width = 133 pixels (11.3 mm)

Bontrager All Weather 28-622 at 31 kg, 100 psi (0.6895 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 76936 square pixels (551.5 mm^2)
 which yields a pressure of 0.5515 N/mm^2 (80% of inflation pressure)
Convex hull ellipse area = 90517.05 square pixels (648.8 (mm^2))
 which yields a pressure of 0.4687 N/mm^2 (68% of inflation pressure)
Convex hull ellipse length = 929 pixels (78.7 mm) and width = 124 pixels (10.5 mm)

Contact patch on the drum

Image area = 71754 square pixels (514.3 mm^2)
 which yields a pressure of 0.5913 N/mm^2 (85.8% of inflation pressure)
Convex hull ellipse area = 84182.82 square pixels (603.4 (mm^2))
 which yields a pressure of 0.504 N/mm^2 (73.1% of inflation pressure)
Convex hull ellipse length = 842.6 pixels (71.3 mm) and width = 127 pixels (10.8 mm)

Bontrager All Weather 28-622 at 31 kg, 120 psi (0.8274 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 72462 square pixels (519.4 mm^2)
which yields a pressure of 0.5855 N/mm^2 (70.8% of inflation pressure)
Convex hull ellipse area = 83426.99 square pixels (598 (mm^2))
which yields a pressure of 0.5085 N/mm^2 (61.5% of inflation pressure)
Convex hull ellipse length = 872.9 pixels (73.9 mm) and width = 122 pixels (10.3 mm)

Contact patch on the drum

Image area = 72499 square pixels (519.7 mm^2)
which yields a pressure of 0.5852 N/mm^2 (70.7% of inflation pressure)
Convex hull ellipse area = 88130.57 square pixels (631.7 (mm^2))
which yields a pressure of 0.4814 N/mm^2 (58.2% of inflation pressure)
Convex hull ellipse length = 822.6 pixels (69.6 mm) and width = 136 pixels (11.5 mm)

Bontrager All Weather 28-622 at 53.5 kg, 100 psi (0.6895 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 107651 square pixels (771.6 mm^2)
which yields a pressure of 0.6802 N/mm^2 (98.6% of inflation pressure)
Convex hull ellipse area = 130342 square pixels (934.3 (mm^2))
which yields a pressure of 0.5618 N/mm^2 (81.5% of inflation pressure)
Convex hull ellipse length = 1136 pixels (96.1 mm) and width = 146 pixels (12.4 mm)

Contact patch on the drum

Image area = 108160 square pixels (775.3 mm^2)
which yields a pressure of 0.677 N/mm^2 (98.2% of inflation pressure)
Convex hull ellipse area = 135965 square pixels (974.6 (mm^2))
which yields a pressure of 0.5385 N/mm^2 (78.1% of inflation pressure)
Convex hull ellipse length = 1046 pixels (88.6 mm) and width = 165 pixels (14 mm)

Bontrager All Weather 28-622 at 53.5 kg, 120 psi (0.8274 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 100026 square pixels (717 mm^2)

which yields a pressure of 0.732 N/mm^2 (88.5% of inflation pressure)

Convex hull ellipse area = 118923.8 square pixels (852.4 (mm^2))

which yields a pressure of 0.6157 N/mm^2 (74.4% of inflation pressure)

Convex hull ellipse length = 1080 pixels (91.5 mm) and width = 140 pixels (11.9 mm)

Contact patch on the drum

Image area = 95316 square pixels (683.2 mm^2)

which yields a pressure of 0.7682 N/mm^2 (92.8% of inflation pressure)

Convex hull ellipse area = 114480.8 square pixels (820.6 (mm^2))

which yields a pressure of 0.6396 N/mm^2 (77.3% of inflation pressure)

Convex hull ellipse length = 963 pixels (81.5 mm) and width = 151 pixels (12.8 mm)

Bontrager All Weather 28-622 at 74.5 kg, 120 psi (0.8274 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 128010 square pixels (917.6 mm^2)
which yields a pressure of 0.7965 N/mm^2 (96.3% of inflation pressure)
Convex hull ellipse area = 157181.7 square pixels (1127 (mm^2))
which yields a pressure of 0.6487 N/mm^2 (78.4% of inflation pressure)
Convex hull ellipse length = 1211 pixels (103 mm) and width = 165 pixels (14 mm)

Contact patch on the drum

Image area = 120981 square pixels (867.2 mm^2)
which yields a pressure of 0.8428 N/mm^2 (102% of inflation pressure)
Convex hull ellipse area = 144930.2 square pixels (1039 (mm^2))
which yields a pressure of 0.7035 N/mm^2 (85% of inflation pressure)
Convex hull ellipse length = 1097 pixels (92.9 mm) and width = 168 pixels (14.2 mm)

Cheng Shin Classic Zeppelin 50-622 at 33 kg, 29 psi (0.1999 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 157978 square pixels (1132 mm^2)
which yields a pressure of 0.2859 N/mm^2 (143% of inflation pressure)
Convex hull ellipse area = 252006.1 square pixels (1806 (mm^2))
which yields a pressure of 0.1792 N/mm^2 (89.6% of inflation pressure)
Convex hull ellipse length = 1324 pixels (112 mm) and width = 242 pixels (20.5 mm)

Contact patch on the drum

Image area = 164444 square pixels (1179 mm^2)
which yields a pressure of 0.2746 N/mm^2 (137% of inflation pressure)
Convex hull ellipse area = 265001.3 square pixels (1900 (mm^2))
which yields a pressure of 0.1704 N/mm^2 (85.2% of inflation pressure)
Convex hull ellipse length = 1212 pixels (103 mm) and width = 278 pixels (23.6 mm)

Cheng Shin Classic Zeppelin 50-622 at 33 kg, 43.5 psi (0.2999 N/mm^2), raw image plus bounding ellipse

Contact patch on the drum

Image area = 136055 square pixels (975.2 mm^2)
which yields a pressure of 0.332 N/mm^2 (111% of inflation pressure)
Convex hull ellipse area = 198693.6 square pixels (1424 (mm^2)
which yields a pressure of 0.2273 N/mm^2 (75.8% of inflation pressure)
Convex hull ellipse length = 1075 pixels (91 mm) and width = 235 pixels (19.9 mm)

Contact patch on the plate

Image area = 134415 square pixels (963.5 mm^2)
which yields a pressure of 0.336 N/mm^2 (112% of inflation pressure)
Convex hull ellipse area = 189468.5 square pixels (1358 (mm^2)
which yields a pressure of 0.2384 N/mm^2 (79.5% of inflation pressure)
Convex hull ellipse length = 1203 pixels (102 mm) and width = 201 pixels (17 mm)

Cheng Shin Classic Zeppelin 50-622 at 33 kg, 58 psi (0.3999 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 87895 square pixels (630 mm^2)
which yields a pressure of 0.5138 N/mm^2 (128% of inflation pressure)
Convex hull ellipse area = 150632.3 square pixels (1080 (mm^2))
which yields a pressure of 0.2998 N/mm^2 (75% of inflation pressure)
Convex hull ellipse length = 1040 pixels (88.1 mm) and width = 184 pixels (15.6 mm)

Contact patch on the drum

Image area = 114508 square pixels (820.8 mm^2)
which yields a pressure of 0.3944 N/mm^2 (98.6% of inflation pressure)
Convex hull ellipse area = 179320.7 square pixels (1285 (mm^2))
which yields a pressure of 0.2519 N/mm^2 (63% of inflation pressure)
Convex hull ellipse length = 994.6 pixels (84.2 mm) and width = 230 pixels (19.4 mm)

Cheng Shin Classic Zeppelin 50-622 at 55 kg, 29 psi (0.1999 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 231812 square pixels (1662 mm^2)
which yields a pressure of 0.3247 N/mm^2 (162% of inflation pressure)
Convex hull ellipse area = 381255.6 square pixels (2733 (mm^2)
which yields a pressure of 0.1974 N/mm^2 (98.7% of inflation pressure)
Convex hull ellipse length = 1598 pixels (135 mm) and width = 304 pixels (25.7 mm)

Contact patch on the drum

Image area = 260127 square pixels (1865 mm^2)
which yields a pressure of 0.2894 N/mm^2 (145% of inflation pressure)
Convex hull ellipse area = 411047.6 square pixels (2946 (mm^2)
which yields a pressure of 0.1831 N/mm^2 (91.6% of inflation pressure)
Convex hull ellipse length = 1461 pixels (124 mm) and width = 358 pixels (30.3 mm)

Cheng Shin Classic Zeppelin 50-622 at 55 kg, 43.5 psi (0.2999 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 196637 square pixels (1409 mm^2)
 which yields a pressure of 0.3828 N/mm^2 (128% of inflation pressure)
Convex hull ellipse area = 298553 square pixels (2140 (mm^2))
 which yields a pressure of 0.2521 N/mm^2 (84.1% of inflation pressure)
Convex hull ellipse length = 1448 pixels (123 mm) and width = 263 pixels (22.2 mm)

Contact patch on the drum

Image area = 212847 square pixels (1526 mm^2)
 which yields a pressure of 0.3536 N/mm^2 (118% of inflation pressure)
Convex hull ellipse area = 337661.3 square pixels (2420 (mm^2))
 which yields a pressure of 0.2229 N/mm^2 (74.3% of inflation pressure)
Convex hull ellipse length = 1328 pixels (112 mm) and width = 324 pixels (27.4 mm)

Cheng Shin Classic Zeppelin 50-622 at 55 kg, 58 psi (0.3999 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 164188 square pixels (1177 mm^2)

which yields a pressure of 0.4585 N/mm^2 (115% of inflation pressure)

Convex hull ellipse area = 247646.3 square pixels (1775 (mm^2))

which yields a pressure of 0.304 N/mm^2 (76% of inflation pressure)

Convex hull ellipse length = 1272 pixels (108 mm) and width = 248 pixels (21 mm)

Contact patch on the drum

Image area = 167851 square pixels (1203 mm^2)

which yields a pressure of 0.4485 N/mm^2 (112% of inflation pressure)

Convex hull ellipse area = 270595.3 square pixels (1940 (mm^2))

which yields a pressure of 0.2782 N/mm^2 (69.6% of inflation pressure)

Convex hull ellipse length = 1195 pixels (101 mm) and width = 288 pixels (24.4 mm)

Cheng Shin Classic Zeppelin 50-622 at 74.5 kg, 29 psi (0.1999 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 351065 square pixels (2516 mm^2)
which yields a pressure of 0.2904 N/mm^2 (145% of inflation pressure)
Convex hull ellipse area = 520114.3 square pixels (3728 (mm^2))
which yields a pressure of 0.196 N/mm^2 (98% of inflation pressure)
Convex hull ellipse length = 1769 pixels (150 mm) and width = 374 pixels (31.7 mm)

Contact patch on the drum

Image area = 363099 square pixels (2603 mm^2)
which yields a pressure of 0.2808 N/mm^2 (140% of inflation pressure)
Convex hull ellipse area = 544239.5 square pixels (3901 (mm^2))
which yields a pressure of 0.1873 N/mm^2 (93.7% of inflation pressure)
Convex hull ellipse length = 1665 pixels (141 mm) and width = 416 pixels (35.2 mm)

Cheng Shin Classic Zeppelin 50-622 at 74.5 kg, 43.5 psi (0.2999 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 244470 square pixels (1752 mm^2)
which yields a pressure of 0.4171 N/mm^2 (139% of inflation pressure)
Convex hull ellipse area = 371906.6 square pixels (2666 (mm^2)
which yields a pressure of 0.2742 N/mm^2 (91.4% of inflation pressure)
Convex hull ellipse length = 1525 pixels (129 mm) and width = 311 pixels (26.3 mm)

Contact patch on the drum

Image area = 282084 square pixels (2022 mm^2)
which yields a pressure of 0.3615 N/mm^2 (121% of inflation pressure)
Convex hull ellipse area = 421598.4 square pixels (3022 (mm^2)
which yields a pressure of 0.2418 N/mm^2 (80.6% of inflation pressure)
Convex hull ellipse length = 1457 pixels (123 mm) and width = 369 pixels (31.2 mm)

Cheng Shin Classic Zeppelin 50-622 at 74.5 kg, 58 psi (0.3999 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 221631 square pixels (1589 mm^2)
which yields a pressure of 0.46 N/mm^2 (115% of inflation pressure)
Convex hull ellipse area = 307777.8 square pixels (2206 (mm^2)
which yields a pressure of 0.3313 N/mm^2 (82.8% of inflation pressure)
Convex hull ellipse length = 1445 pixels (122 mm) and width = 271 pixels (23 mm)

Contact patch on the drum

Image area = 225549 square pixels (1617 mm^2)
which yields a pressure of 0.4521 N/mm^2 (113% of inflation pressure)
Convex hull ellipse area = 325643.4 square pixels (2334 (mm^2)
which yields a pressure of 0.3131 N/mm^2 (78.3% of inflation pressure)
Convex hull ellipse length = 1294 pixels (110 mm) and width = 320 pixels (27.1 mm)

Continental Top Contact Winter 37-622 at 33 kg, 50 psi (0.3447 N/mm^2), raw image plus bounding ellipse

Contact patch on the drum

Image area = 33059 square pixels (237 mm^2)
  which yields a pressure of 1.366 N/mm^2 (396% of inflation pressure)
Convex hull ellipse area = 124768.1 square pixels (894.3 (mm^2))
  which yields a pressure of 0.362 N/mm^2 (105% of inflation pressure)
Convex hull ellipse length = 985.4 pixels (83.4 mm) and width = 161 pixels (13.6 mm)

Contact patch on the plate

Image area = 35154 square pixels (252 mm^2)
  which yields a pressure of 1.285 N/mm^2 (373% of inflation pressure)
Convex hull ellipse area = 135066.9 square pixels (968.2 (mm^2))
  which yields a pressure of 0.3344 N/mm^2 (97% of inflation pressure)
Convex hull ellipse length = 1055 pixels (89.3 mm) and width = 163 pixels (13.8 mm)

Continental Top Contact Winter 37-622 at 33 kg, 58 psi (0.3999 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 38907 square pixels (278.9 mm^2)
which yields a pressure of 1.161 N/mm^2 (290% of inflation pressure)
Convex hull ellipse area = 134613.8 square pixels (964.9 (mm^2))
which yields a pressure of 0.3355 N/mm^2 (83.9% of inflation pressure)
Convex hull ellipse length = 1072 pixels (90.8 mm) and width = 160 pixels (13.5 mm)

Contact patch on the drum

Image area = 37430 square pixels (268.3 mm^2)
which yields a pressure of 1.207 N/mm^2 (302% of inflation pressure)
Convex hull ellipse area = 134405.5 square pixels (963.4 (mm^2))
which yields a pressure of 0.336 N/mm^2 (84% of inflation pressure)
Convex hull ellipse length = 1027 pixels (87 mm) and width = 167 pixels (14.1 mm)

Continental Top Contact Winter 37-622 at 33 kg, 70 psi (0.4826 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 39332 square pixels (281.9 mm^2)
 which yields a pressure of 1.148 N/mm^2 (238% of inflation pressure)
Convex hull ellipse area = 126655.1 square pixels (907.9 (mm^2)
 which yields a pressure of 0.3566 N/mm^2 (73.9% of inflation pressure)
Convex hull ellipse length = 1043 pixels (88.3 mm) and width = 155 pixels (13.1 mm)

Contact patch on the drum

Image area = 35284 square pixels (252.9 mm^2)
 which yields a pressure of 1.28 N/mm^2 (265% of inflation pressure)
Convex hull ellipse area = 122276.4 square pixels (876.5 (mm^2)
 which yields a pressure of 0.3694 N/mm^2 (76.5% of inflation pressure)
Convex hull ellipse length = 951.3 pixels (80.5 mm) and width = 164 pixels (13.9 mm)

Continental Top Contact Winter 37-622 at 33 kg, 90 psi (0.6205 N/mm^2), raw image plus bounding ellipse

Contact patch on the drum

Image area = 34855 square pixels (249.8 mm^2)
which yields a pressure of 1.296 N/mm^2 (209% of inflation pressure)
Convex hull ellipse area = 121011.9 square pixels (867.4 (mm^2)
which yields a pressure of 0.3732 N/mm^2 (60.1% of inflation pressure)
Convex hull ellipse length = 930.9 pixels (78.8 mm) and width = 166 pixels (14 mm)

Contact patch on the plate

Image area = 47445 square pixels (340.1 mm^2)
which yields a pressure of 0.9519 N/mm^2 (153% of inflation pressure)
Convex hull ellipse area = 117395.9 square pixels (841.5 (mm^2)
which yields a pressure of 0.3847 N/mm^2 (62% of inflation pressure)
Convex hull ellipse length = 967.4 pixels (81.9 mm) and width = 155 pixels (13.1 mm)

Continental Top Contact Winter 37-622 at 55 kg, 50 psi (0.3447 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 94123 square pixels (674.7 mm^2)
 which yields a pressure of 0.7997 N/mm^2 (232% of inflation pressure)
Convex hull ellipse area = 202621.8 square pixels (1452 (mm^2))
 which yields a pressure of 0.3715 N/mm^2 (108% of inflation pressure)
Convex hull ellipse length = 1389 pixels (118 mm) and width = 186 pixels (15.7 mm)

Contact patch on the drum

Image area = 101463 square pixels (727.3 mm^2)
 which yields a pressure of 0.7419 N/mm^2 (215% of inflation pressure)
Convex hull ellipse area = 220490.4 square pixels (1580 (mm^2))
 which yields a pressure of 0.3414 N/mm^2 (99% of inflation pressure)
Convex hull ellipse length = 1307 pixels (111 mm) and width = 215 pixels (18.2 mm)

Continental Top Contact Winter 37-622 at 55 kg, 58 psi (0.3999 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 93258 square pixels (668.5 mm^2)
which yields a pressure of 0.8071 N/mm^2 (202% of inflation pressure)
Convex hull ellipse area = 199245.6 square pixels (1428 (mm^2))
which yields a pressure of 0.3778 N/mm^2 (94.5% of inflation pressure)
Convex hull ellipse length = 1314 pixels (111 mm) and width = 193 pixels (16.3 mm)

Contact patch on the drum

Image area = 89270 square pixels (639.9 mm^2)
which yields a pressure of 0.8432 N/mm^2 (211% of inflation pressure)
Convex hull ellipse area = 204031 square pixels (1462 (mm^2))
which yields a pressure of 0.3689 N/mm^2 (92.3% of inflation pressure)
Convex hull ellipse length = 1224 pixels (104 mm) and width = 212 pixels (18 mm)

Continental Top Contact Winter 37-622 at 55 kg, 70 psi (0.4826 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 85666 square pixels (614 mm^2)
which yields a pressure of 0.8787 N/mm^2 (182% of inflation pressure)
Convex hull ellipse area = 187544.3 square pixels (1344 (mm^2))
which yields a pressure of 0.4014 N/mm^2 (83.2% of inflation pressure)
Convex hull ellipse length = 1253 pixels (106 mm) and width = 191 pixels (16.1 mm)

Contact patch on the drum

Image area = 79992 square pixels (573.4 mm^2)
which yields a pressure of 0.941 N/mm^2 (195% of inflation pressure)
Convex hull ellipse area = 178074.9 square pixels (1276 (mm^2))
which yields a pressure of 0.4227 N/mm^2 (87.6% of inflation pressure)
Convex hull ellipse length = 1165 pixels (98.6 mm) and width = 195 pixels (16.5 mm)

Continental Top Contact Winter 37-622 at 55 kg, 90 psi (0.6205 N/mm^2), raw image plus bounding ellipse

Contact patch on the drum

Image area = 79303 square pixels (568.4 mm^2)
 which yields a pressure of 0.9492 N/mm^2 (153% of inflation pressure)
Convex hull ellipse area = 160312.3 square pixels (1149 (mm^2))
 which yields a pressure of 0.4695 N/mm^2 (75.7% of inflation pressure)
Convex hull ellipse length = 1086 pixels (91.9 mm) and width = 188 pixels (15.9 mm)

Contact patch on the plate

Image area = 77478 square pixels (555.4 mm^2)
 which yields a pressure of 0.9715 N/mm^2 (157% of inflation pressure)
Convex hull ellipse area = 157918.7 square pixels (1132 (mm^2))
 which yields a pressure of 0.4767 N/mm^2 (76.8% of inflation pressure)
Convex hull ellipse length = 1105 pixels (93.5 mm) and width = 182 pixels (15.4 mm)

Continental Top Contact Winter 37-622 at 74.5 kg, 50 psi (0.3447 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 152176 square pixels (1091 mm^2)
which yields a pressure of 0.67 N/mm^2 (194% of inflation pressure)
Convex hull ellipse area = 284698.8 square pixels (2041 (mm^2))
which yields a pressure of 0.3581 N/mm^2 (104% of inflation pressure)
Convex hull ellipse length = 1637 pixels (139 mm) and width = 221 pixels (18.8 mm)

Contact patch on the drum

Image area = 103852 square pixels (744.4 mm^2)
which yields a pressure of 0.9818 N/mm^2 (285% of inflation pressure)
Convex hull ellipse area = 242186.5 square pixels (1736 (mm^2))
which yields a pressure of 0.421 N/mm^2 (122% of inflation pressure)
Convex hull ellipse length = 1323 pixels (112 mm) and width = 233 pixels (19.7 mm)

Continental Top Contact Winter 37-622 at 74.5 kg, 58 psi (0.3999 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 117595 square pixels (842.9 mm^2)
 which yields a pressure of 0.867 N/mm^2 (217% of inflation pressure)
Convex hull ellipse area = 237973 square pixels (1706 (mm^2))
 which yields a pressure of 0.4285 N/mm^2 (107% of inflation pressure)
Convex hull ellipse length = 1369 pixels (116 mm) and width = 221 pixels (18.7 mm)

Contact patch on the drum

Image area = 138965 square pixels (996.1 mm^2)
 which yields a pressure of 0.7337 N/mm^2 (183% of inflation pressure)
Convex hull ellipse area = 256998.8 square pixels (1842 (mm^2))
 which yields a pressure of 0.3967 N/mm^2 (99.2% of inflation pressure)
Convex hull ellipse length = 1439 pixels (122 mm) and width = 227 pixels (19.3 mm)

Continental Top Contact Winter 37-622 at 74.5 kg, 70 psi (0.4826 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 114367 square pixels (819.8 mm^2)
which yields a pressure of 0.8915 N/mm^2 (185% of inflation pressure)
Convex hull ellipse area = 235794.3 square pixels (1690 (mm^2))
which yields a pressure of 0.4324 N/mm^2 (89.6% of inflation pressure)
Convex hull ellipse length = 1305 pixels (110 mm) and width = 230 pixels (19.5 mm)

Contact patch on the drum

Image area = 103696 square pixels (743.3 mm^2)
which yields a pressure of 0.9833 N/mm^2 (204% of inflation pressure)
Convex hull ellipse area = 211515.4 square pixels (1516 (mm^2))
which yields a pressure of 0.482 N/mm^2 (99.9% of inflation pressure)
Convex hull ellipse length = 1214 pixels (103 mm) and width = 222 pixels (18.8 mm)

Continental Top Contact Winter 37-622 at 74.5 kg, 90 psi (0.6205 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 128509 square pixels (921.1 mm^2)
which yields a pressure of 0.7934 N/mm^2 (128% of inflation pressure)
Convex hull ellipse area = 217643.1 square pixels (1560 (mm^2))
which yields a pressure of 0.4685 N/mm^2 (75.5% of inflation pressure)
Convex hull ellipse length = 1365 pixels (116 mm) and width = 203 pixels (17.2 mm)

Contact patch on the drum

Image area = 116305 square pixels (833.7 mm^2)
which yields a pressure of 0.8767 N/mm^2 (141% of inflation pressure)
Convex hull ellipse area = 207731.1 square pixels (1489 (mm^2))
which yields a pressure of 0.4908 N/mm^2 (79.1% of inflation pressure)
Convex hull ellipse length = 1248 pixels (106 mm) and width = 212 pixels (17.9 mm)

Maxxis radial 22-622 at 31 kg, 100 psi (0.6895 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 66420 square pixels (476.1 mm^2)
which yields a pressure of 0.6388 N/mm^2 (92.6% of inflation pressure)
Convex hull ellipse area = 76533.94 square pixels (548.6 (mm^2))
which yields a pressure of 0.5543 N/mm^2 (80.4% of inflation pressure)
Convex hull ellipse length = 935.5 pixels (79.2 mm) and width = 104 pixels (8.82 mm)

Contact patch on the drum

Image area = 63042 square pixels (451.9 mm^2)
which yields a pressure of 0.673 N/mm^2 (97.6% of inflation pressure)
Convex hull ellipse area = 72221.41 square pixels (517.7 (mm^2))
which yields a pressure of 0.5875 N/mm^2 (85.2% of inflation pressure)
Convex hull ellipse length = 852.6 pixels (72.2 mm) and width = 108 pixels (9.13 mm)

Maxxis radial 22-622 at 31 kg, 120 psi (0.8274 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 59341 square pixels (425.4 mm^2)
 which yields a pressure of 0.715 N/mm^2 (86.4% of inflation pressure)
Convex hull ellipse area = 67564.78 square pixels (484.3 (mm^2))
 which yields a pressure of 0.6279 N/mm^2 (75.9% of inflation pressure)
Convex hull ellipse length = 888.3 pixels (75.2 mm) and width = 96.8 pixels (8.2 mm)

Contact patch on the drum

Image area = 59015 square pixels (423 mm^2)
 which yields a pressure of 0.7189 N/mm^2 (86.9% of inflation pressure)
Convex hull ellipse area = 68366.57 square pixels (490 (mm^2))
 which yields a pressure of 0.6206 N/mm^2 (75% of inflation pressure)
Convex hull ellipse length = 825.2 pixels (69.9 mm) and width = 105 pixels (8.93 mm)

Maxxis radial 22-622 at 53.5 kg, 120 psi (0.8274 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 87719 square pixels (628.8 mm^2)
  which yields a pressure of 0.8347 N/mm^2 (101% of inflation pressure)
Convex hull ellipse area = 98909.41 square pixels (709 (mm^2))
  which yields a pressure of 0.7403 N/mm^2 (89.5% of inflation pressure)
Convex hull ellipse length = 1078 pixels (91.3 mm) and width = 117 pixels (9.89 mm)

Contact patch on the drum

Image area = 87332 square pixels (626 mm^2)
  which yields a pressure of 0.8384 N/mm^2 (101% of inflation pressure)
Convex hull ellipse area = 97743.21 square pixels (700.6 (mm^2))
  which yields a pressure of 0.7491 N/mm^2 (90.5% of inflation pressure)
Convex hull ellipse length = 1007 pixels (85.2 mm) and width = 124 pixels (10.5 mm)

Maxxis radial 22-622 at 74.5 kg, 120 psi (0.8274 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 112192 square pixels (804.2 mm^2)
 which yields a pressure of 0.9088 N/mm^2 (110% of inflation pressure)
Convex hull ellipse area = 125322 square pixels (898.3 (mm^2))
 which yields a pressure of 0.8136 N/mm^2 (98.3% of inflation pressure)
Convex hull ellipse length = 1167 pixels (98.8 mm) and width = 137 pixels (11.6 mm)

Contact patch on the drum

Image area = 114115 square pixels (818 mm^2)
 which yields a pressure of 0.8935 N/mm^2 (108% of inflation pressure)
Convex hull ellipse area = 125897.8 square pixels (902.4 (mm^2))
 which yields a pressure of 0.8099 N/mm^2 (97.9% of inflation pressure)
Convex hull ellipse length = 1105 pixels (93.6 mm) and width = 145 pixels (12.3 mm)

Maxxis radial 22-622 at 74.5 kg, 140 psi (0.9653 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 104377 square pixels (748.2 mm^2)
which yields a pressure of 0.9768 N/mm^2 (101% of inflation pressure)
Convex hull ellipse area = 116378.1 square pixels (834.2 (mm^2))
which yields a pressure of 0.8761 N/mm^2 (90.8% of inflation pressure)
Convex hull ellipse length = 1142 pixels (96.6 mm) and width = 130 pixels (11 mm)

Contact patch on the drum

Image area = 103677 square pixels (743.1 mm^2)
which yields a pressure of 0.9834 N/mm^2 (102% of inflation pressure)
Convex hull ellipse area = 113587.6 square pixels (814.2 (mm^2))
which yields a pressure of 0.8976 N/mm^2 (93% of inflation pressure)
Convex hull ellipse length = 1073 pixels (90.9 mm) and width = 135 pixels (11.4 mm)

Schwalbe Big Apple 55-622 at 33 kg, 29 psi (0.1999 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 134517 square pixels (964.2 mm^2)
which yields a pressure of 0.3357 N/mm^2 (168% of inflation pressure)
Convex hull ellipse area = 176025.3 square pixels (1262 (mm^2))
which yields a pressure of 0.2566 N/mm^2 (128% of inflation pressure)
Convex hull ellipse length = 1164 pixels (98.5 mm) and width = 193 pixels (16.3 mm)

Contact patch on the drum

Image area = 140153 square pixels (1005 mm^2)
which yields a pressure of 0.3222 N/mm^2 (161% of inflation pressure)
Convex hull ellipse area = 182420.4 square pixels (1308 (mm^2))
which yields a pressure of 0.2476 N/mm^2 (124% of inflation pressure)
Convex hull ellipse length = 1085 pixels (91.9 mm) and width = 214 pixels (18.1 mm)

Schwalbe Big Apple 55-622 at 33 kg, 43.5 psi (0.2999 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 116986 square pixels (838.5 mm^2)
which yields a pressure of 0.3861 N/mm^2 (129% of inflation pressure)
Convex hull ellipse area = 152112.6 square pixels (1090 (mm^2))
which yields a pressure of 0.2969 N/mm^2 (99% of inflation pressure)
Convex hull ellipse length = 1062 pixels (89.9 mm) and width = 182 pixels (15.4 mm)

Contact patch on the drum

Image area = 120032 square pixels (860.4 mm^2)
which yields a pressure of 0.3763 N/mm^2 (125% of inflation pressure)
Convex hull ellipse area = 158997.8 square pixels (1140 (mm^2))
which yields a pressure of 0.2841 N/mm^2 (94.7% of inflation pressure)
Convex hull ellipse length = 981.2 pixels (83.1 mm) and width = 206 pixels (17.5 mm)

Schwalbe Big Apple 55-622 at 33 kg, 58 psi (0.3999 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 105588 square pixels (756.8 mm^2)
which yields a pressure of 0.4277 N/mm^2 (107% of inflation pressure)
Convex hull ellipse area = 137438.6 square pixels (985.1 (mm^2))
which yields a pressure of 0.3286 N/mm^2 (82.2% of inflation pressure)
Convex hull ellipse length = 1022 pixels (86.5 mm) and width = 171 pixels (14.5 mm)

Contact patch on the drum

Image area = 108806 square pixels (779.9 mm^2)
which yields a pressure of 0.4151 N/mm^2 (104% of inflation pressure)
Convex hull ellipse area = 141359.1 square pixels (1013 (mm^2))
which yields a pressure of 0.3195 N/mm^2 (79.9% of inflation pressure)
Convex hull ellipse length = 924.5 pixels (78.3 mm) and width = 195 pixels (16.5 mm)

Schwalbe Big Apple 55-622 at 55 kg, 29 psi (0.1999 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 212824 square pixels (1526 mm^2)
which yields a pressure of 0.3537 N/mm^2 (177% of inflation pressure)
Convex hull ellipse area = 340640.9 square pixels (2442 (mm^2)
which yields a pressure of 0.221 N/mm^2 (111% of inflation pressure)
Convex hull ellipse length = 1476 pixels (125 mm) and width = 294 pixels (24.9 mm)

Contact patch on the drum

Image area = 214309 square pixels (1536 mm^2)
which yields a pressure of 0.3512 N/mm^2 (176% of inflation pressure)
Convex hull ellipse area = 349314.2 square pixels (2504 (mm^2)
which yields a pressure of 0.2155 N/mm^2 (108% of inflation pressure)
Convex hull ellipse length = 1321 pixels (112 mm) and width = 337 pixels (28.5 mm)

Schwalbe Big Apple 55-622 at 55 kg, 43.5 psi (0.2999 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 195699 square pixels (1403 mm^2)
 which yields a pressure of 0.3846 N/mm^2 (128% of inflation pressure)
Convex hull ellipse area = 244888.3 square pixels (1755 (mm^2))
 which yields a pressure of 0.3074 N/mm^2 (102% of inflation pressure)
Convex hull ellipse length = 1344 pixels (114 mm) and width = 232 pixels (19.6 mm)

Contact patch on the drum

Image area = 190710 square pixels (1367 mm^2)
 which yields a pressure of 0.3947 N/mm^2 (132% of inflation pressure)
Convex hull ellipse area = 292699.2 square pixels (2098 (mm^2))
 which yields a pressure of 0.2572 N/mm^2 (85.7% of inflation pressure)
Convex hull ellipse length = 1219 pixels (103 mm) and width = 306 pixels (25.9 mm)

Schwalbe Big Apple 55-622 at 55 kg, 58 psi (0.3999 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 160805 square pixels (1153 mm^2)

which yields a pressure of 0.4681 N/mm^2 (117% of inflation pressure)

Convex hull ellipse area = 201945.5 square pixels (1448 (mm^2))

which yields a pressure of 0.3727 N/mm^2 (93.2% of inflation pressure)

Convex hull ellipse length = 1243 pixels (105 mm) and width = 207 pixels (17.5 mm)

Contact patch on the drum

Image area = 165533 square pixels (1187 mm^2)

which yields a pressure of 0.4547 N/mm^2 (114% of inflation pressure)

Convex hull ellipse area = 232635.4 square pixels (1668 (mm^2))

which yields a pressure of 0.3236 N/mm^2 (80.9% of inflation pressure)

Convex hull ellipse length = 1143 pixels (96.8 mm) and width = 259 pixels (21.9 mm)

Schwalbe Big Apple 55-622 at 74.5 kg, 29 psi (0.1999 N/mm^2), raw image plus bounding ellipse

Contact patch on the drum

Image area = 299994 square pixels (2150 mm^2)
 which yields a pressure of 0.3399 N/mm^2 (170% of inflation pressure)
Convex hull ellipse area = 458048.3 square pixels (3283 (mm^2)
 which yields a pressure of 0.2226 N/mm^2 (111% of inflation pressure)
Convex hull ellipse length = 1564 pixels (132 mm) and width = 373 pixels (31.6 mm)

Contact patch on the plate

Image area = 305438 square pixels (2189 mm^2)
 which yields a pressure of 0.3338 N/mm^2 (167% of inflation pressure)
Convex hull ellipse area = 451487.3 square pixels (3236 (mm^2)
 which yields a pressure of 0.2258 N/mm^2 (113% of inflation pressure)
Convex hull ellipse length = 1719 pixels (146 mm) and width = 334 pixels (28.3 mm)

Schwalbe Big Apple 55-622 at 74.5 kg, 43.5 psi (0.2999 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 244371 square pixels (1752 mm^2)
which yields a pressure of 0.4172 N/mm^2 (139% of inflation pressure)
Convex hull ellipse area = 331900.2 square pixels (2379 (mm^2)
which yields a pressure of 0.3072 N/mm^2 (102% of inflation pressure)
Convex hull ellipse length = 1530 pixels (130 mm) and width = 276 pixels (23.4 mm)

Contact patch on the drum

Image area = 251676 square pixels (1804 mm^2)
which yields a pressure of 0.4051 N/mm^2 (135% of inflation pressure)
Convex hull ellipse area = 368774.1 square pixels (2643 (mm^2)
which yields a pressure of 0.2765 N/mm^2 (92.2% of inflation pressure)
Convex hull ellipse length = 1403 pixels (119 mm) and width = 335 pixels (28.3 mm)

Schwalbe Big Apple 55-622 at 74.5 kg, 58 psi (0.3999 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 215362 square pixels (1544 mm^2)
which yields a pressure of 0.4734 N/mm^2 (118% of inflation pressure)
Convex hull ellipse area = 264588.6 square pixels (1897 (mm^2)
which yields a pressure of 0.3854 N/mm^2 (96.4% of inflation pressure)
Convex hull ellipse length = 1414 pixels (120 mm) and width = 238 pixels (20.2 mm)

Contact patch on the drum

Image area = 207477 square pixels (1487 mm^2)
which yields a pressure of 0.4914 N/mm^2 (123% of inflation pressure)
Convex hull ellipse area = 296894.2 square pixels (2128 (mm^2)
which yields a pressure of 0.3434 N/mm^2 (85.9% of inflation pressure)
Convex hull ellipse length = 1266 pixels (107 mm) and width = 299 pixels (25.3 mm)

Schwalbe Kojak 35-622 at 33 kg, 50 psi (0.3447 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 84692 square pixels (607.1 mm^2)
which yields a pressure of 0.5333 N/mm^2 (155% of inflation pressure)
Convex hull ellipse area = 103636.6 square pixels (742.9 (mm^2)
which yields a pressure of 0.4358 N/mm^2 (126% of inflation pressure)
Convex hull ellipse length = 1136 pixels (96.2 mm) and width = 116 pixels (9.83 mm)

Contact patch on the drum

Image area = 87261 square pixels (625.5 mm^2)
which yields a pressure of 0.5176 N/mm^2 (150% of inflation pressure)
Convex hull ellipse area = 111841.6 square pixels (801.7 (mm^2)
which yields a pressure of 0.4038 N/mm^2 (117% of inflation pressure)
Convex hull ellipse length = 1004 pixels (85 mm) and width = 142 pixels (12 mm)

Schwalbe Kojak 35-622 at 33 kg, 58 psi (0.3999 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 77487 square pixels (555.4 mm^2)

which yields a pressure of 0.5829 N/mm^2 (146% of inflation pressure)

Convex hull ellipse area = 95764.12 square pixels (686.4 (mm^2))

which yields a pressure of 0.4716 N/mm^2 (118% of inflation pressure)

Convex hull ellipse length = 1066 pixels (90.2 mm) and width = 114 pixels (9.69 mm)

Contact patch on the drum

Image area = 90458 square pixels (648.4 mm^2)

which yields a pressure of 0.4993 N/mm^2 (125% of inflation pressure)

Convex hull ellipse area = 109479.9 square pixels (784.7 (mm^2))

which yields a pressure of 0.4125 N/mm^2 (103% of inflation pressure)

Convex hull ellipse length = 980.2 pixels (83 mm) and width = 142 pixels (12 mm)

Schwalbe Kojak 35-622 at 33 kg, 70 psi (0.4826 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 74895 square pixels (536.8 mm^2)
 which yields a pressure of 0.603 N/mm^2 (125% of inflation pressure)
Convex hull ellipse area = 89567.07 square pixels (642 (mm^2))
 which yields a pressure of 0.5042 N/mm^2 (104% of inflation pressure)
Convex hull ellipse length = 934.9 pixels (79.1 mm) and width = 122 pixels (10.3 mm)

Contact patch on the drum

Image area = 88064 square pixels (631.2 mm^2)
 which yields a pressure of 0.5129 N/mm^2 (106% of inflation pressure)
Convex hull ellipse area = 105215.6 square pixels (754.2 (mm^2))
 which yields a pressure of 0.4292 N/mm^2 (88.9% of inflation pressure)
Convex hull ellipse length = 908.5 pixels (76.9 mm) and width = 147 pixels (12.5 mm)

Schwalbe Kojak 35-622 at 33 kg, 90 psi (0.6205 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 76148 square pixels (545.8 mm^2)
which yields a pressure of 0.5931 N/mm^2 (95.6% of inflation pressure)
Convex hull ellipse area = 87072.09 square pixels (624.1 (mm^2))
which yields a pressure of 0.5187 N/mm^2 (83.6% of inflation pressure)
Convex hull ellipse length = 895.9 pixels (75.9 mm) and width = 124 pixels (10.5 mm)

Contact patch on the drum

Image area = 81863 square pixels (586.8 mm^2)
which yields a pressure of 0.5517 N/mm^2 (88.9% of inflation pressure)
Convex hull ellipse area = 93392.06 square pixels (669.4 (mm^2))
which yields a pressure of 0.4836 N/mm^2 (77.9% of inflation pressure)
Convex hull ellipse length = 834.6 pixels (70.7 mm) and width = 142 pixels (12.1 mm)

Schwalbe Kojak 35-622 at 55 kg, 50 psi (0.3447 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 149825 square pixels (1074 mm^2)

  which yields a pressure of 0.5024 N/mm^2 (146% of inflation pressure)

Convex hull ellipse area = 174419.7 square pixels (1250 (mm^2))

  which yields a pressure of 0.4316 N/mm^2 (125% of inflation pressure)

Convex hull ellipse length = 1354 pixels (115 mm) and width = 164 pixels (13.9 mm)

Contact patch on the drum

Image area = 165196 square pixels (1184 mm^2)

  which yields a pressure of 0.4557 N/mm^2 (132% of inflation pressure)

Convex hull ellipse area = 197180.5 square pixels (1413 (mm^2))

  which yields a pressure of 0.3817 N/mm^2 (111% of inflation pressure)

Convex hull ellipse length = 1252 pixels (106 mm) and width = 200 pixels (17 mm)

Schwalbe Kojak 35-622 at 55 kg, 58 psi (0.3999 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 131337 square pixels (941.4 mm^2)
 which yields a pressure of 0.5731 N/mm^2 (143% of inflation pressure)
Convex hull ellipse area = 148884.9 square pixels (1067 (mm^2))
 which yields a pressure of 0.5056 N/mm^2 (126% of inflation pressure)
Convex hull ellipse length = 1232 pixels (104 mm) and width = 154 pixels (13 mm)

Contact patch on the drum

Image area = 146001 square pixels (1047 mm^2)
 which yields a pressure of 0.5156 N/mm^2 (129% of inflation pressure)
Convex hull ellipse area = 168111.3 square pixels (1205 (mm^2))
 which yields a pressure of 0.4478 N/mm^2 (112% of inflation pressure)
Convex hull ellipse length = 1167 pixels (98.8 mm) and width = 183 pixels (15.5 mm)

Schwalbe Kojak 35-622 at 55 kg, 70 psi (0.4826 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 129358 square pixels (927.2 mm^2)
which yields a pressure of 0.5819 N/mm^2 (121% of inflation pressure)
Convex hull ellipse area = 143188.2 square pixels (1026 (mm^2)
which yields a pressure of 0.5257 N/mm^2 (109% of inflation pressure)
Convex hull ellipse length = 1199 pixels (101 mm) and width = 152 pixels (12.9 mm)

Contact patch on the drum

Image area = 144392 square pixels (1035 mm^2)
which yields a pressure of 0.5213 N/mm^2 (108% of inflation pressure)
Convex hull ellipse area = 168937.1 square pixels (1211 (mm^2)
which yields a pressure of 0.4456 N/mm^2 (92.3% of inflation pressure)
Convex hull ellipse length = 1122 pixels (95 mm) and width = 192 pixels (16.2 mm)

Schwalbe Kojak 35-622 at 55 kg, 90 psi (0.6205 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 108613 square pixels (778.5 mm^2)

which yields a pressure of 0.693 N/mm^2 (112% of inflation pressure)

Convex hull ellipse area = 124485.5 square pixels (892.3 (mm^2))

which yields a pressure of 0.6047 N/mm^2 (97.4% of inflation pressure)

Convex hull ellipse length = 1096 pixels (92.8 mm) and width = 145 pixels (12.2 mm)

Contact patch on the drum

Image area = 114552 square pixels (821.1 mm^2)

which yields a pressure of 0.6571 N/mm^2 (106% of inflation pressure)

Convex hull ellipse area = 134788.3 square pixels (966.2 (mm^2))

which yields a pressure of 0.5585 N/mm^2 (90% of inflation pressure)

Convex hull ellipse length = 1012 pixels (85.7 mm) and width = 170 pixels (14.4 mm)

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Schwalbe Kojak 35-622 at 74.5 kg, 50 psi (0.3447 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 213060 square pixels (1527 mm^2)
  which yields a pressure of 0.4786 N/mm^2 (139% of inflation pressure)
Convex hull ellipse area = 236673.5 square pixels (1696 (mm^2))
  which yields a pressure of 0.4308 N/mm^2 (125% of inflation pressure)
Convex hull ellipse length = 1579 pixels (134 mm) and width = 191 pixels (16.2 mm)

Contact patch on the drum

Image area = 237552 square pixels (1703 mm^2)
  which yields a pressure of 0.4292 N/mm^2 (125% of inflation pressure)
Convex hull ellipse area = 271112.9 square pixels (1943 (mm^2))
  which yields a pressure of 0.3761 N/mm^2 (109% of inflation pressure)
Convex hull ellipse length = 1454 pixels (123 mm) and width = 237 pixels (20.1 mm)

Schwalbe Kojak 35-622 at 74.5 kg, 58 psi (0.3999 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 191417 square pixels (1372 mm^2)
   which yields a pressure of 0.5327 N/mm^2 (133% of inflation pressure)
Convex hull ellipse area = 212395.3 square pixels (1522 (mm^2))
   which yields a pressure of 0.4801 N/mm^2 (120% of inflation pressure)
Convex hull ellipse length = 1468 pixels (124 mm) and width = 184 pixels (15.6 mm)

Contact patch on the drum

Image area = 205617 square pixels (1474 mm^2)
   which yields a pressure of 0.4959 N/mm^2 (124% of inflation pressure)
Convex hull ellipse area = 231823.9 square pixels (1662 (mm^2))
   which yields a pressure of 0.4398 N/mm^2 (110% of inflation pressure)
Convex hull ellipse length = 1354 pixels (115 mm) and width = 218 pixels (18.5 mm)

Schwalbe Kojak 35-622 at 74.5 kg, 70 psi (0.4826 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 167181 square pixels (1198 mm^2)
  which yields a pressure of 0.6099 N/mm^2 (126% of inflation pressure)
Convex hull ellipse area = 186320.6 square pixels (1336 (mm^2))
  which yields a pressure of 0.5472 N/mm^2 (113% of inflation pressure)
Convex hull ellipse length = 1381 pixels (117 mm) and width = 172 pixels (14.5 mm)

Contact patch on the drum

Image area = 183712 square pixels (1317 mm^2)
  which yields a pressure of 0.555 N/mm^2 (115% of inflation pressure)
Convex hull ellipse area = 212264.6 square pixels (1521 (mm^2))
  which yields a pressure of 0.4803 N/mm^2 (99.5% of inflation pressure)
Convex hull ellipse length = 1271 pixels (108 mm) and width = 213 pixels (18 mm)

Schwalbe Kojak 35-622 at 74.5 kg, 90 psi (0.6205 N/mm^2), raw image plus bounding ellipse

Contact patch on the drum

Image area = 159041 square pixels (1140 mm^2)
which yields a pressure of 0.6411 N/mm^2 (103% of inflation pressure)
Convex hull ellipse area = 176515.7 square pixels (1265 (mm^2))
which yields a pressure of 0.5776 N/mm^2 (93.1% of inflation pressure)
Convex hull ellipse length = 1154 pixels (97.7 mm) and width = 195 pixels (16.5 mm)

Contact patch on the plate

Image area = 144350 square pixels (1035 mm^2)
which yields a pressure of 0.7063 N/mm^2 (114% of inflation pressure)
Convex hull ellipse area = 166790.6 square pixels (1196 (mm^2))
which yields a pressure of 0.6113 N/mm^2 (98.5% of inflation pressure)
Convex hull ellipse length = 1244 pixels (105 mm) and width = 171 pixels (14.5 mm)

Schwalbe Marathon Plus 37-622 at 33 kg, 50 psi (0.3447 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 69956 square pixels (501.4 mm^2)
which yields a pressure of 0.6456 N/mm^2 (187% of inflation pressure)
Convex hull ellipse area = 126562.8 square pixels (907.2 (mm^2))
which yields a pressure of 0.3568 N/mm^2 (104% of inflation pressure)
Convex hull ellipse length = 1038 pixels (87.9 mm) and width = 155 pixels (13.1 mm)

Contact patch on the drum

Image area = 76673 square pixels (549.6 mm^2)
which yields a pressure of 0.589 N/mm^2 (171% of inflation pressure)
Convex hull ellipse area = 124013.5 square pixels (888.9 (mm^2))
which yields a pressure of 0.3642 N/mm^2 (106% of inflation pressure)
Convex hull ellipse length = 996.6 pixels (84.4 mm) and width = 158 pixels (13.4 mm)

Schwalbe Marathon Plus 37-622 at 33 kg, 58 psi (0.3999 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 75213 square pixels (539.1 mm^2)
which yields a pressure of 0.6005 N/mm^2 (150% of inflation pressure)
Convex hull ellipse area = 124848 square pixels (894.9 (mm^2))
which yields a pressure of 0.3617 N/mm^2 (90.5% of inflation pressure)
Convex hull ellipse length = 1088 pixels (92.1 mm) and width = 146 pixels (12.4 mm)

Contact patch on the drum

Image area = 78647 square pixels (563.7 mm^2)
which yields a pressure of 0.5743 N/mm^2 (144% of inflation pressure)
Convex hull ellipse area = 134110.1 square pixels (961.3 (mm^2))
which yields a pressure of 0.3368 N/mm^2 (84.2% of inflation pressure)
Convex hull ellipse length = 1030 pixels (87.2 mm) and width = 166 pixels (14 mm)

Schwalbe Marathon Plus 37-622 at 33 kg, 70 psi (0.4826 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 76089 square pixels (545.4 mm^2)
which yields a pressure of 0.5936 N/mm^2 (123% of inflation pressure)
Convex hull ellipse area = 121285.9 square pixels (869.4 (mm^2))
which yields a pressure of 0.3724 N/mm^2 (77.2% of inflation pressure)
Convex hull ellipse length = 998 pixels (84.5 mm) and width = 155 pixels (13.1 mm)

Contact patch on the drum

Image area = 78246 square pixels (560.9 mm^2)
which yields a pressure of 0.5772 N/mm^2 (120% of inflation pressure)
Convex hull ellipse area = 122416.6 square pixels (877.5 (mm^2))
which yields a pressure of 0.3689 N/mm^2 (76.4% of inflation pressure)
Convex hull ellipse length = 910.9 pixels (77.1 mm) and width = 171 pixels (14.5 mm)

Schwalbe Marathon Plus 37-622 at 33 kg, 90 psi (0.6205 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 74908 square pixels (536.9 mm^2)
which yields a pressure of 0.6029 N/mm^2 (97.2% of inflation pressure)
Convex hull ellipse area = 119224.8 square pixels (854.6 (mm^2))
which yields a pressure of 0.3788 N/mm^2 (61% of inflation pressure)
Convex hull ellipse length = 959.2 pixels (81.2 mm) and width = 158 pixels (13.4 mm)

Contact patch on the drum

Image area = 75850 square pixels (543.7 mm^2)
which yields a pressure of 0.5954 N/mm^2 (96% of inflation pressure)
Convex hull ellipse area = 118428.7 square pixels (848.9 (mm^2))
which yields a pressure of 0.3814 N/mm^2 (61.5% of inflation pressure)
Convex hull ellipse length = 895 pixels (75.8 mm) and width = 168 pixels (14.3 mm)

Schwalbe Marathon Plus 37-622 at 55 kg, 50 psi (0.3447 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 106702 square pixels (764.8 mm^2)
which yields a pressure of 0.7054 N/mm^2 (205% of inflation pressure)
Convex hull ellipse area = 171483.5 square pixels (1229 (mm^2))
which yields a pressure of 0.439 N/mm^2 (127% of inflation pressure)
Convex hull ellipse length = 1287 pixels (109 mm) and width = 170 pixels (14.4 mm)

Contact patch on the drum

Image area = 103240 square pixels (740 mm^2)
which yields a pressure of 0.7291 N/mm^2 (211% of inflation pressure)
Convex hull ellipse area = 167605 square pixels (1201 (mm^2))
which yields a pressure of 0.4491 N/mm^2 (130% of inflation pressure)
Convex hull ellipse length = 1195 pixels (101 mm) and width = 179 pixels (15.1 mm)

Schwalbe Marathon Plus 37-622 at 55 kg, 58 psi (0.3999 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 111506 square pixels (799.3 mm^2)
  which yields a pressure of 0.6751 N/mm^2 (169% of inflation pressure)
Convex hull ellipse area = 170722 square pixels (1224 (mm^2))
  which yields a pressure of 0.4409 N/mm^2 (110% of inflation pressure)
Convex hull ellipse length = 1250 pixels (106 mm) and width = 174 pixels (14.7 mm)

Contact patch on the drum

Image area = 113556 square pixels (814 mm^2)
  which yields a pressure of 0.6629 N/mm^2 (166% of inflation pressure)
Convex hull ellipse area = 177416.7 square pixels (1272 (mm^2))
  which yields a pressure of 0.4243 N/mm^2 (106% of inflation pressure)
Convex hull ellipse length = 1176 pixels (99.6 mm) and width = 192 pixels (16.3 mm)

Schwalbe Marathon Plus 37-622 at 55 kg, 70 psi (0.4826 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 109637 square pixels (785.9 mm^2)
which yields a pressure of 0.6866 N/mm^2 (142% of inflation pressure)
Convex hull ellipse area = 162299.3 square pixels (1163 (mm^2))
which yields a pressure of 0.4638 N/mm^2 (96.1% of inflation pressure)
Convex hull ellipse length = 1203 pixels (102 mm) and width = 172 pixels (14.5 mm)

Contact patch on the drum

Image area = 107957 square pixels (773.8 mm^2)
which yields a pressure of 0.6972 N/mm^2 (144% of inflation pressure)
Convex hull ellipse area = 157989.7 square pixels (1132 (mm^2))
which yields a pressure of 0.4764 N/mm^2 (98.7% of inflation pressure)
Convex hull ellipse length = 1136 pixels (96.2 mm) and width = 177 pixels (15 mm)

Schwalbe Marathon Plus 37-622 at 55 kg, 90 psi (0.6205 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 102704 square pixels (736.2 mm^2)
which yields a pressure of 0.7329 N/mm^2 (118% of inflation pressure)
Convex hull ellipse area = 159428.4 square pixels (1143 (mm^2)
which yields a pressure of 0.4721 N/mm^2 (76.1% of inflation pressure)
Convex hull ellipse length = 1130 pixels (95.7 mm) and width = 180 pixels (15.2 mm)

Contact patch on the drum

Image area = 107568 square pixels (771 mm^2)
which yields a pressure of 0.6998 N/mm^2 (113% of inflation pressure)
Convex hull ellipse area = 161250.2 square pixels (1156 (mm^2)
which yields a pressure of 0.4668 N/mm^2 (75.2% of inflation pressure)
Convex hull ellipse length = 1081 pixels (91.5 mm) and width = 190 pixels (16.1 mm)

Schwalbe Marathon Plus 37-622 at 74.5 kg, 50 psi (0.3447 N/mm^2), raw image plus bounding ellipse

Contact patch on the drum

Image area = 127376 square pixels (913 mm^2)
which yields a pressure of 0.8005 N/mm^2 (232% of inflation pressure)
Convex hull ellipse area = 209937.1 square pixels (1505 (mm^2))
which yields a pressure of 0.4857 N/mm^2 (141% of inflation pressure)
Convex hull ellipse length = 1277 pixels (108 mm) and width = 209 pixels (17.7 mm)

Contact patch on the plate

Image area = 157443 square pixels (1129 mm^2)
which yields a pressure of 0.6476 N/mm^2 (188% of inflation pressure)
Convex hull ellipse area = 245920.6 square pixels (1763 (mm^2))
which yields a pressure of 0.4146 N/mm^2 (120% of inflation pressure)
Convex hull ellipse length = 1359 pixels (115 mm) and width = 230 pixels (19.5 mm)

Schwalbe Marathon Plus 37-622 at 74.5 kg, 58 psi (0.3999 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 144181 square pixels (1033 mm^2)
  which yields a pressure of 0.7072 N/mm^2 (177% of inflation pressure)
Convex hull ellipse area = 236535.3 square pixels (1695 (mm^2)
  which yields a pressure of 0.4311 N/mm^2 (108% of inflation pressure)
Convex hull ellipse length = 1460 pixels (124 mm) and width = 206 pixels (17.5 mm)

Contact patch on the drum

Image area = 146423 square pixels (1050 mm^2)
  which yields a pressure of 0.6963 N/mm^2 (174% of inflation pressure)
Convex hull ellipse area = 249124 square pixels (1786 (mm^2)
  which yields a pressure of 0.4093 N/mm^2 (102% of inflation pressure)
Convex hull ellipse length = 1335 pixels (113 mm) and width = 238 pixels (20.1 mm)

Schwalbe Marathon Plus 37-622 at 74.5 kg, 70 psi (0.4826 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 145089 square pixels (1040 mm^2)
which yields a pressure of 0.7027 N/mm^2 (146% of inflation pressure)
Convex hull ellipse area = 223615.3 square pixels (1603 (mm^2))
which yields a pressure of 0.456 N/mm^2 (94.5% of inflation pressure)
Convex hull ellipse length = 1374 pixels (116 mm) and width = 207 pixels (17.5 mm)

Contact patch on the drum

Image area = 143439 square pixels (1028 mm^2)
which yields a pressure of 0.7108 N/mm^2 (147% of inflation pressure)
Convex hull ellipse area = 216315.4 square pixels (1551 (mm^2))
which yields a pressure of 0.4714 N/mm^2 (97.7% of inflation pressure)
Convex hull ellipse length = 1245 pixels (105 mm) and width = 221 pixels (18.7 mm)

Schwalbe Marathon Plus 37-622 at 74.5 kg, 90 psi (0.6205 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 123305 square pixels (883.8 mm^2)
  which yields a pressure of 0.8269 N/mm^2 (133% of inflation pressure)
Convex hull ellipse area = 198728.3 square pixels (1424 (mm^2))
  which yields a pressure of 0.5131 N/mm^2 (82.7% of inflation pressure)
Convex hull ellipse length = 1187 pixels (100 mm) and width = 213 pixels (18 mm)

Contact patch on the drum

Image area = 83986 square pixels (602 mm^2)
  which yields a pressure of 1.214 N/mm^2 (196% of inflation pressure)
Convex hull ellipse area = 135179.7 square pixels (969 (mm^2))
  which yields a pressure of 0.7543 N/mm^2 (122% of inflation pressure)
Convex hull ellipse length = 969.9 pixels (82.1 mm) and width = 177 pixels (15 mm)

Vittoria Randonneur Hyper 37-622 at 33 kg, 50 psi (0.3447 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 109980 square pixels (788.3 mm^2)
  which yields a pressure of 0.4107 N/mm^2 (119% of inflation pressure)
Convex hull ellipse area = 132803.9 square pixels (951.9 (mm^2))
  which yields a pressure of 0.3401 N/mm^2 (98.6% of inflation pressure)
Convex hull ellipse length = 1088 pixels (92.1 mm) and width = 155 pixels (13.2 mm)

Contact patch on the drum

Image area = 113722 square pixels (815.2 mm^2)
  which yields a pressure of 0.3971 N/mm^2 (115% of inflation pressure)
Convex hull ellipse area = 142122 square pixels (1019 (mm^2))
  which yields a pressure of 0.3178 N/mm^2 (92.2% of inflation pressure)
Convex hull ellipse length = 997.2 pixels (84.4 mm) and width = 181 pixels (15.4 mm)

Vittoria Randonneur Hyper 37-622 at 33 kg, 58 psi (0.3999 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 103978 square pixels (745.3 mm^2)
which yields a pressure of 0.4344 N/mm^2 (109% of inflation pressure)
Convex hull ellipse area = 123632.8 square pixels (886.2 (mm^2))
which yields a pressure of 0.3653 N/mm^2 (91.3% of inflation pressure)
Convex hull ellipse length = 1080 pixels (91.5 mm) and width = 146 pixels (12.3 mm)

Contact patch on the drum

Image area = 114453 square pixels (820.4 mm^2)
which yields a pressure of 0.3946 N/mm^2 (98.7% of inflation pressure)
Convex hull ellipse area = 142600.9 square pixels (1022 (mm^2))
which yields a pressure of 0.3167 N/mm^2 (79.2% of inflation pressure)
Convex hull ellipse length = 981.6 pixels (83.1 mm) and width = 185 pixels (15.7 mm)

Vittoria Randonneur Hyper 37-622 at 33 kg, 70 psi (0.4826 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 85436 square pixels (612.4 mm^2)
which yields a pressure of 0.5286 N/mm^2 (110% of inflation pressure)
Convex hull ellipse area = 108814 square pixels (780 (mm^2))
which yields a pressure of 0.4151 N/mm^2 (86% of inflation pressure)
Convex hull ellipse length = 980.1 pixels (83 mm) and width = 141 pixels (12 mm)

Contact patch on the drum

Image area = 97517 square pixels (699 mm^2)
which yields a pressure of 0.4631 N/mm^2 (96% of inflation pressure)
Convex hull ellipse area = 126028.2 square pixels (903.4 (mm^2))
which yields a pressure of 0.3584 N/mm^2 (74.3% of inflation pressure)
Convex hull ellipse length = 916.3 pixels (77.6 mm) and width = 175 pixels (14.8 mm)

Vittoria Randonneur Hyper 37-622 at 33 kg, 90 psi (0.6205 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 84651 square pixels (606.8 mm^2)
which yields a pressure of 0.5335 N/mm^2 (86% of inflation pressure)
Convex hull ellipse area = 101546.9 square pixels (727.9 (mm^2))
which yields a pressure of 0.4448 N/mm^2 (71.7% of inflation pressure)
Convex hull ellipse length = 922.7 pixels (78.1 mm) and width = 140 pixels (11.9 mm)

Contact patch on the drum

Image area = 90309 square pixels (647.3 mm^2)
which yields a pressure of 0.5001 N/mm^2 (80.6% of inflation pressure)
Convex hull ellipse area = 110506.4 square pixels (792.1 (mm^2))
which yields a pressure of 0.4087 N/mm^2 (65.9% of inflation pressure)
Convex hull ellipse length = 850.2 pixels (72 mm) and width = 165 pixels (14 mm)

Vittoria Randonneur Hyper 37-622 at 55 kg, 50 psi (0.3447 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 189034 square pixels (1355 mm^2)
which yields a pressure of 0.3982 N/mm^2 (116% of inflation pressure)
Convex hull ellipse area = 222128.4 square pixels (1592 (mm^2)
which yields a pressure of 0.3389 N/mm^2 (98.3% of inflation pressure)
Convex hull ellipse length = 1381 pixels (117 mm) and width = 205 pixels (17.3 mm)

Contact patch on the drum

Image area = 202059 square pixels (1448 mm^2)
which yields a pressure of 0.3725 N/mm^2 (108% of inflation pressure)
Convex hull ellipse area = 242787.8 square pixels (1740 (mm^2)
which yields a pressure of 0.31 N/mm^2 (89.9% of inflation pressure)
Convex hull ellipse length = 1281 pixels (108 mm) and width = 241 pixels (20.4 mm)

Vittoria Randonneur Hyper 37-622 at 55 kg, 58 psi (0.3999 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 165141 square pixels (1184 mm^2)
which yields a pressure of 0.4558 N/mm^2 (114% of inflation pressure)
Convex hull ellipse area = 203561.9 square pixels (1459 (mm^2))
which yields a pressure of 0.3698 N/mm^2 (92.5% of inflation pressure)
Convex hull ellipse length = 1290 pixels (109 mm) and width = 201 pixels (17 mm)

Contact patch on the drum

Image area = 184841 square pixels (1325 mm^2)
which yields a pressure of 0.4072 N/mm^2 (102% of inflation pressure)
Convex hull ellipse area = 227500.8 square pixels (1631 (mm^2))
which yields a pressure of 0.3309 N/mm^2 (82.7% of inflation pressure)
Convex hull ellipse length = 1203 pixels (102 mm) and width = 241 pixels (20.4 mm)

Vittoria Randonneur Hyper 37-622 at 55 kg, 70 psi (0.4826 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 155642 square pixels (1116 mm^2)
 which yields a pressure of 0.4836 N/mm^2 (100% of inflation pressure)
Convex hull ellipse area = 183903.4 square pixels (1318 (mm^2)
 which yields a pressure of 0.4093 N/mm^2 (84.8% of inflation pressure)
Convex hull ellipse length = 1232 pixels (104 mm) and width = 190 pixels (16.1 mm)

Contact patch on the drum

Image area = 161314 square pixels (1156 mm^2)
 which yields a pressure of 0.4666 N/mm^2 (96.7% of inflation pressure)
Convex hull ellipse area = 193313.2 square pixels (1386 (mm^2)
 which yields a pressure of 0.3894 N/mm^2 (80.7% of inflation pressure)
Convex hull ellipse length = 1124 pixels (95.2 mm) and width = 219 pixels (18.5 mm)

Vittoria Randonneur Hyper 37-622 at 55 kg, 90 psi (0.6205 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 124835 square pixels (894.8 mm^2)
which yields a pressure of 0.603 N/mm^2 (97.2% of inflation pressure)
Convex hull ellipse area = 154871.8 square pixels (1110 (mm^2))
which yields a pressure of 0.486 N/mm^2 (78.3% of inflation pressure)
Convex hull ellipse length = 1122 pixels (95 mm) and width = 176 pixels (14.9 mm)

Contact patch on the drum

Image area = 132105 square pixels (946.9 mm^2)
which yields a pressure of 0.5698 N/mm^2 (91.8% of inflation pressure)
Convex hull ellipse area = 167246.5 square pixels (1199 (mm^2))
which yields a pressure of 0.4501 N/mm^2 (72.5% of inflation pressure)
Convex hull ellipse length = 1036 pixels (87.7 mm) and width = 205 pixels (17.4 mm)

Vittoria Randonneur Hyper 37-622 at 74.5 kg, 50 psi (0.3447 N/mm²), raw image plus bounding ellipse

Contact patch on the plate

Image area = 258311 square pixels (1852 mm²)
which yields a pressure of 0.3947 N/mm² (114% of inflation pressure)
Convex hull ellipse area = 305236.5 square pixels (2188 (mm²)
which yields a pressure of 0.334 N/mm² (96.9% of inflation pressure)
Convex hull ellipse length = 1553 pixels (131 mm) and width = 250 pixels (21.2 mm)

Contact patch on the drum

Image area = 275590 square pixels (1975 mm²)
which yields a pressure of 0.37 N/mm² (107% of inflation pressure)
Convex hull ellipse area = 334391.6 square pixels (2397 (mm²)
which yields a pressure of 0.3049 N/mm² (88.4% of inflation pressure)
Convex hull ellipse length = 1428 pixels (121 mm) and width = 298 pixels (25.2 mm)

Vittoria Randonneur Hyper 37-622 at 74.5 kg, 58 psi (0.3999 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 225009 square pixels (1613 mm^2)
  which yields a pressure of 0.4531 N/mm^2 (113% of inflation pressure)
Convex hull ellipse area = 272680 square pixels (1955 (mm^2)
  which yields a pressure of 0.3739 N/mm^2 (93.5% of inflation pressure)
Convex hull ellipse length = 1449 pixels (123 mm) and width = 240 pixels (20.3 mm)

Contact patch on the drum

Image area = 234469 square pixels (1681 mm^2)
  which yields a pressure of 0.4349 N/mm^2 (109% of inflation pressure)
Convex hull ellipse area = 293502.2 square pixels (2104 (mm^2)
  which yields a pressure of 0.3474 N/mm^2 (86.9% of inflation pressure)
Convex hull ellipse length = 1328 pixels (112 mm) and width = 281 pixels (23.8 mm)

Vittoria Randonneur Hyper 37-622 at 74.5 kg, 70 psi (0.4826 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 197343 square pixels (1415 mm^2)
which yields a pressure of 0.5167 N/mm^2 (107% of inflation pressure)
Convex hull ellipse area = 234279.5 square pixels (1679 (mm^2))
which yields a pressure of 0.4352 N/mm^2 (90.2% of inflation pressure)
Convex hull ellipse length = 1357 pixels (115 mm) and width = 220 pixels (18.6 mm)

Contact patch on the drum

Image area = 217010 square pixels (1556 mm^2)
which yields a pressure of 0.4698 N/mm^2 (97.3% of inflation pressure)
Convex hull ellipse area = 264720.2 square pixels (1897 (mm^2))
which yields a pressure of 0.3852 N/mm^2 (79.8% of inflation pressure)
Convex hull ellipse length = 1255 pixels (106 mm) and width = 269 pixels (22.7 mm)

Vittoria Randonneur Hyper 37-622 at 74.5 kg, 90 psi (0.6205 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 176023 square pixels (1262 mm^2)

which yields a pressure of 0.5792 N/mm^2 (93.3% of inflation pressure)

Convex hull ellipse area = 205161.9 square pixels (1471 (mm^2))

which yields a pressure of 0.497 N/mm^2 (80.1% of inflation pressure)

Convex hull ellipse length = 1260 pixels (107 mm) and width = 207 pixels (17.6 mm)

Contact patch on the drum

Image area = 185226 square pixels (1328 mm^2)

which yields a pressure of 0.5505 N/mm^2 (88.7% of inflation pressure)

Convex hull ellipse area = 222237.8 square pixels (1593 (mm^2))

which yields a pressure of 0.4588 N/mm^2 (73.9% of inflation pressure)

Convex hull ellipse length = 1172 pixels (99.2 mm) and width = 242 pixels (20.4 mm)

Vredestein Perfect Tour 37-622 at 31 kg, 50 psi (0.3447 N/mm^2), raw image plus bounding ellipse

Contact patch on the drum

Image area = 111766 square pixels (801.1 mm^2)
  which yields a pressure of 0.3796 N/mm^2 (110% of inflation pressure)
Convex hull ellipse area = 182318.4 square pixels (1307 (mm^2))
  which yields a pressure of 0.2327 N/mm^2 (67.5% of inflation pressure)
Convex hull ellipse length = 1048 pixels (88.7 mm) and width = 222 pixels (18.8 mm)

Contact patch on the plate

Image area = 107393 square pixels (769.8 mm^2)
  which yields a pressure of 0.3951 N/mm^2 (115% of inflation pressure)
Convex hull ellipse area = 177636.2 square pixels (1273 (mm^2))
  which yields a pressure of 0.2388 N/mm^2 (69.3% of inflation pressure)
Convex hull ellipse length = 1112 pixels (94.1 mm) and width = 203 pixels (17.2 mm)

Vredestein Perfect Tour 37-622 at 31 kg, 58 psi (0.3999 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 87142 square pixels (624.6 mm^2)
which yields a pressure of 0.4869 N/mm^2 (122% of inflation pressure)
Convex hull ellipse area = 156599.7 square pixels (1122 (mm^2))
which yields a pressure of 0.2709 N/mm^2 (67.7% of inflation pressure)
Convex hull ellipse length = 1046 pixels (88.6 mm) and width = 191 pixels (16.1 mm)

Contact patch on the drum

Image area = 94909 square pixels (680.3 mm^2)
which yields a pressure of 0.447 N/mm^2 (112% of inflation pressure)
Convex hull ellipse area = 160494.4 square pixels (1150 (mm^2))
which yields a pressure of 0.2643 N/mm^2 (66.1% of inflation pressure)
Convex hull ellipse length = 984.4 pixels (83.3 mm) and width = 208 pixels (17.6 mm)

Vredestein Perfect Tour 37-622 at 31 kg, 70 psi (0.4826 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 97081 square pixels (695.9 mm^2)
which yields a pressure of 0.437 N/mm^2 (90.5% of inflation pressure)
Convex hull ellipse area = 142636.4 square pixels (1022 (mm^2))
which yields a pressure of 0.2974 N/mm^2 (61.6% of inflation pressure)
Convex hull ellipse length = 980.1 pixels (83 mm) and width = 185 pixels (15.7 mm)

Contact patch on the drum

Image area = 87719 square pixels (628.8 mm^2)
which yields a pressure of 0.4837 N/mm^2 (100% of inflation pressure)
Convex hull ellipse area = 146595.1 square pixels (1051 (mm^2))
which yields a pressure of 0.2894 N/mm^2 (60% of inflation pressure)
Convex hull ellipse length = 912.2 pixels (77.2 mm) and width = 205 pixels (17.3 mm)

Vredestein Perfect Tour 37-622 at 31 kg, 90 psi (0.6205 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 99377 square pixels (712.3 mm^2)
which yields a pressure of 0.4269 N/mm^2 (68.8% of inflation pressure)
Convex hull ellipse area = 148185.1 square pixels (1062 (mm^2))
which yields a pressure of 0.2863 N/mm^2 (46.1% of inflation pressure)
Convex hull ellipse length = 1019 pixels (86.3 mm) and width = 185 pixels (15.7 mm)

Contact patch on the drum

Image area = 109753 square pixels (786.7 mm^2)
which yields a pressure of 0.3866 N/mm^2 (62.3% of inflation pressure)
Convex hull ellipse area = 152004.2 square pixels (1090 (mm^2))
which yields a pressure of 0.2791 N/mm^2 (45% of inflation pressure)
Convex hull ellipse length = 976.9 pixels (82.7 mm) and width = 198 pixels (16.8 mm)

Vredestein Perfect Tour 37-622 at 53.5 kg, 50 psi (0.3447 N/mm²), raw image plus bounding ellipse

Contact patch on the plate

Image area = 178866 square pixels (1282 mm²)
which yields a pressure of 0.4094 N/mm² (119% of inflation pressure)
Convex hull ellipse area = 260299.5 square pixels (1866 (mm²)
which yields a pressure of 0.2813 N/mm² (81.6% of inflation pressure)
Convex hull ellipse length = 1381 pixels (117 mm) and width = 240 pixels (20.3 mm)

Contact patch on the drum

Image area = 188435 square pixels (1351 mm²)
which yields a pressure of 0.3886 N/mm² (113% of inflation pressure)
Convex hull ellipse area = 273188.4 square pixels (1958 (mm²)
which yields a pressure of 0.268 N/mm² (77.7% of inflation pressure)
Convex hull ellipse length = 1309 pixels (111 mm) and width = 266 pixels (22.5 mm)

Vredestein Perfect Tour 37-622 at 53.5 kg, 58 psi (0.3999 N/mm^2), raw image plus bounding ellipse

Contact patch on the drum

Image area = 175433 square pixels (1257 mm^2)

which yields a pressure of 0.4174 N/mm^2 (104% of inflation pressure)

Convex hull ellipse area = 261480.5 square pixels (1874 (mm^2))

which yields a pressure of 0.28 N/mm^2 (70% of inflation pressure)

Convex hull ellipse length = 1240 pixels (105 mm) and width = 268 pixels (22.7 mm)

Contact patch on the plate

Image area = 159372 square pixels (1142 mm^2)

which yields a pressure of 0.4594 N/mm^2 (115% of inflation pressure)

Convex hull ellipse area = 240604.6 square pixels (1725 (mm^2))

which yields a pressure of 0.3043 N/mm^2 (76.1% of inflation pressure)

Convex hull ellipse length = 1354 pixels (115 mm) and width = 226 pixels (19.2 mm)

Vredestein Perfect Tour 37-622 at 53.5 kg, 70 psi (0.4826 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 143512 square pixels (1029 mm^2)
which yields a pressure of 0.5102 N/mm^2 (106% of inflation pressure)
Convex hull ellipse area = 222784.2 square pixels (1597 (mm^2))
which yields a pressure of 0.3287 N/mm^2 (68.1% of inflation pressure)
Convex hull ellipse length = 1247 pixels (106 mm) and width = 228 pixels (19.3 mm)

Contact patch on the drum

Image area = 148699 square pixels (1066 mm^2)
which yields a pressure of 0.4924 N/mm^2 (102% of inflation pressure)
Convex hull ellipse area = 218793.2 square pixels (1568 (mm^2))
which yields a pressure of 0.3347 N/mm^2 (69.3% of inflation pressure)
Convex hull ellipse length = 1128 pixels (95.5 mm) and width = 247 pixels (20.9 mm)

Vredestein Perfect Tour 37-622 at 53.5 kg, 90 psi (0.6205 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 130143 square pixels (932.9 mm^2)
which yields a pressure of 0.5626 N/mm^2 (90.7% of inflation pressure)
Convex hull ellipse area = 181928 square pixels (1304 (mm^2)
which yields a pressure of 0.4025 N/mm^2 (64.9% of inflation pressure)
Convex hull ellipse length = 1082 pixels (91.6 mm) and width = 214 pixels (18.1 mm)

Contact patch on the drum

Image area = 134037 square pixels (960.8 mm^2)
which yields a pressure of 0.5463 N/mm^2 (88% of inflation pressure)
Convex hull ellipse area = 187335.1 square pixels (1343 (mm^2)
which yields a pressure of 0.3908 N/mm^2 (63% of inflation pressure)
Convex hull ellipse length = 1024 pixels (86.7 mm) and width = 233 pixels (19.7 mm)

Vredestein Perfect Tour 37-622 at 74.5 kg, 50 psi (0.3447 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 234074 square pixels (1678 mm^2)
 which yields a pressure of 0.4356 N/mm^2 (126% of inflation pressure)
Convex hull ellipse area = 322052.2 square pixels (2308 (mm^2))
 which yields a pressure of 0.3166 N/mm^2 (91.8% of inflation pressure)
Convex hull ellipse length = 1513 pixels (128 mm) and width = 271 pixels (23 mm)

Contact patch on the drum

Image area = 194990 square pixels (1398 mm^2)
 which yields a pressure of 0.5229 N/mm^2 (152% of inflation pressure)
Convex hull ellipse area = 309220.5 square pixels (2216 (mm^2))
 which yields a pressure of 0.3297 N/mm^2 (95.6% of inflation pressure)
Convex hull ellipse length = 1232 pixels (104 mm) and width = 320 pixels (27.1 mm)

Vredestein Perfect Tour 37-622 at 74.5 kg, 58 psi (0.3999 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 214232 square pixels (1536 mm^2)
which yields a pressure of 0.4759 N/mm^2 (119% of inflation pressure)
Convex hull ellipse area = 291336.8 square pixels (2088 (mm^2))
which yields a pressure of 0.35 N/mm^2 (87.5% of inflation pressure)
Convex hull ellipse length = 1421 pixels (120 mm) and width = 261 pixels (22.1 mm)

Contact patch on the drum

Image area = 239028 square pixels (1713 mm^2)
which yields a pressure of 0.4266 N/mm^2 (107% of inflation pressure)
Convex hull ellipse area = 320419.2 square pixels (2297 (mm^2))
which yields a pressure of 0.3182 N/mm^2 (79.6% of inflation pressure)
Convex hull ellipse length = 1336 pixels (113 mm) and width = 305 pixels (25.9 mm)

Vredestein Perfect Tour 37-622 at 74.5 kg, 70 psi (0.4826 N/mm^2), raw image plus bounding ellipse

Contact patch on the plate

Image area = 210969 square pixels (1512 mm^2)
which yields a pressure of 0.4833 N/mm^2 (100% of inflation pressure)
Convex hull ellipse area = 276936.8 square pixels (1985 (mm^2))
which yields a pressure of 0.3682 N/mm^2 (76.3% of inflation pressure)
Convex hull ellipse length = 1397 pixels (118 mm) and width = 252 pixels (21.4 mm)

Contact patch on the drum

Image area = 212658 square pixels (1524 mm^2)
which yields a pressure of 0.4795 N/mm^2 (99.3% of inflation pressure)
Convex hull ellipse area = 301454.3 square pixels (2161 (mm^2))
which yields a pressure of 0.3382 N/mm^2 (70.1% of inflation pressure)
Convex hull ellipse length = 1243 pixels (105 mm) and width = 309 pixels (26.2 mm)

Vredestein Perfect Tour 37-622 at 74.5 kg, 90 psi (0.6205 N/mm^2), raw image plus bounding ellipse

Contact patch on the drum

Image area = 194355 square pixels (1393 mm^2)
which yields a pressure of 0.5246 N/mm^2 (84.5% of inflation pressure)
Convex hull ellipse area = 256395.6 square pixels (1838 (mm^2))
which yields a pressure of 0.3977 N/mm^2 (64.1% of inflation pressure)
Convex hull ellipse length = 1186 pixels (100 mm) and width = 275 pixels (23.3 mm)

Contact patch on the plate

Image area = 178369 square pixels (1279 mm^2)
which yields a pressure of 0.5716 N/mm^2 (92.1% of inflation pressure)
Convex hull ellipse area = 255753.7 square pixels (1833 (mm^2))
which yields a pressure of 0.3987 N/mm^2 (64.2% of inflation pressure)
Convex hull ellipse length = 1312 pixels (111 mm) and width = 248 pixels (21 mm)

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Bontrager All Weather 25-622
contact patch "pressure" on a flat plate vs inflation pressure

- vertical load = 304 N
- vertical load = 525 N
- vertical load = 731 N
- unity: inflation pressure = vertical load / ellipse area

vertical load (N) / ellipse area (mm^2)

inflation pressure (N/mm^2)

Bontrager All Weather 28-622
contact patch "pressure" on a flat plate vs inflation pressure

- vertical load = 304 N
- vertical load = 525 N
- vertical load = 731 N
- unity: inflation pressure = vertical load / ellipse area

vertical load (N) / ellipse area (mm^2)

inflation pressure (N/mm^2)

Cheng Shin Classic Zeppelin 50-622
contact patch "pressure" on a flat plate vs inflation pressure

- vertical load = 324 N
- vertical load = 540 N
- vertical load = 731 N
- unity: inflation pressure = vertical load / ellipse area

vertical load (N) / ellipse area (mm^2)

inflation pressure (N/mm^2)

Continental Top Contact Winter 37-622
contact patch "pressure" on a flat plate vs inflation pressure

- vertical load = 324 N
- vertical load = 540 N
- vertical load = 731 N
- unity: inflation pressure = vertical load / ellipse area

vertical load (N) / ellipse area (mm^2)

inflation pressure (N/mm^2)

Maxxis radial 22-622
contact patch "pressure" on a flat plate vs inflation pressure

- vertical load = 304 N
- vertical load = 525 N
- vertical load = 731 N
- unity: inflation pressure = vertical load / ellipse area

vertical load (N) / ellipse area (mm^2)

inflation pressure (N/mm^2)

Schwalbe Big Apple 55-622
contact patch "pressure" on a flat plate vs inflation pressure

- vertical load = 324 N
- vertical load = 540 N
- vertical load = 731 N
- unity: inflation pressure = vertical load / ellipse area

vertical load (N) / ellipse area (mm^2)

inflation pressure (N/mm^2)

Schwalbe Kojak 35-622
contact patch "pressure" on a flat plate vs inflation pressure

vertical load = 324 N
vertical load = 540 N
vertical load = 731 N
unity: inflation pressure = vertical load / ellipse area

Schwalbe Marathon Plus 37-622
contact patch "pressure" on a flat plate vs inflation pressure

vertical load = 324 N
vertical load = 540 N
vertical load = 731 N
unity: inflation pressure = vertical load / ellipse area

Vredestein Perfect Tour 37-622
contact patch "pressure" on a flat plate vs inflation pressure

vertical load = 304 N
vertical load = 525 N
vertical load = 731 N
unity: inflation pressure = vertical load / ellipse area

Vittoria Randonneur Hyper 37-622
contact patch "pressure" on a flat plate vs inflation pressure

vertical load = 324 N
vertical load = 540 N
vertical load = 731 N
unity: inflation pressure = vertical load / ellipse area

Bontrager All Weather 25-622
vertical load vs contact patch area on a flat plate

inflation pressure = 0.552 N/mm²
inflation pressure = 0.689 N/mm²
inflation pressure = 0.827 N/mm²

Bontrager All Weather 28-622
vertical load vs contact patch area on a flat plate

inflation pressure = 0.552 N/mm²
inflation pressure = 0.689 N/mm²
inflation pressure = 0.827 N/mm²

Cheng Shin Classic Zeppelin 50-622
vertical load vs contact patch area on a flat plate

inflation pressure = 0.2 N/mm²
inflation pressure = 0.3 N/mm²
inflation pressure = 0.4 N/mm²

Continental Top Contact Winter 37-622
vertical load vs contact patch area on a flat plate

inflation pressure = 0.345 N/mm²
inflation pressure = 0.4 N/mm²
inflation pressure = 0.483 N/mm²
inflation pressure = 0.621 N/mm²

Maxxis radial 22-622
vertical load vs contact patch area on a flat plate

inflation pressure = 0.689 N/mm²
inflation pressure = 0.827 N/mm²
inflation pressure = 0.965 N/mm²

Schwalbe Big Apple 55-622
vertical load vs contact patch area on a flat plate

inflation pressure = 0.2 N/mm²
inflation pressure = 0.3 N/mm²
inflation pressure = 0.4 N/mm²

Schwalbe Kojak 35-622
vertical load vs contact patch area on a flat plate

Schwalbe Marathon Plus 37-622
vertical load vs contact patch area on a flat plate

Vittoria Randonneur Hyper 37-622
vertical load vs contact patch area on a flat plate

Vredestein Perfect Tour 37-622
vertical load vs contact patch area on a flat plate

Contact patch area on the drum as a function of load/pressure (Fn/p) by tire

Legend:
- Bontrager All Weather 25-622
- Bontrager All Weather 28-622
- Cheng Shin Classic Zeppelin 50-622
- Continental Top Contact Winter 37-622
- Maxxis radial 22-622
- Schwalbe Big Apple 55-622
- Schwalbe Kojak 35-622
- Schwalbe Marathon Plus 37-622
- Vittoria Randonneur Hyper 37-622
- Vredestein Perfect Tour 37-622

Contact patch area on the drum vs by plate by tire

Contact patch area on the plate as a function of load/pressure (Fn/p) by tire

load/pressure (mm²)

ellipse area (mm²)

Bontrager All Weather 25-622
Bontrager All Weather 28-622
Cheng Shin Classic Zeppelin 50-622
Continental Top Contact Winter 37-622
Maxxis radial 22-622
Schwalbe Big Apple 55-622
Schwalbe Kojak 35-622
Schwalbe Marathon Plus 37-622
Vittoria Randonneur Hyper 37-622
Vredestein Perfect Tour 37-622

Contact patch length/relaxation length vs load/pressure (Fn/p) by tire

Relaxation length/contact patch length vs load/pressure (Fn/p) by tire

APPENDIX B

INDIVIDUAL TIRE TEST RESULTS

Bontrager All Weather 23-622
tire radius = 11.5 mm, rim width = 13.4 mm

at 5.52 bar (80 psi) and
under 304 N (31 kg, 68.3 lb)

Cornering stiffness = 0.259/° (78.6 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.012/° (3.65 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
−0.00309 m/° (−0.939 N•m/°)

Twisting torque stiffness =
3.77e-005 m/° (0.0115 N•m/°)

Mean pneumatic trail = 5.33 mm

Static lateral stiffness = 0.123/mm (37.4 N/mm)
Calculated relaxation length = 120 mm

Bontrager All Weather 23-622
tire radius = 11.5 mm, rim width = 13.4 mm

at 5.52 bar (80 psi) and
under 304 N (31 kg, 68.3 lb)

Cornering stiffness = 0.26/° (79.2 N/°)

Solid line represents
extracted stiffness value

Normalized lateral force
slip angle [°]

Camber stiffness = 0.0119/° (3.63 N/°)

Dashed line represents
tangent of camber angle

Normalized lateral force
camber [°]

Self-aligning torque stiffness =
-0.00312 m/° (-0.948 N•m/°)

Normalized self-aligning torque [m]
slip angle [°]

Twisting torque stiffness =
8.91e-005 m/° (0.0271 N•m/°)

Normalized twisting torque [m]
camber angle [°]

Mean pneumatic trail = 5.11 mm

Pneumatic trail [mm]
slip angle [°]

Static lateral stiffness = 0.123/mm (37.4 N/mm)
Calculated relaxation length = 121 mm

Normalized lateral force
displacement [mm]

Bontrager All Weather 23-622
tire radius = 11.5 mm, rim width = 13.4 mm

at 5.52 bar (80 psi) and
under 304 N (31 kg, 68.3 lb)

Cornering stiffness = 0.26/° (79 N/°)

Solid line represents
extracted stiffness value

Normalized lateral force

slip angle [°]

Camber stiffness = 0.0122/° (3.71 N/°)

Dashed line represents
tangent of camber angle

Normalized lateral force

camber [°]

Self-aligning torque stiffness =
-0.00287 m/° (-0.872 N•m/°)

Normalized self-aligning torque [m]

slip angle [°]

Twisting torque stiffness =
3.15e-005 m/° (0.00958 N•m/°)

Normalized twisting torque [m]

camber angle [°]

Mean pneumatic trail = 5.31 mm

Pneumatic trail [mm]

slip angle [°]

Static lateral stiffness = 0.123/mm (37.4 N/mm)
Calculated relaxation length = 121 mm

Normalized lateral force

displacement [mm]

Bontrager All Weather 23-622
tire radius = 11.5 mm, rim width = 13.4 mm

at 6.89 bar (100 psi) and
under 304 N (31 kg, 68.3 lb)

Cornering stiffness = 0.252/° (76.8 N/°)

Solid line represents
extracted stiffness value

Normalized lateral force vs *slip angle [°]*

Camber stiffness = 0.0117/° (3.55 N/°)

Dashed line represents
tangent of camber angle

Normalized lateral force vs *camber [°]*

Self-aligning torque stiffness =
-0.00273 m/° (-0.829 N•m/°)

Normalized self-aligning torque [m] vs *slip angle [°]*

Twisting torque stiffness =
6.68e-005 m/° (0.0203 N•m/°)

Normalized twisting torque [m] vs *camber angle [°]*

Mean pneumatic trail = 5.37 mm

Pneumatic trail [mm] vs *slip angle [°]*

Static lateral stiffness = 0.133/mm (40.4 N/mm)
Calculated relaxation length = 109 mm

Normalized lateral force vs *displacement [mm]*

Bontrager All Weather 23-622
tire radius = 11.5 mm, rim width = 13.4 mm

at 8.27 bar (120 psi) and
under 304 N (31 kg, 68.3 lb)

Cornering stiffness = 0.241/° (73.2 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.0115/° (3.51 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00245 m/° (-0.746 N•m/°)

Twisting torque stiffness =
5.75e-005 m/° (0.0175 N•m/°)

Mean pneumatic trail = 4.6 mm

Static lateral stiffness = 0.139/mm (42.4 N/mm)
Calculated relaxation length = 98.9 mm

Bontrager All Weather 23-622
tire radius = 11.5 mm, rim width = 13.4 mm
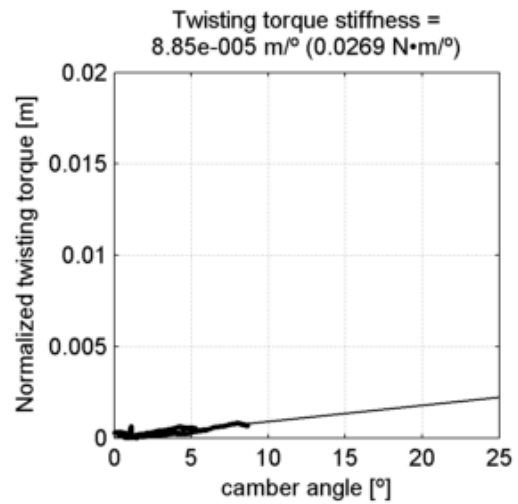
at 8.27 bar (120 psi) and
under 525 N (53.5 kg, 118 lb)

Cornering stiffness = 0.193/° (101 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.0135/° (7.07 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00248 m/° (-1.3 N·m/°)

Twisting torque stiffness =
5.85e-005 m/° (0.0307 N·m/°)

Mean pneumatic trail = 5.14 mm

Static lateral stiffness = 0.0803/mm (42.1 N/mm)
Calculated relaxation length = 138 mm

Bontrager All Weather 23-622
tire radius = 11.5 mm, rim width = 13.4 mm

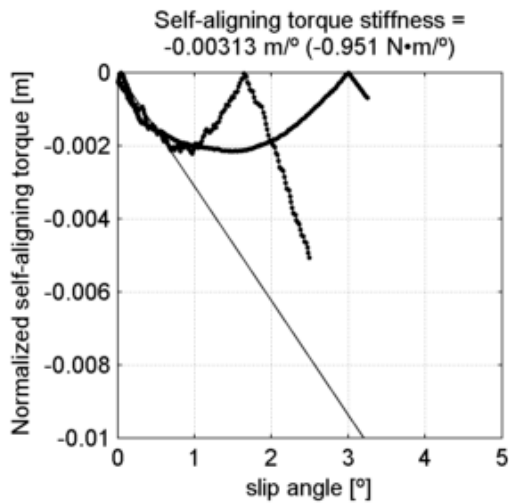at 8.27 bar (120 psi) and
under 731 N (74.5 kg, 164 lb)

Cornering stiffness = 0.157/° (115 N/°)

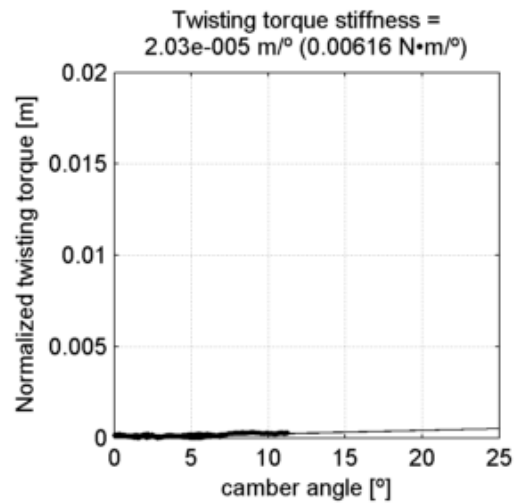Solid line represents
extracted stiffness value

Normalized lateral force vs *slip angle [°]*

Camber stiffness = 0.0144/° (10.6 N/°)

Dashed line represents
tangent of camber angle

Normalized lateral force vs *camber [°]*

Self-aligning torque stiffness =
−0.00249 m/° (−1.82 N•m/°)

Normalized self-aligning torque [m] vs *slip angle [°]*

Twisting torque stiffness =
6.65e-005 m/° (0.0486 N•m/°)

Normalized twisting torque [m] vs *camber angle [°]*

Mean pneumatic trail = 8.49 mm

Pneumatic trail [mm] vs *slip angle [°]*

Static lateral stiffness = 0.0562/mm (41.1 N/mm)
Calculated relaxation length = 160 mm

Normalized lateral force vs *displacement [mm]*

Bontrager All Weather 25-622
tire radius = 12.5 mm, rim width = 13.4 mm
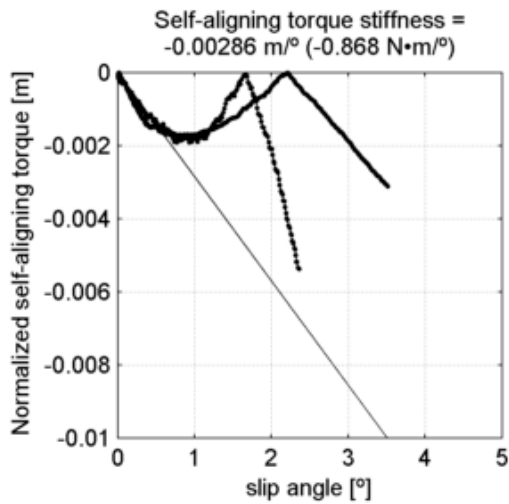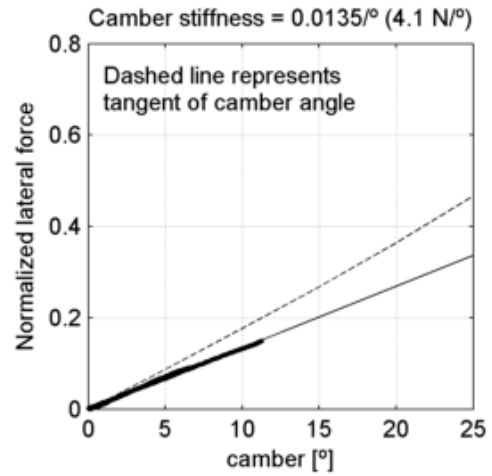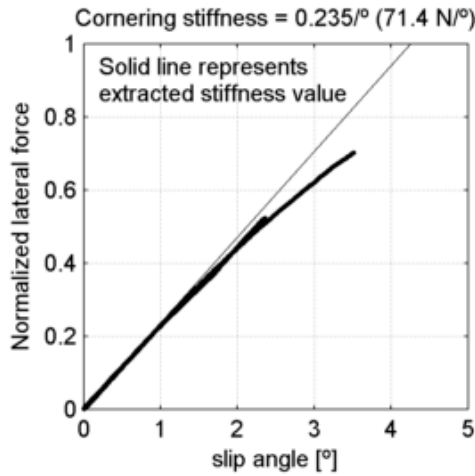
at 5.17 bar (75 psi) and
under 304 N (31 kg, 68.3 lb)

Cornering stiffness = 0.289/° (87.8 N/°)

Solid line represents
extracted stiffness value

Normalized lateral force vs *slip angle [°]*

Camber stiffness = 0.0127/° (3.87 N/°)

Dashed line represents
tangent of camber angle

Normalized lateral force vs *camber [°]*

Self-aligning torque stiffness =
−0.0032 m/° (−0.974 N•m/°)

Normalized self-aligning torque [m] vs *slip angle [°]*

Twisting torque stiffness =
0.000131 m/° (0.0398 N•m/°)

Normalized twisting torque [m] vs *camber angle [°]*

Mean pneumatic trail = 5.12 mm

Pneumatic trail [mm] vs *slip angle [°]*

Bontrager All Weather 25-622
tire radius = 12.5 mm, rim width = 13.4 mm

at 5.52 bar (80 psi) and
under 304 N (31 kg, 68.3 lb)

Cornering stiffness = 0.291/° (88.6 N/°)
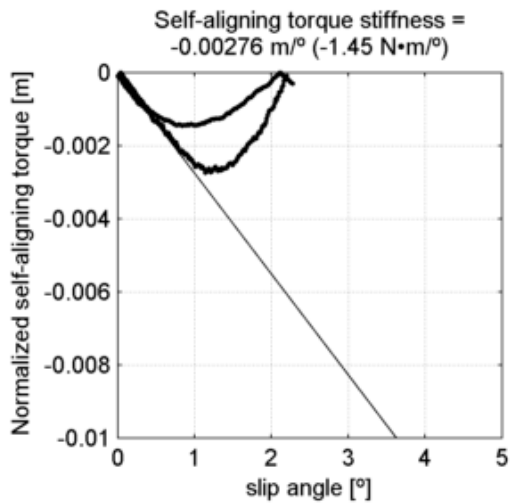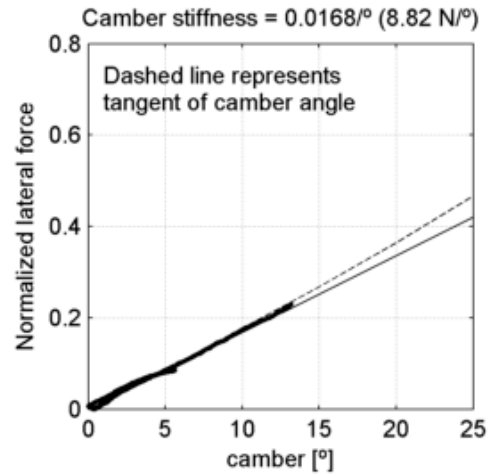
Solid line represents
extracted stiffness value

Camber stiffness = 0.0131/° (3.99 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00322 m/° (-0.978 N•m/°)

Twisting torque stiffness =
0.000119 m/° (0.0363 N•m/°)

Mean pneumatic trail = 4.95 mm

Static lateral stiffness = 0.121/mm (36.8 N/mm)
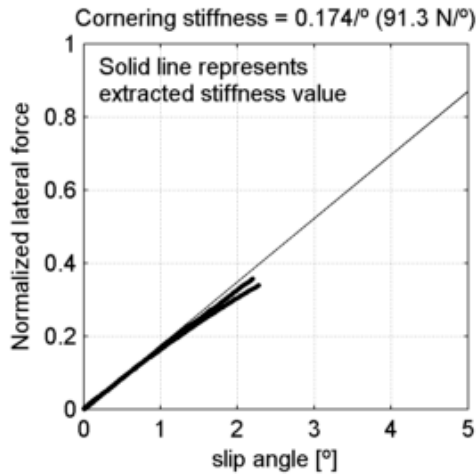Calculated relaxation length = 138 mm

Bontrager All Weather 25-622
tire radius = 12.5 mm, rim width = 13.4 mm

at 6.89 bar (100 psi) and
under 304 N (31 kg, 68.3 lb)

Cornering stiffness = 0.275/° (83.6 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.0128/° (3.89 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00224 m/° (-0.68 N•m/°)

Twisting torque stiffness =
9.04e-005 m/° (0.0275 N•m/°)

Mean pneumatic trail = 4.4 mm

Static lateral stiffness = 0.126/mm (38.2 N/mm)
Calculated relaxation length = 126 mm

Bontrager All Weather 25-622
tire radius = 12.5 mm, rim width = 13.4 mm
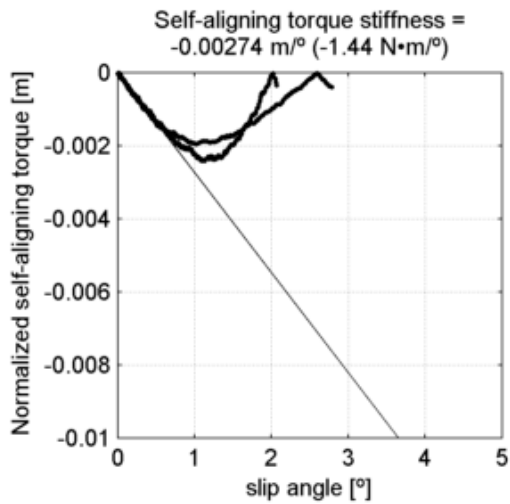at 8.27 bar (120 psi) and
under 304 N (31 kg, 68.3 lb)

Bontrager All Weather 25-622
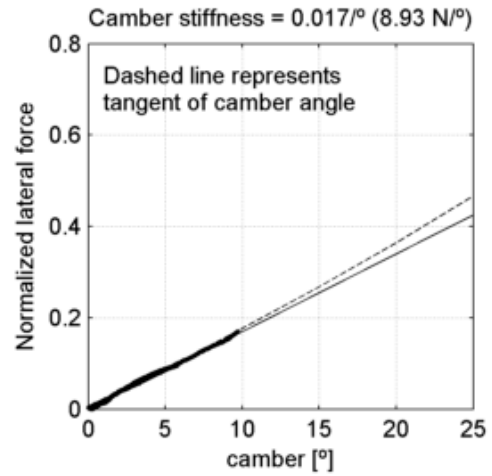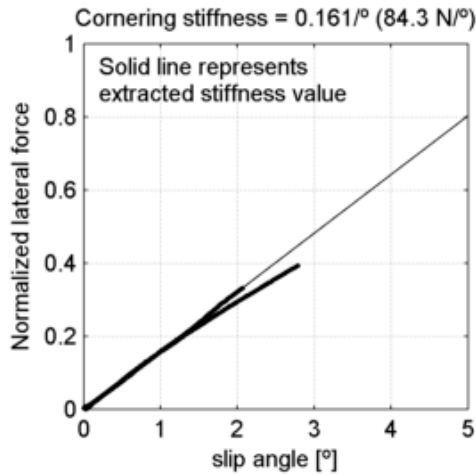tire radius = 12.5 mm, rim width = 13.4 mm

at 8.27 bar (120 psi) and
under 525 N (53.5 kg, 118 lb)

Cornering stiffness = 0.208/° (109 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.0144/° (7.58 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
−0.0023 m/° (−1.21 N•m/°)

Twisting torque stiffness =
3.27e-005 m/° (0.0172 N•m/°)

Mean pneumatic trail = 4.46 mm

Static lateral stiffness = 0.0827/mm (43.4 N/mm)
Calculated relaxation length = 144 mm

Bontrager All Weather 25-622
tire radius = 12.5 mm, rim width = 13.4 mm

at 5.52 bar (80 psi) and
under 731 N (74.5 kg, 164 lb)

Cornering stiffness = 0.169/° (124 N/°)

Solid line represents
extracted stiffness value

x-axis: slip angle [°]
y-axis: Normalized lateral force

Camber stiffness = 0.0168/° (12.3 N/°)

Dashed line represents
tangent of camber angle

x-axis: camber [°]
y-axis: Normalized lateral force

Self-aligning torque stiffness =
-0.00336 m/° (-2.45 N•m/°)

x-axis: slip angle [°]
y-axis: Normalized self-aligning torque [m]

Twisting torque stiffness =
3.34e-005 m/° (0.0244 N•m/°)

x-axis: camber angle [°]
y-axis: Normalized twisting torque [m]

Mean pneumatic trail = 13.7 mm

x-axis: slip angle [°]
y-axis: Pneumatic trail [mm]

Static lateral stiffness = 0.0511/mm (37.4 N/mm)
Calculated relaxation length = 190 mm

x-axis: displacement [mm]
y-axis: Normalized lateral force

Bontrager All Weather 25-622
tire radius = 12.5 mm, rim width = 13.4 mm

at 6.89 bar (100 psi) and
under 731 N (74.5 kg, 164 lb)

Cornering stiffness = 0.171/° (125 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.0163/° (11.9 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00276 m/° (-2.02 N•m/°)

Twisting torque stiffness =
4.6e-005 m/° (0.0336 N•m/°)

Mean pneumatic trail = 9.84 mm

Static lateral stiffness = 0.0529/mm (38.7 N/mm)
Calculated relaxation length = 185 mm

Bontrager All Weather 25-622
tire radius = 12.5 mm, rim width = 13.4 mm

at 8.27 bar (120 psi) and
under 731 N (74.5 kg, 164 lb)

Cornering stiffness = 0.171/° (125 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.0155/° (11.4 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00263 m/° (-1.92 N•m/°)

Twisting torque stiffness =
7.24e-005 m/° (0.0529 N•m/°)

Mean pneumatic trail = 7.86 mm

Static lateral stiffness = 0.0547/mm (40 N/mm)
Calculated relaxation length = 179 mm

Bontrager All Weather 28-622
tire radius = 14 mm, rim width = 13.4 mm

at 5.52 bar (80 psi) and
under 304 N (31 kg, 68.3 lb)

Cornering stiffness = 0.278/° (84.6 N/°)

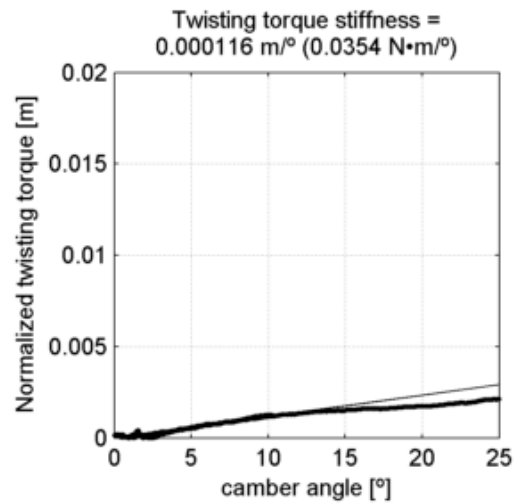Solid line represents
extracted stiffness value

Camber stiffness = 0.0142/° (4.33 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00129 m/° (-0.393 N•m/°)

Twisting torque stiffness =
5.02e-005 m/° (0.0153 N•m/°)

Mean pneumatic trail = 4.27 mm

Static lateral stiffness = 0.138/mm (42 N/mm)
Calculated relaxation length = 115 mm

Bontrager All Weather 28-622
tire radius = 14 mm, rim width = 13.4 mm

at 6.89 bar (100 psi) and
under 304 N (31 kg, 68.3 lb)

Bontrager All Weather 28-622
tire radius = 14 mm, rim width = 13.4 mm

at 8.27 bar (120 psi) and
under 304 N (31 kg, 68.3 lb)

Cornering stiffness = 0.26/° (79 N/°)

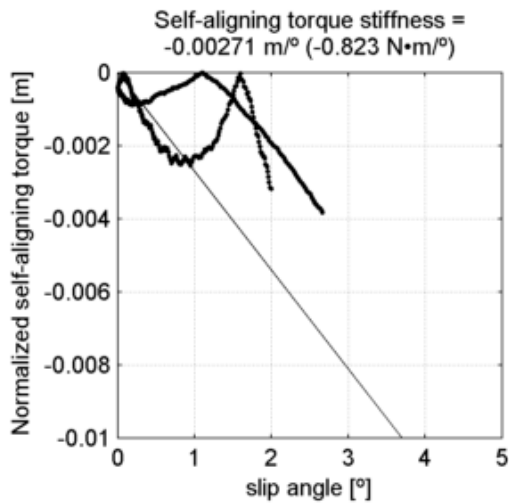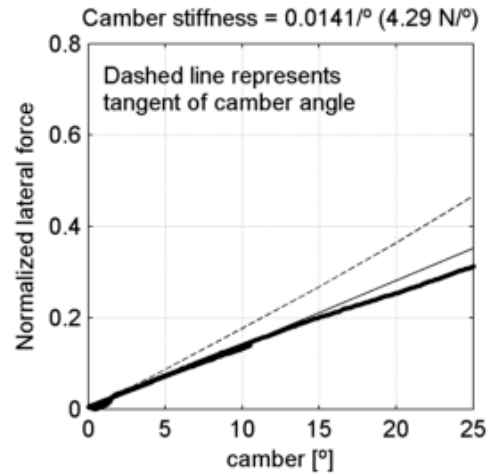Solid line represents
extracted stiffness value

Camber stiffness = 0.0151/° (4.61 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00118 m/° (-0.358 N•m/°)

Twisting torque stiffness =
7.55e-005 m/° (0.0229 N•m/°)

Mean pneumatic trail = 5.39 mm

Static lateral stiffness = 0.146/mm (44.4 N/mm)
Calculated relaxation length = 102 mm

Bontrager All Weather 28-622
tire radius = 14 mm, rim width = 13.4 mm

at 6.89 bar (100 psi) and
under 525 N (53.5 kg, 118 lb)

Cornering stiffness = 0.203/° (106 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.0161/° (8.46 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00155 m/° (-0.814 N•m/°)

Twisting torque stiffness =
8.38e-005 m/° (0.044 N•m/°)

Mean pneumatic trail = 4.89 mm

Static lateral stiffness = 0.0725/mm (38.1 N/mm)
Calculated relaxation length = 160 mm

Bontrager All Weather 28-622
tire radius = 14 mm, rim width = 13.4 mm

at 8.27 bar (120 psi) and
under 525 N (53.5 kg, 118 lb)

Cornering stiffness = 0.205/° (107 N/°)

Solid line represents
extracted stiffness value

Normalized lateral force

slip angle [°]

Camber stiffness = 0.018/° (9.44 N/°)

Dashed line represents
tangent of camber angle

Normalized lateral force

camber [°]

Self-aligning torque stiffness =
−0.0011 m/° (−0.576 N•m/°)

Normalized self-aligning torque [m]

slip angle [°]

Twisting torque stiffness =
0.00011 m/° (0.0578 N•m/°)

Normalized twisting torque [m]

camber angle [°]

Mean pneumatic trail = 3.9 mm

Pneumatic trail [mm]

slip angle [°]

Static lateral stiffness = 0.078/mm (40.9 N/mm)
Calculated relaxation length = 150 mm

Normalized lateral force

displacement [mm]

Bontrager All Weather 28-622
tire radius = 14 mm, rim width = 13.4 mm

at 8.27 bar (120 psi) and
under 731 N (74.5 kg, 164 lb)

Cornering stiffness = 0.172/° (126 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.0204/° (14.9 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.0018 m/° (-1.32 N•m/°)

Twisting torque stiffness =
0.000118 m/° (0.0861 N•m/°)

Mean pneumatic trail = 6.39 mm

Static lateral stiffness = 0.0563/mm (41.1 N/mm)
Calculated relaxation length = 175 mm

Cheng Shin Classic Zeppelin 50-622
tire radius = 22.1 mm, rim width = 18.7 mm

at 2 bar (29 psi) and
under 324 N (33 kg, 72.8 lb)

Cornering stiffness = 0.299/° (96.8 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.0135/° (4.38 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.0063 m/° (-2.04 N•m/°)

Twisting torque stiffness =
0.00018 m/° (0.0582 N•m/°)

Mean pneumatic trail = 14.5 mm

Static lateral stiffness = 0.0694/mm (22.5 N/mm)
Calculated relaxation length = 247 mm

Cheng Shin Classic Zeppelin 50-622
tire radius = 22.1 mm, rim width = 18.7 mm

at 2 bar (29 psi) and
under 540 N (55 kg, 121 lb)

Cornering stiffness = 0.194/° (105 N/°)

Solid line represents extracted stiffness value

Camber stiffness = 0.0136/° (7.36 N/°)

Dashed line represents tangent of camber angle

Self-aligning torque stiffness = -0.00606 m/° (-3.27 N•m/°)

Twisting torque stiffness = 0.000174 m/° (0.0938 N•m/°)

Mean pneumatic trail = 24.7 mm

Static lateral stiffness = 0.0415/mm (22.4 N/mm)
Calculated relaxation length = 267 mm

Cheng Shin Classic Zeppelin 50-622
tire radius = 22.1 mm, rim width = 18.7 mm

at 2 bar (29 psi) and
under 731 N (74.5 kg, 164 lb)

Cornering stiffness = 0.13/° (95.2 N/°)
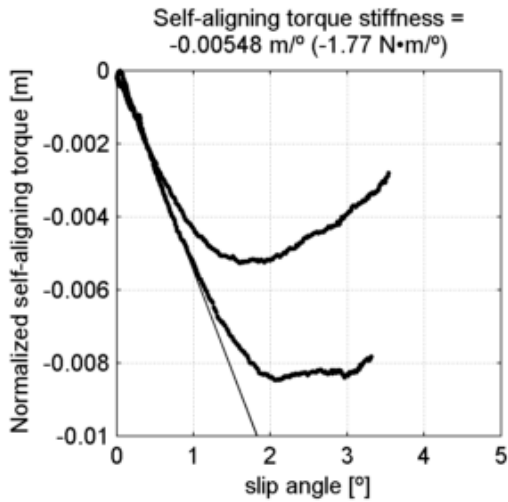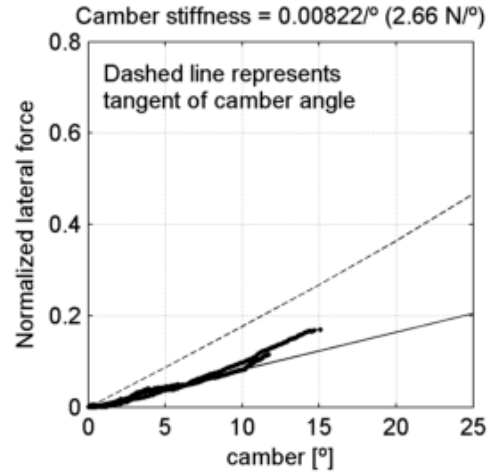
Camber stiffness = 0.0115/° (8.4 N/°)

Self-aligning torque stiffness = -0.00548 m/° (-4 N•m/°)

Twisting torque stiffness = 0.000153 m/° (0.112 N•m/°)

Mean pneumatic trail = 36.5 mm

Static lateral stiffness = 0.0327/mm (23.9 N/mm)
Calculated relaxation length = 228 mm

Cheng Shin Classic Zeppelin 50-622
tire radius = 22.1 mm, rim width = 18.7 mm

at 3.03 bar (44 psi) and
under 324 N (33 kg, 72.8 lb)

Cornering stiffness = 0.311/° (101 N/°)

Solid line represents
extracted stiffness value

Normalized lateral force — slip angle [°]

Camber stiffness = 0.0102/° (3.29 N/°)

Dashed line represents
tangent of camber angle

Normalized lateral force — camber [°]

Self-aligning torque stiffness =
-0.0057 m/° (-1.85 N•m/°)

Normalized self-aligning torque [m] — slip angle [°]

Twisting torque stiffness =
0.000108 m/° (0.0349 N•m/°)

Normalized twisting torque [m] — camber angle [°]

Mean pneumatic trail = 11.1 mm

Pneumatic trail [mm] — slip angle [°]

Static lateral stiffness = 0.0924/mm (29.9 N/mm)
Calculated relaxation length = 193 mm

Normalized lateral force — displacement [mm]

Cheng Shin Classic Zeppelin 50-622
tire radius = 22.1 mm, rim width = 18.7 mm

at 3.03 bar (44 psi) and
under 540 N (55 kg, 121 lb)

Cornering stiffness = 0.218/° (117 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.0122/° (6.61 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00545 m/° (-2.94 N•m/°)

Twisting torque stiffness =
0.000131 m/° (0.0706 N•m/°)

Mean pneumatic trail = 18.4 mm

Static lateral stiffness = 0.05/mm (27 N/mm)
Calculated relaxation length = 249 mm

Cheng Shin Classic Zeppelin 50-622
tire radius = 22.1 mm, rim width = 18.7 mm
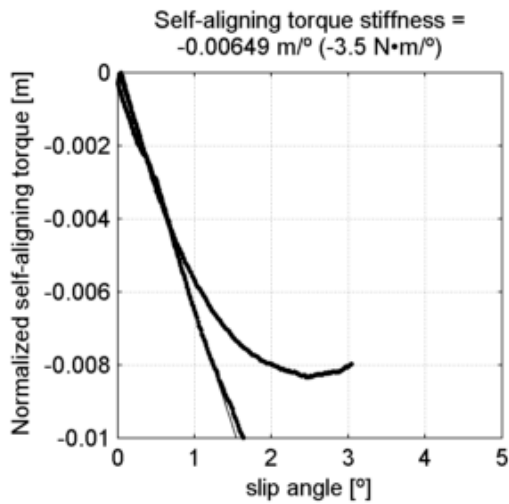
at 3.03 bar (44 psi) and
under 731 N (74.5 kg, 164 lb)

Cornering stiffness = 0.169/° (124 N/°)

Solid line represents
extracted stiffness value

x-axis: slip angle [°], y-axis: Normalized lateral force

Camber stiffness = 0.0126/° (9.18 N/°)

Dashed line represents
tangent of camber angle

x-axis: camber [°], y-axis: Normalized lateral force

Self-aligning torque stiffness =
-0.00536 m/° (-3.92 N•m/°)

x-axis: slip angle [°], y-axis: Normalized self-aligning torque [m]

Twisting torque stiffness =
0.000127 m/° (0.0931 N•m/°)

x-axis: camber angle [°], y-axis: Normalized twisting torque [m]

Mean pneumatic trail = 26.3 mm

x-axis: slip angle [°], y-axis: Pneumatic trail [mm]

Static lateral stiffness = 0.0373/mm (27.3 N/mm)
Calculated relaxation length = 260 mm

x-axis: displacement [mm], y-axis: Normalized lateral force

Cheng Shin Classic Zeppelin 50-622
tire radius = 22.1 mm, rim width = 18.7 mm

at 4 bar (58 psi) and
under 324 N (33 kg, 72.8 lb)

Cheng Shin Classic Zeppelin 50-622
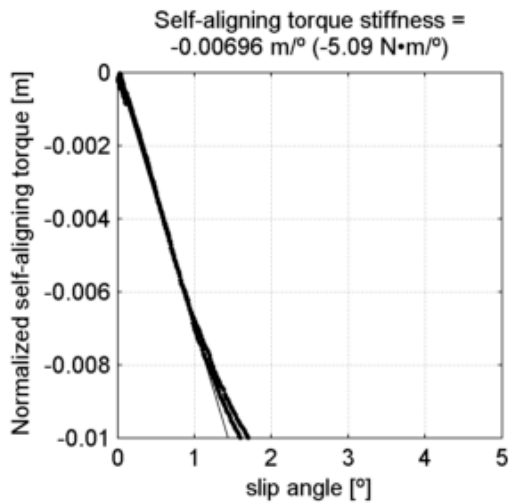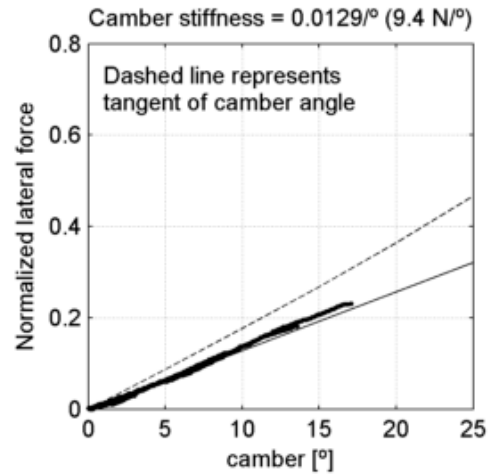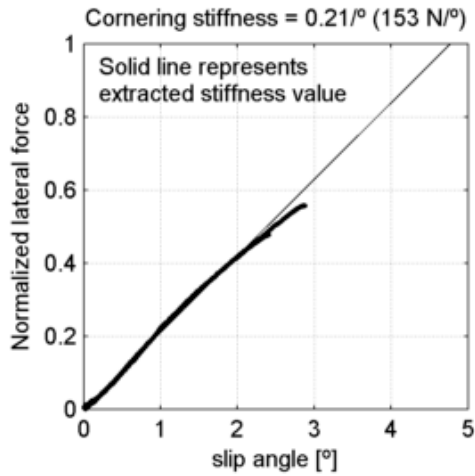tire radius = 22.1 mm, rim width = 18.7 mm

at 4 bar (58 psi) and
under 540 N (55 kg, 121 lb)

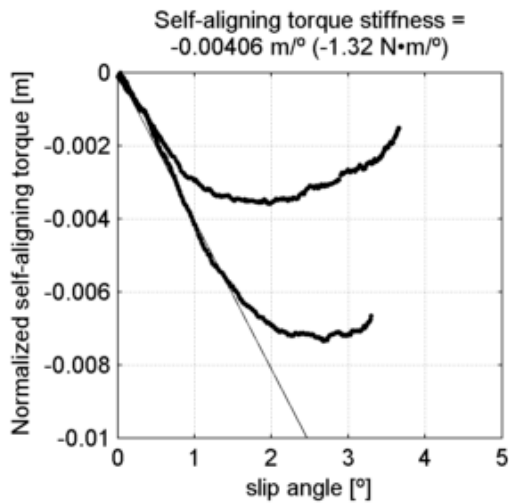Cornering stiffness = 0.242/° (131 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.0119/° (6.44 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00495 m/° (-2.67 N•m/°)

Twisting torque stiffness =
0.000105 m/° (0.0565 N•m/°)

Mean pneumatic trail = 15.2 mm

Static lateral stiffness = 0.0628/mm (33.9 N/mm)
Calculated relaxation length = 221 mm
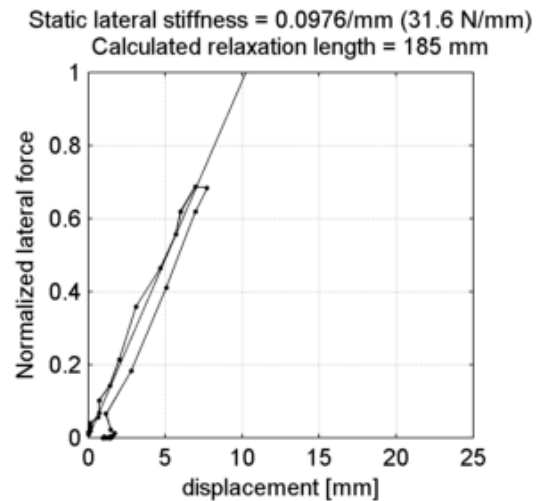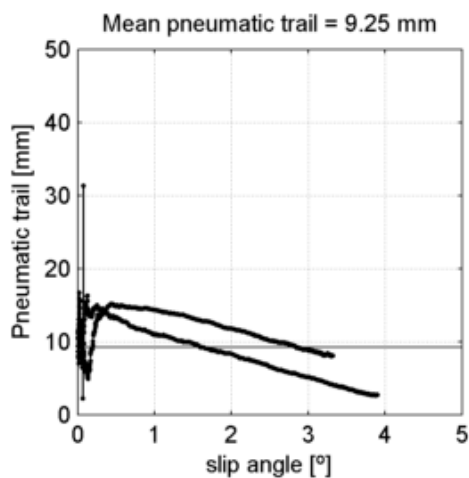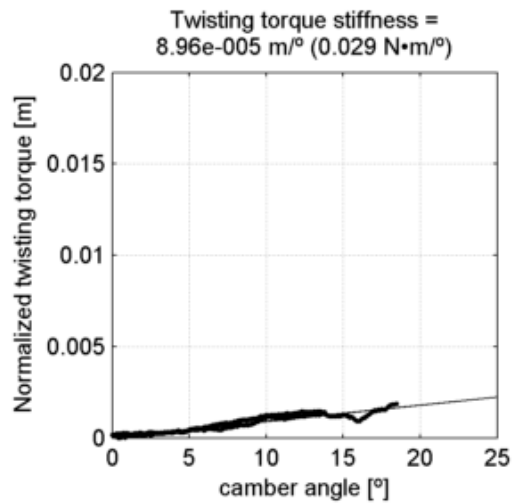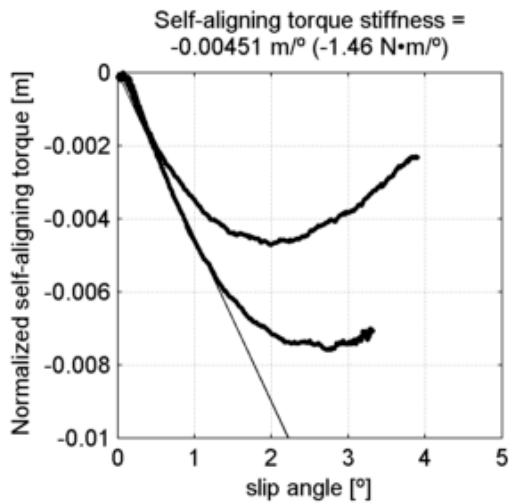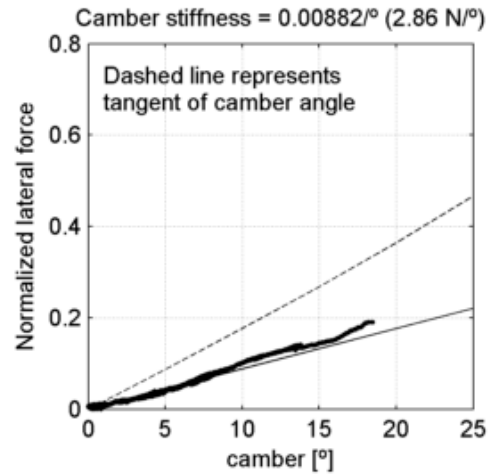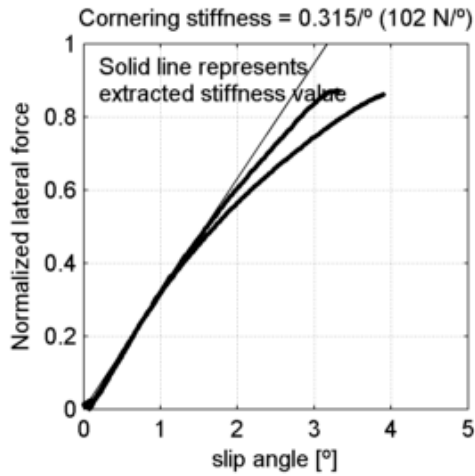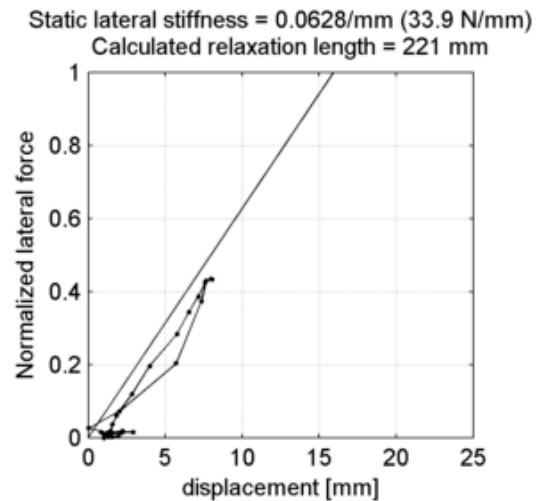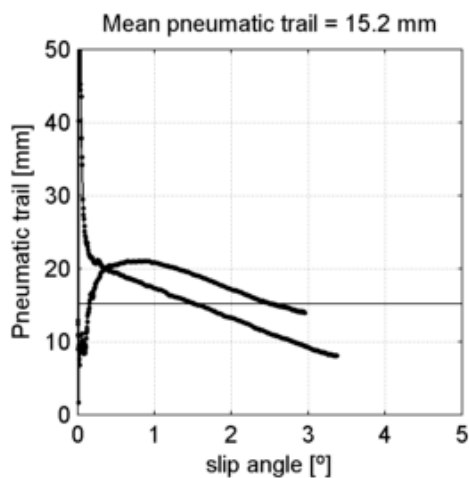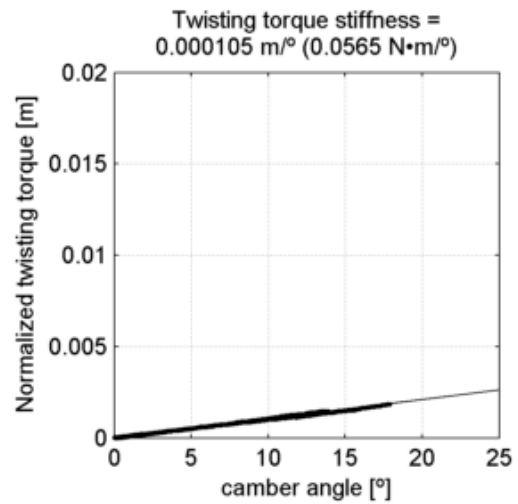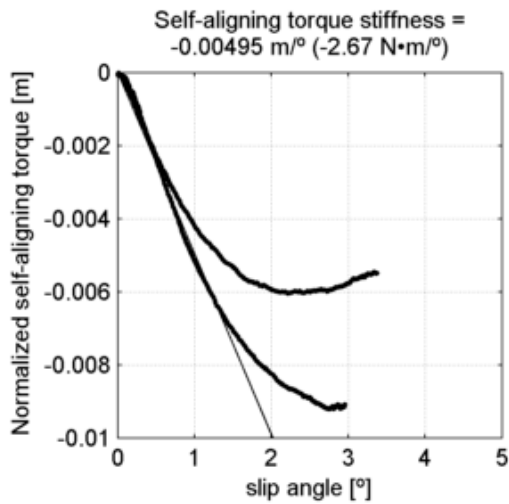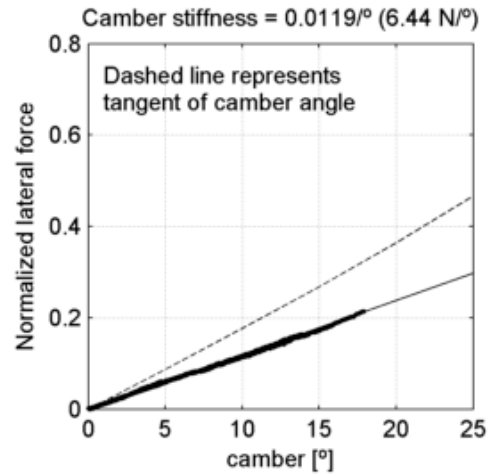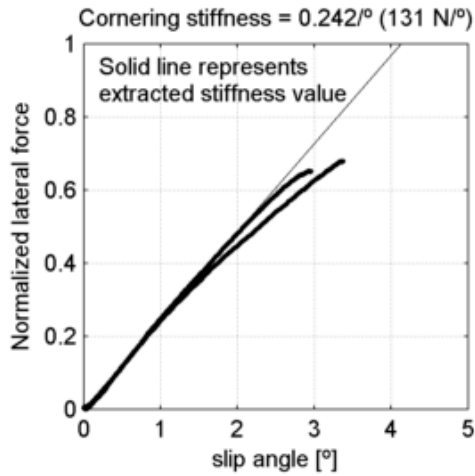
Cheng Shin Classic Zeppelin 50-622
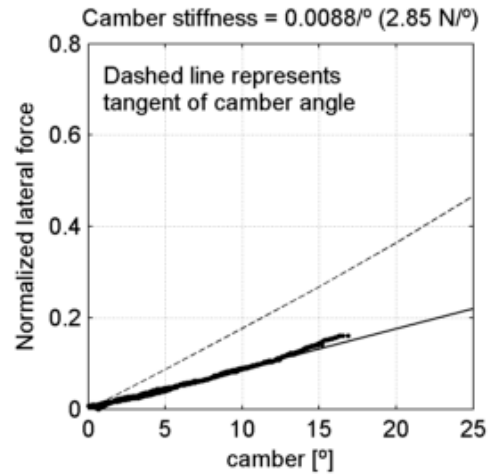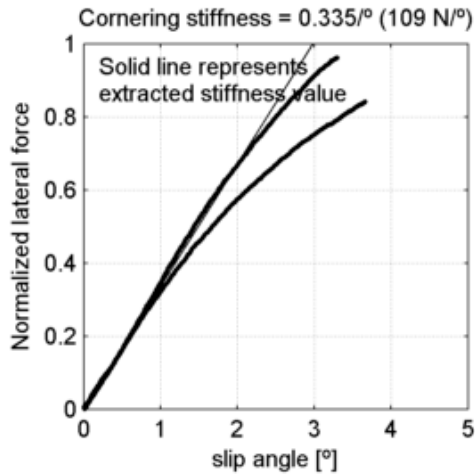tire radius = 22.1 mm, rim width = 18.7 mm

at 4 bar (58 psi) and
under 731 N (74.5 kg, 164 lb)

Continental Top Contact Winter 37-622
tire radius = 15.6 mm, rim width = 18.7 mm

at 4 bar (58 psi) and
under 324 N (33 kg, 72.8 lb)

Cornering stiffness = 0.183/° (59.4 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.00694/° (2.25 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00368 m/° (-1.19 N•m/°)

Twisting torque stiffness =
8.74e-005 m/° (0.0283 N•m/°)

Mean pneumatic trail = 16.9 mm

Static lateral stiffness = 0.0882/mm (28.6 N/mm)
Calculated relaxation length = 119 mm
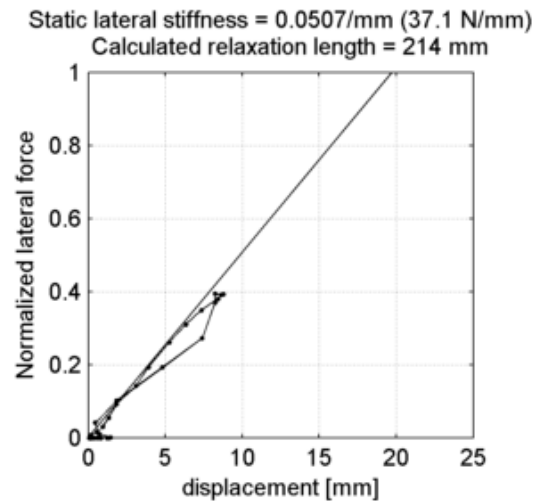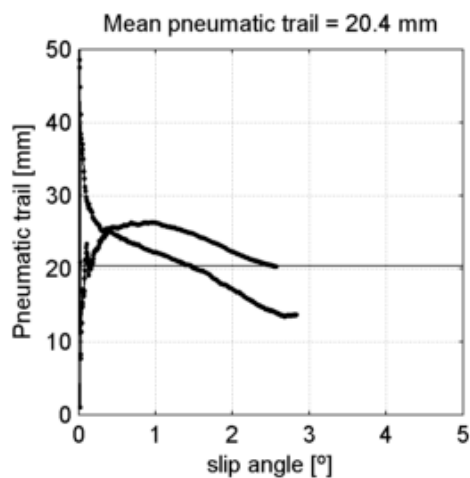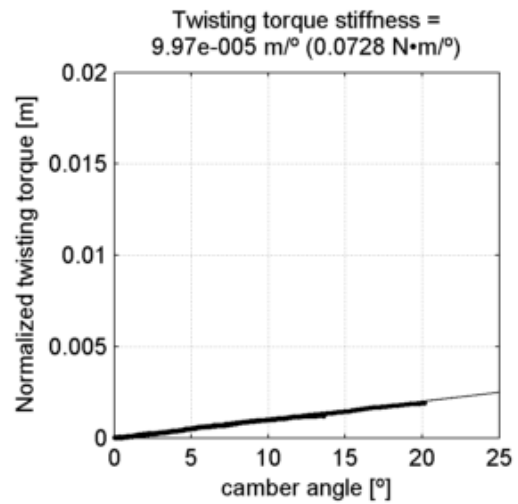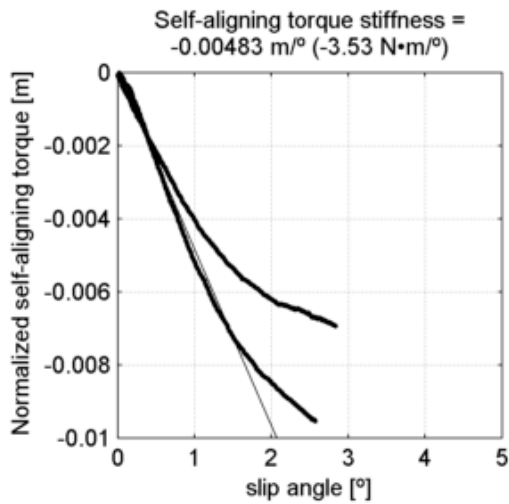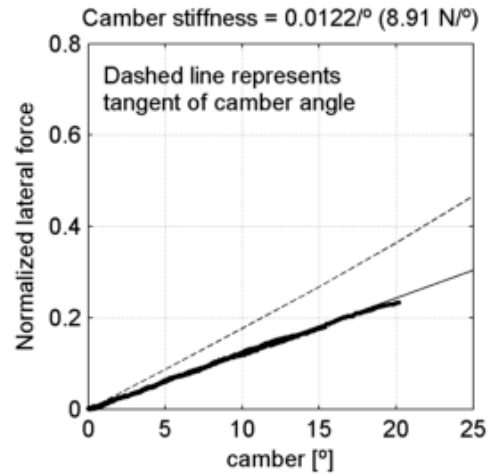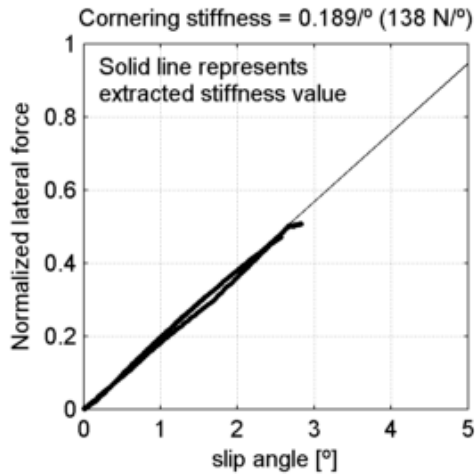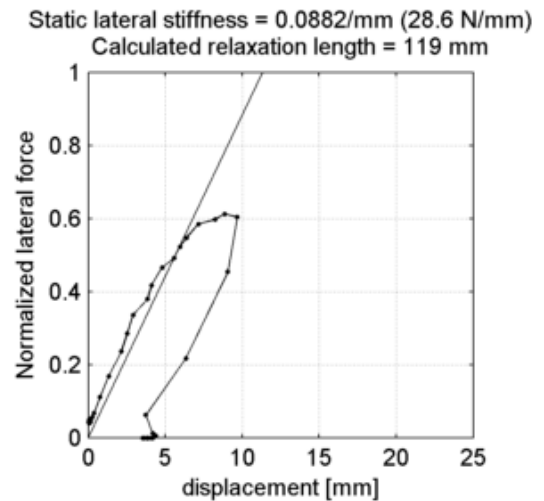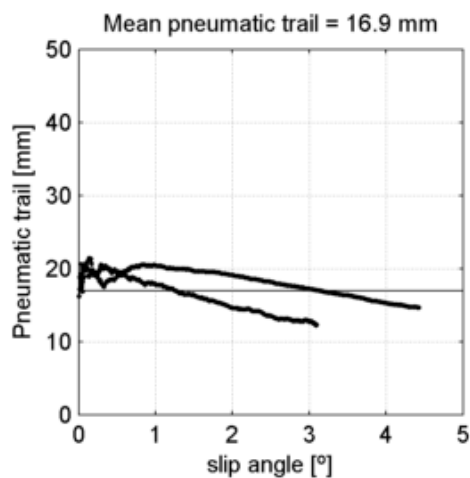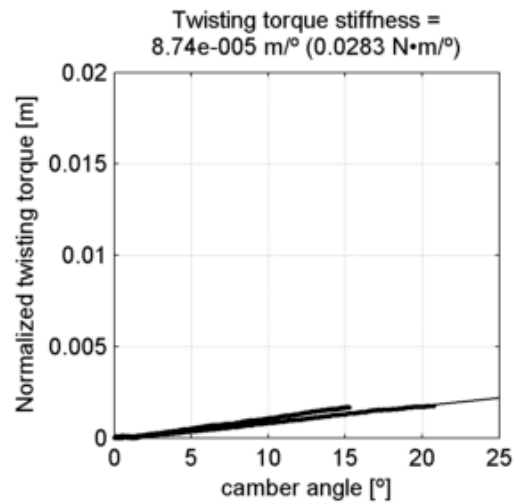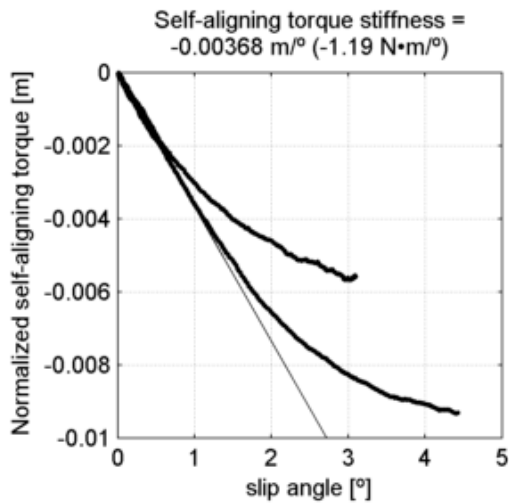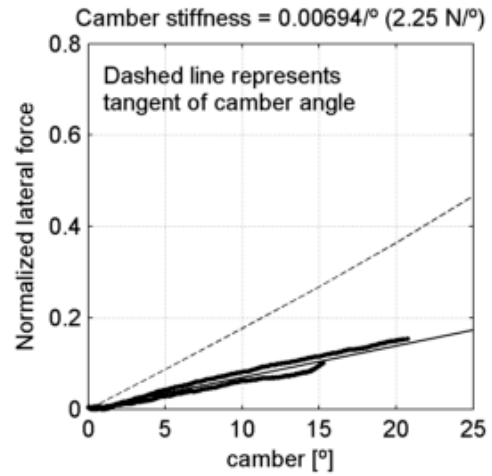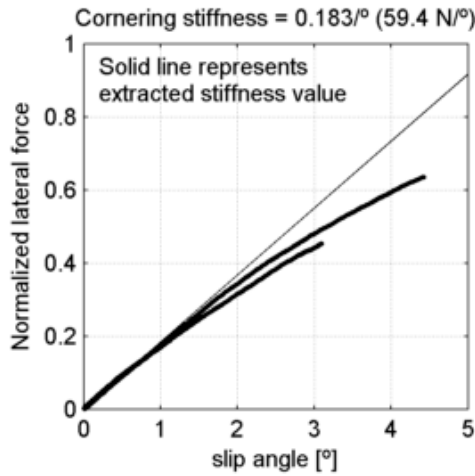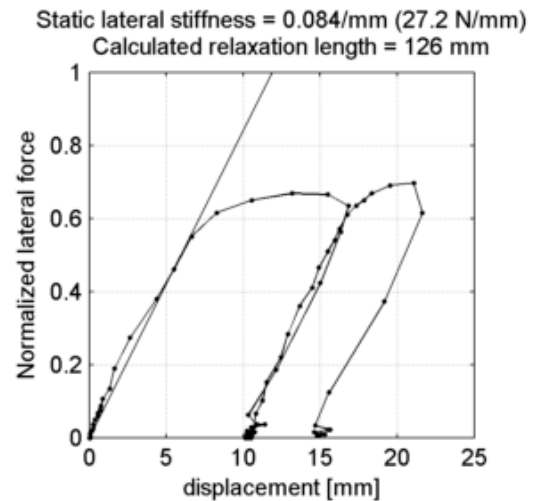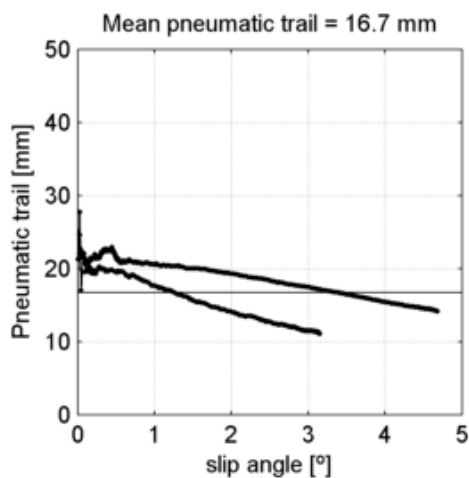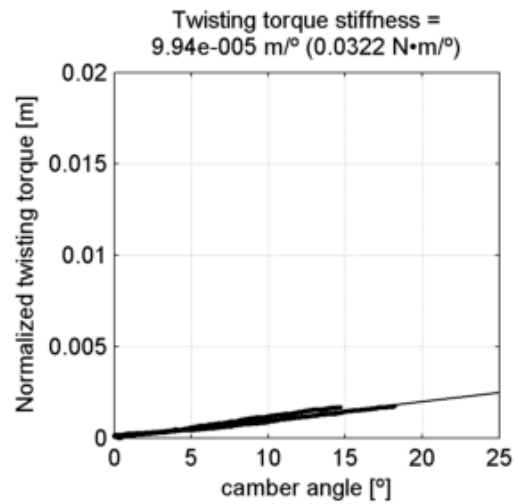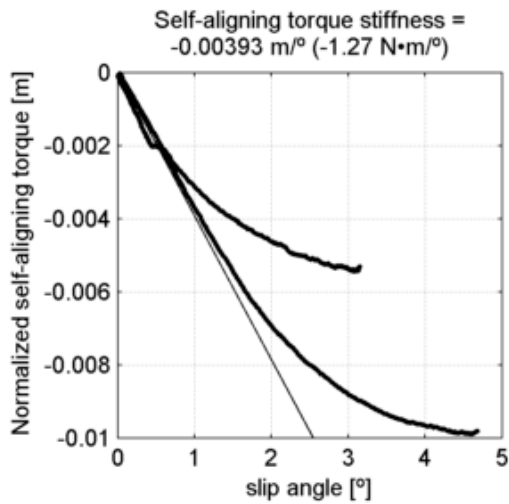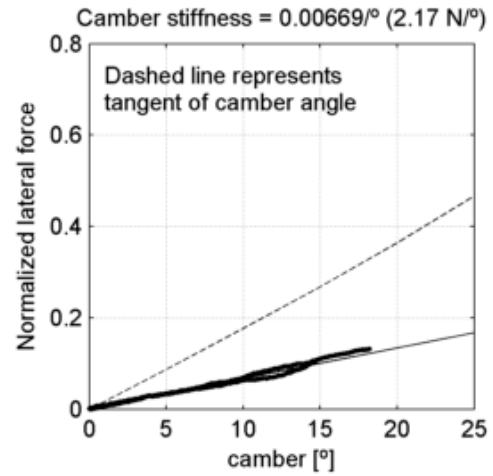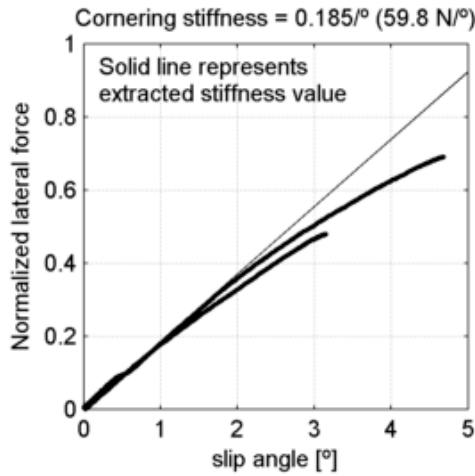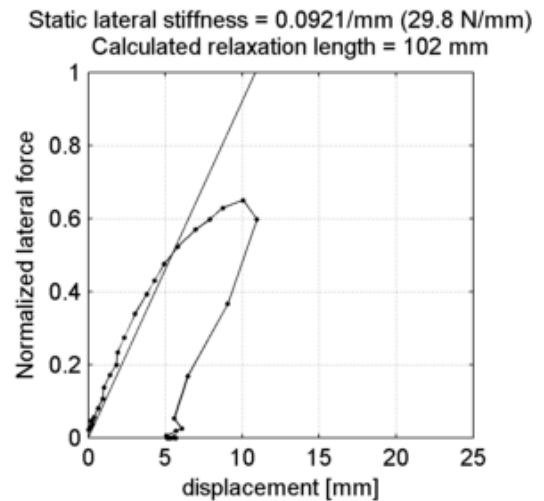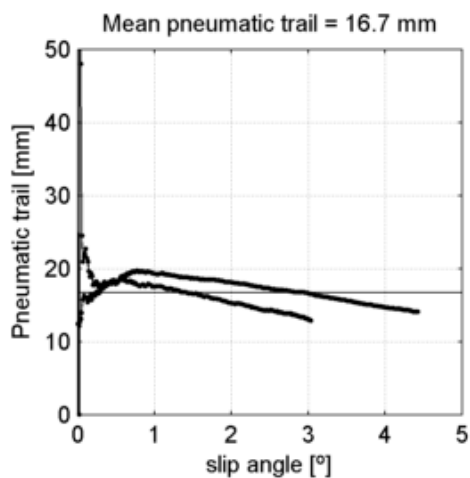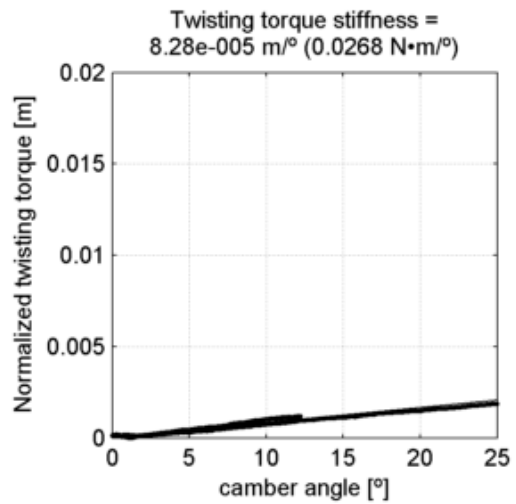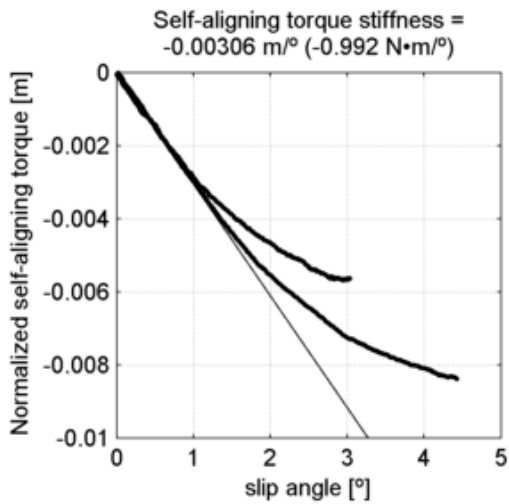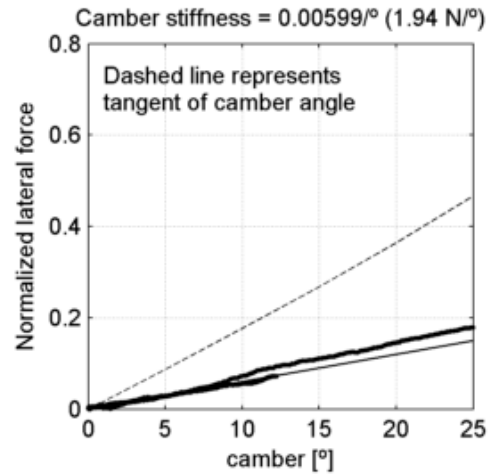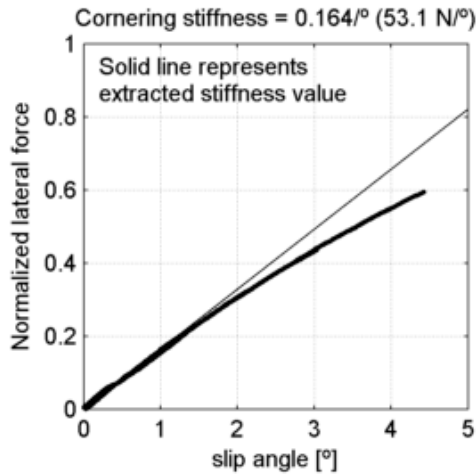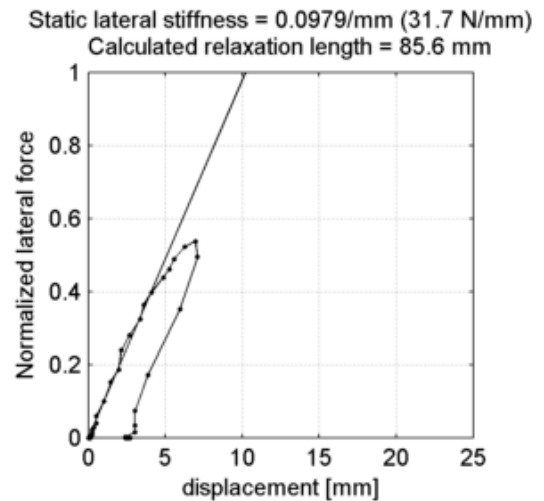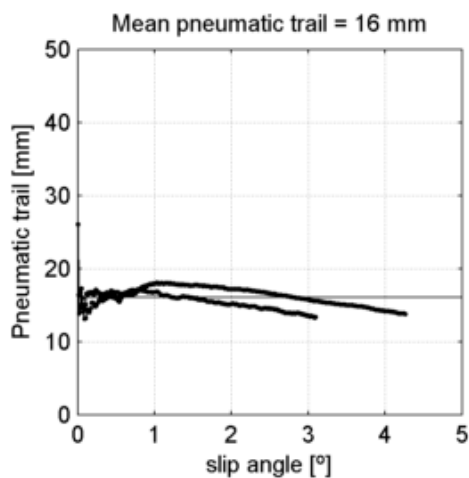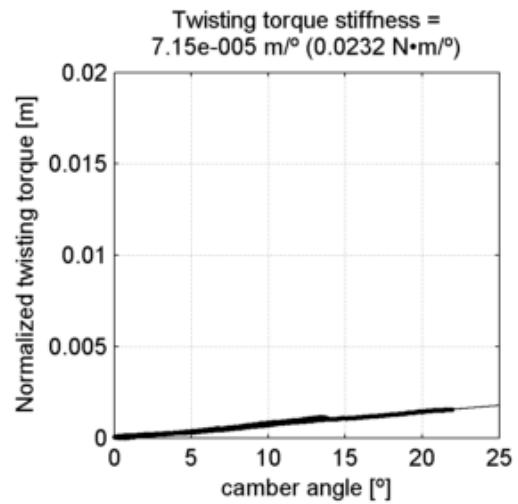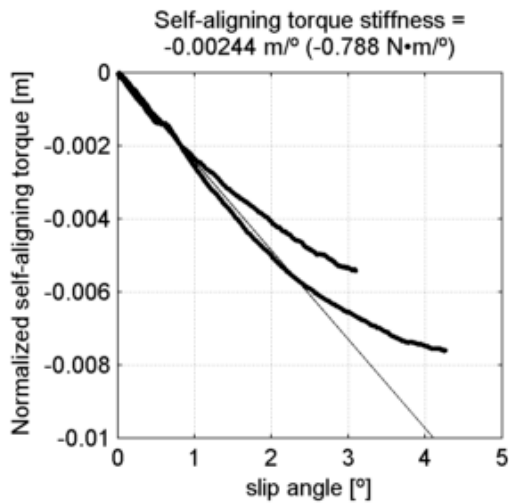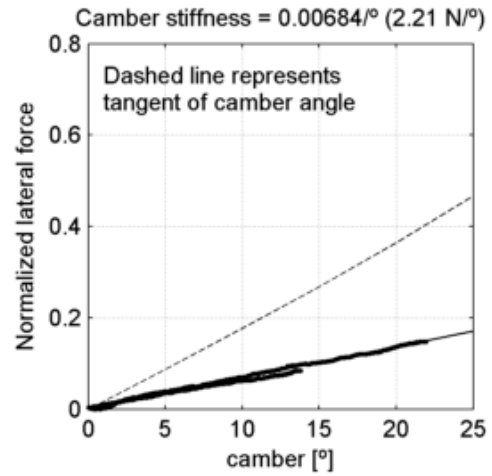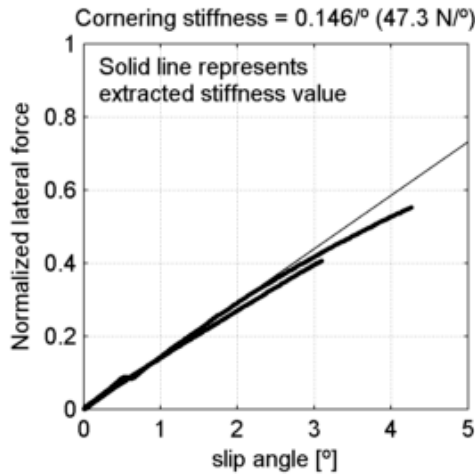
Continental Top Contact Winter 37-622
tire radius = 15.6 mm, rim width = 18.7 mm — at 4 bar (58 psi) and under 540 N (55 kg, 121 lb)

Continental Top Contact Winter 37-622
tire radius = 15.6 mm, rim width = 18.7 mm

at 4 bar (58 psi) and
under 731 N (74.5 kg, 164 lb)

Cornering stiffness = 0.131/° (96 N/°)
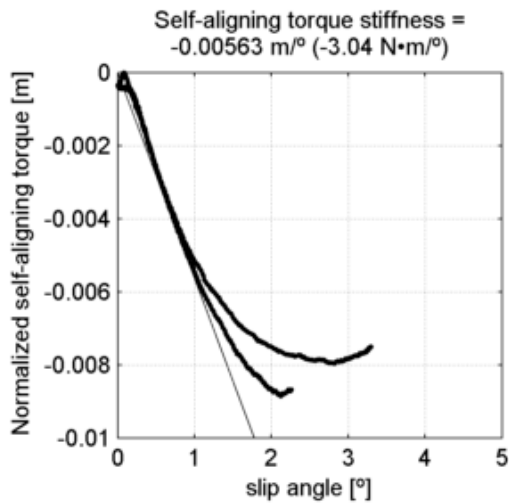
Solid line represents
extracted stiffness value

Camber stiffness = 0.0089/° (6.5 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
−0.00417 m/° (−3.05 N•m/°)

Twisting torque stiffness =
8.29e-005 m/° (0.0606 N•m/°)

Mean pneumatic trail = 28.1 mm

Static lateral stiffness = 0.0477/mm (34.8 N/mm)
Calculated relaxation length = 158 mm

Continental Top Contact Winter 37-622
tire radius = 15.6 mm, rim width = 18.7 mm

at 3.45 bar (50 psi) and
under 324 N (33 kg, 72.8 lb)

Cornering stiffness = 0.185/° (59.8 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.00669/° (2.17 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00393 m/° (-1.27 N•m/°)

Twisting torque stiffness =
9.94e-005 m/° (0.0322 N•m/°)

Mean pneumatic trail = 16.7 mm

Static lateral stiffness = 0.084/mm (27.2 N/mm)
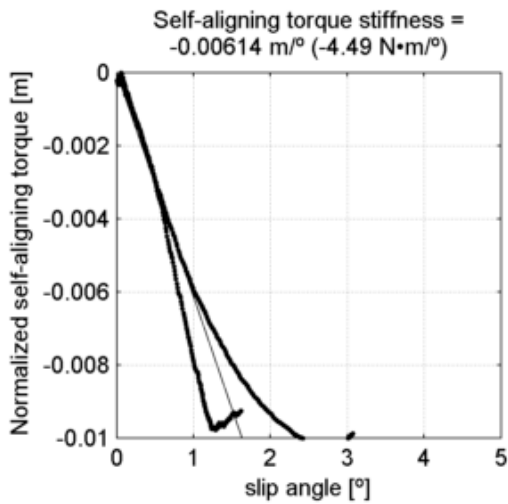Calculated relaxation length = 126 mm

Continental Top Contact Winter 37-622
tire radius = 15.6 mm, rim width = 18.7 mm

at 3.45 bar (50 psi) and
under 540 N (55 kg, 121 lb)

Cornering stiffness = 0.151/° (81.6 N/°)

Solid line represents
extracted stiffness value

Normalized lateral force vs slip angle [°]

Camber stiffness = 0.0088/° (4.75 N/°)

Dashed line represents
tangent of camber angle

Normalized lateral force vs camber [°]

Self-aligning torque stiffness =
-0.00393 m/° (-2.12 N•m/°)

Normalized self-aligning torque [m] vs slip angle [°]

Twisting torque stiffness =
6.67e-005 m/° (0.036 N•m/°)

Normalized twisting torque [m] vs camber angle [°]

Mean pneumatic trail = 22.6 mm

Pneumatic trail [mm] vs slip angle [°]

Static lateral stiffness = 0.0565/mm (30.5 N/mm)
Calculated relaxation length = 153 mm

Normalized lateral force vs displacement [mm]

Continental Top Contact Winter 37-622
tire radius = 15.6 mm, rim width = 18.7 mm

at 3.45 bar (50 psi) and
under 731 N (74.5 kg, 164 lb)

Cornering stiffness = 0.133/° (96.9 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.00976/° (7.13 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00423 m/° (-3.09 N•m/°)

Twisting torque stiffness =
4.91e-005 m/° (0.0359 N•m/°)

Mean pneumatic trail = 28.3 mm

Static lateral stiffness = 0.0445/mm (32.5 N/mm)
Calculated relaxation length = 171 mm

Continental Top Contact Winter 37-622
tire radius = 15.6 mm, rim width = 18.7 mm

at 4.83 bar (70 psi) and
under 324 N (33 kg, 72.8 lb)

Cornering stiffness = 0.164/° (53.1 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.00599/° (1.94 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00306 m/° (-0.992 N•m/°)

Twisting torque stiffness =
8.28e-005 m/° (0.0268 N•m/°)

Mean pneumatic trail = 16.7 mm

Static lateral stiffness = 0.0921/mm (29.8 N/mm)
Calculated relaxation length = 102 mm

Continental Top Contact Winter 37-622
tire radius = 15.6 mm, rim width = 18.7 mm

at 4.83 bar (70 psi) and
under 540 N (55 kg, 121 lb)

Cornering stiffness = 0.133/° (71.8 N/°)

Solid line represents
extracted stiffness value

Normalized lateral force

slip angle [°]

Camber stiffness = 0.00896/° (4.83 N/°)

Dashed line represents
tangent of camber angle

Normalized lateral force

camber [°]

Self-aligning torque stiffness =
-0.0031 m/° (-1.67 N•m/°)

Normalized self-aligning torque [m]

slip angle [°]

Twisting torque stiffness =
4.79e-005 m/° (0.0259 N•m/°)

Normalized twisting torque [m]

camber angle [°]

Mean pneumatic trail = 21.2 mm

Pneumatic trail [mm]

slip angle [°]

Static lateral stiffness = 0.0594/mm (32 N/mm)
Calculated relaxation length = 129 mm

Normalized lateral force

displacement [mm]

Continental Top Contact Winter 37-622
tire radius = 15.6 mm, rim width = 18.7 mm

at 4.83 bar (70 psi) and
under 731 N (74.5 kg, 164 lb)

Cornering stiffness = 0.127/° (92.6 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.00902/° (6.59 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
−0.00354 m/° (−2.59 N·m/°)

Twisting torque stiffness =
4.57e-005 m/° (0.0334 N·m/°)

Mean pneumatic trail = 24.3 mm

Static lateral stiffness = 0.0464/mm (33.9 N/mm)
Calculated relaxation length = 156 mm

Continental Top Contact Winter 37-622
tire radius = 15.6 mm, rim width = 18.7 mm

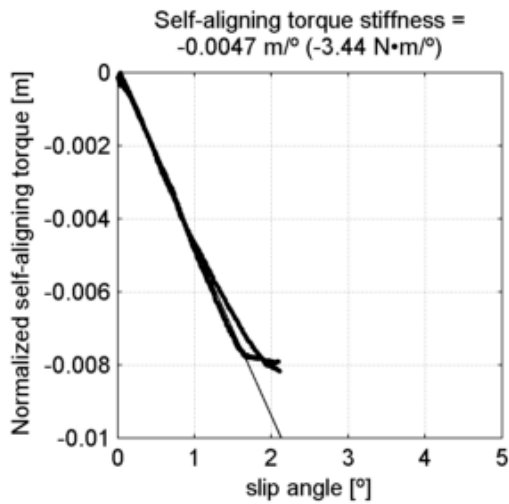at 6.21 bar (90 psi) and
under 324 N (33 kg, 72.8 lb)

Cornering stiffness = 0.146/° (47.3 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.00684/° (2.21 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00244 m/° (-0.788 N•m/°)

Twisting torque stiffness =
7.15e-005 m/° (0.0232 N•m/°)

Mean pneumatic trail = 16 mm

Static lateral stiffness = 0.0979/mm (31.7 N/mm)
Calculated relaxation length = 85.6 mm

Continental Top Contact Winter 37-622
tire radius = 15.6 mm, rim width = 18.7 mm

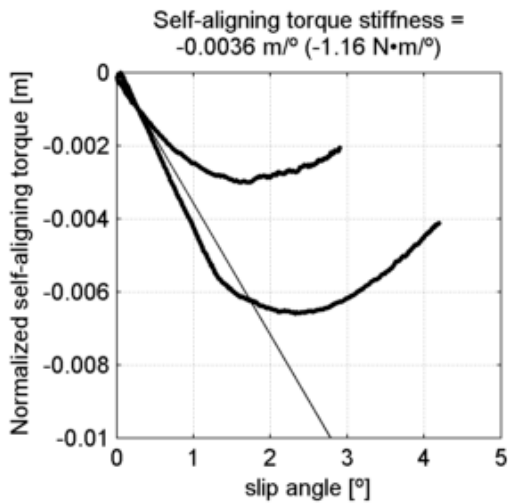at 6.21 bar (90 psi) and
under 540 N (55 kg, 121 lb)

Cornering stiffness = 0.135/° (72.6 N/°)

Solid line represents
extracted stiffness value

(x-axis: slip angle [°], y-axis: Normalized lateral force)

Camber stiffness = 0.00754/° (4.07 N/°)

Dashed line represents
tangent of camber angle

(x-axis: camber [°], y-axis: Normalized lateral force)

**Self-aligning torque stiffness =
-0.0029 m/° (-1.56 N•m/°)**

(x-axis: slip angle [°], y-axis: Normalized self-aligning torque [m])

**Twisting torque stiffness =
7.02e-005 m/° (0.0379 N•m/°)**

(x-axis: camber angle [°], y-axis: Normalized twisting torque [m])

Mean pneumatic trail = 19.3 mm

(x-axis: slip angle [°], y-axis: Pneumatic trail [mm])

**Static lateral stiffness = 0.0697/mm (37.6 N/mm)
Calculated relaxation length = 111 mm**

(x-axis: displacement [mm], y-axis: Normalized lateral force)

Continental Top Contact Winter 37-622
tire radius = 15.6 mm, rim width = 18.7 mm

at 6.21 bar (90 psi) and
under 731 N (74.5 kg, 164 lb)

Cornering stiffness = 0.113/° (82.3 N/°)
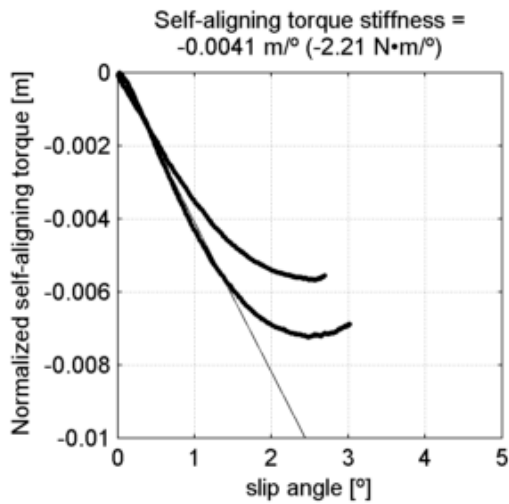
Solid line represents
extracted stiffness value

(y-axis: Normalized lateral force, x-axis: slip angle [°])

Camber stiffness = 0.00807/° (5.9 N/°)

Dashed line represents
tangent of camber angle

(y-axis: Normalized lateral force, x-axis: camber [°])

Self-aligning torque stiffness =
-0.00308 m/° (-2.25 N•m/°)

(y-axis: Normalized self-aligning torque [m], x-axis: slip angle [°])

Twisting torque stiffness =
7e-005 m/° (0.0512 N•m/°)

(y-axis: Normalized twisting torque [m], x-axis: camber angle [°])

Mean pneumatic trail = 22.3 mm

(y-axis: Pneumatic trail [mm], x-axis: slip angle [°])

Static lateral stiffness = 0.053/mm (38.7 N/mm)
Calculated relaxation length = 122 mm

(y-axis: Normalized lateral force, x-axis: displacement [mm])

Maxis Radial 22-622
tire radius = 11 mm, rim width = 13.4 mm

at 6.89 bar (100 psi) and
under 304 N (31 kg, 68.3 lb)

Cornering stiffness = 0.217/° (66 N/°)

Solid line represents
extracted stiffness value

x-axis: slip angle [°], y-axis: Normalized lateral force

Camber stiffness = 0.0158/° (4.81 N/°)

Dashed line represents
tangent of camber angle

x-axis: camber [°], y-axis: Normalized lateral force

Self-aligning torque stiffness =
−0.0035 m/° (−1.06 N•m/°)

x-axis: slip angle [°], y-axis: Normalized self-aligning torque [m]

Twisting torque stiffness =
7.36e-005 m/° (0.0224 N•m/°)

x-axis: camber angle [°], y-axis: Normalized twisting torque [m]

Mean pneumatic trail = 6.56 mm

x-axis: slip angle [°], y-axis: Pneumatic trail [mm]

Static lateral stiffness = 0.146/mm (44.3 N/mm)
Calculated relaxation length = 85.3 mm

x-axis: displacement [mm], y-axis: Normalized lateral force

Maxis Radial 22-622
tire radius = 11 mm, rim width = 13.4 mm

at 8.27 bar (120 psi) and
under 304 N (31 kg, 68.3 lb)

Cornering stiffness = 0.229/° (69.8 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.0142/° (4.31 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00313 m/° (-0.951 N•m/°)

Twisting torque stiffness =
8.85e-005 m/° (0.0269 N•m/°)

Mean pneumatic trail = 6.28 mm

Static lateral stiffness = 0.146/mm (44.4 N/mm)
Calculated relaxation length = 90.1 mm

Maxis Radial 22-622
tire radius = 11 mm, rim width = 13.4 mm

at 9.65 bar (140 psi) and
under 304 N (31 kg, 68.3 lb)

Cornering stiffness = 0.235/° (71.4 N/°)

Solid line represents
extracted stiffness value

Normalized lateral force vs *slip angle [°]*

Camber stiffness = 0.0135/° (4.1 N/°)

Dashed line represents
tangent of camber angle

Normalized lateral force vs *camber [°]*

Self-aligning torque stiffness =
-0.00286 m/° (-0.868 N•m/°)

Normalized self-aligning torque [m] vs *slip angle [°]*

Twisting torque stiffness =
2.03e-005 m/° (0.00616 N•m/°)

Normalized twisting torque [m] vs *camber angle [°]*

Mean pneumatic trail = 5.35 mm

Pneumatic trail [mm] vs *slip angle [°]*

Static lateral stiffness = 0.156/mm (47.3 N/mm)
Calculated relaxation length = 86.5 mm

Normalized lateral force vs *displacement [mm]*

Maxis Radial 22-622
tire radius = 11 mm, rim width = 13.4 mm

at 9.65 bar (140 psi) and
under 525 N (53.5 kg, 118 lb)

Cornering stiffness = 0.174/º (91.3 N/º)

Solid line represents
extracted stiffness value

Camber stiffness = 0.0168/º (8.82 N/º)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00276 m/º (-1.45 N•m/º)

Twisting torque stiffness =
1.66e-005 m/º (0.00869 N•m/º)

Mean pneumatic trail = 8.44 mm

Static lateral stiffness = 0.0964/mm (50.6 N/mm)
Calculated relaxation length = 103 mm

Maxis Radial 22-622
tire radius = 11 mm, rim width = 13.4 mm

at 9.65 bar (140 psi) and
under 525 N (53.5 kg, 118 lb)

Cornering stiffness = 0.161/° (84.3 N/°)

Solid line represents
extracted stiffness value

[plot: Normalized lateral force vs slip angle [°]]

Camber stiffness = 0.017/° (8.93 N/°)

Dashed line represents
tangent of camber angle

[plot: Normalized lateral force vs camber [°]]

Self-aligning torque stiffness =
−0.00274 m/° (−1.44 N•m/°)

[plot: Normalized self-aligning torque [m] vs slip angle [°]]

Twisting torque stiffness =
1.6e-005 m/° (0.00839 N•m/°)

[plot: Normalized twisting torque [m] vs camber angle [°]]

Mean pneumatic trail = 8.57 mm

[plot: Pneumatic trail [mm] vs slip angle [°]]

Static lateral stiffness = 0.0964/mm (50.6 N/mm)
Calculated relaxation length = 95.4 mm

[plot: Normalized lateral force vs displacement [mm]]

Maxis Radial 22-622
tire radius = 11 mm, rim width = 13.4 mm

at 9.65 bar (140 psi) and
under 525 N (53.5 kg, 118 lb)

Cornering stiffness = 0.156/° (82 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.0154/° (8.09 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00257 m/° (-1.35 N•m/°)

Twisting torque stiffness =
3.05e-005 m/° (0.016 N•m/°)

Mean pneumatic trail = 8.54 mm

Static lateral stiffness = 0.0964/mm (50.6 N/mm)
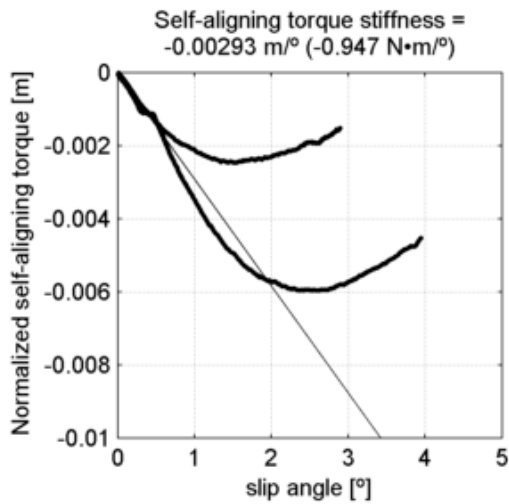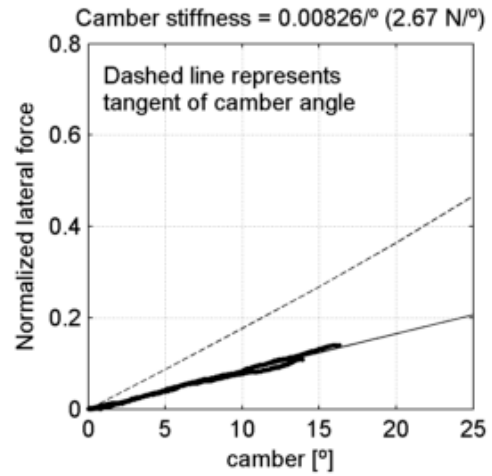Calculated relaxation length = 92.9 mm

Maxis Radial 22-622
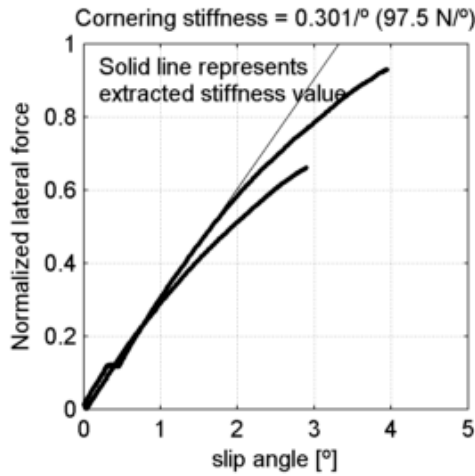tire radius = 11 mm, rim width = 13.4 mm

at 9.65 bar (140 psi) and
under 731 N (74.5 kg, 164 lb)

Cornering stiffness = 0.118/° (86.2 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.0155/° (11.3 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00256 m/° (-1.87 N•m/°)

Twisting torque stiffness =
7.12e-006 m/° (0.0052 N•m/°)

Mean pneumatic trail = 14.2 mm

Static lateral stiffness = 0.0663/mm (48.4 N/mm)
Calculated relaxation length = 102 mm

Michelin Dynamic xx-622
tire radius = 12.5 mm, rim width = 13.4 mm

at 4.14 bar (60 psi) and
under 304 N (31 kg, 68.3 lb)

Cornering stiffness = 0.348/° (106 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.0154/° (4.68 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00383 m/° (-1.16 N•m/°)

Twisting torque stiffness =
0.000116 m/° (0.0354 N•m/°)

Mean pneumatic trail = 4.81 mm

Michelin Dynamic xx-622
tire radius = 12.5 mm, rim width = 13.4 mm

at 6.21 bar (90 psi) and
under 304 N (31 kg, 68.3 lb)

Cornering stiffness = 0.357/° (109 N/°)

Solid line represents
extracted stiffness value

x-axis: slip angle [°]
y-axis: Normalized lateral force

Camber stiffness = 0.0141/° (4.29 N/°)

Dashed line represents
tangent of camber angle

x-axis: camber [°]
y-axis: Normalized lateral force

Self-aligning torque stiffness = -0.00271 m/° (-0.823 N•m/°)

x-axis: slip angle [°]
y-axis: Normalized self-aligning torque [m]

Twisting torque stiffness = 7.74e-005 m/° (0.0235 N•m/°)

x-axis: camber angle [°]
y-axis: Normalized twisting torque [m]

Mean pneumatic trail = 3.7 mm

x-axis: slip angle [°]
y-axis: Pneumatic trail [mm]

Michelin Dynamic xx-622
tire radius = 12.5 mm, rim width = 13.4 mm

at 8.27 bar (120 psi) and
under 304 N (31 kg, 68.3 lb)

Cornering stiffness = 0.379/° (115 N/°)

Solid line represents
extracted stiffness value

(Plot: Normalized lateral force vs slip angle [°])

Camber stiffness = 0.0139/° (4.23 N/°)

Dashed line represents
tangent of camber angle

(Plot: Normalized lateral force vs camber [°])

Self-aligning torque stiffness = -0.00232 m/° (-0.705 N•m/°)

(Plot: Normalized self-aligning torque [m] vs slip angle [°])

Twisting torque stiffness = 5.56e-005 m/° (0.0169 N•m/°)

(Plot: Normalized twisting torque [m] vs camber angle [°])

Mean pneumatic trail = 3.45 mm

(Plot: Pneumatic trail [mm] vs slip angle [°])

Michelin Dynamic xx-622
tire radius = 12.5 mm, rim width = 13.4 mm

at 8.27 bar (120 psi) and
under 535 N (54.5 kg, 120 lb)

Cornering stiffness = 0.265/° (142 N/°)

Solid line represents
extracted stiffness value

Normalized lateral force

slip angle [°]

Camber stiffness = 0.0141/° (7.54 N/°)

Dashed line represents
tangent of camber angle

Normalized lateral force

camber [°]

Self-aligning torque stiffness =
-0.00288 m/° (-1.54 N•m/°)

Normalized self-aligning torque [m]

slip angle [°]

Twisting torque stiffness =
2.14e-005 m/° (0.0115 N•m/°)

Normalized twisting torque [m]

camber angle [°]

Mean pneumatic trail = 5.01 mm

Pneumatic trail [mm]

slip angle [°]

Michelin Dynamic xx-622
tire radius = 12.5 mm, rim width = 13.4 mm

at 8.27 bar (120 psi) and
under 731 N (74.5 kg, 164 lb)

Cornering stiffness = 0.204/° (149 N/°)

Solid line represents
extracted stiffness value

Normalized lateral force

slip angle [°]

Camber stiffness = 0.0141/° (10.3 N/°)

Dashed line represents
tangent of camber angle

Normalized lateral force

camber [°]

Self-aligning torque stiffness =
-0.00296 m/° (-2.16 N•m/°)

Normalized self-aligning torque [m]

slip angle [°]

Twisting torque stiffness =
4.12e-006 m/° (0.00301 N•m/°)

Normalized twisting torque [m]

camber angle [°]

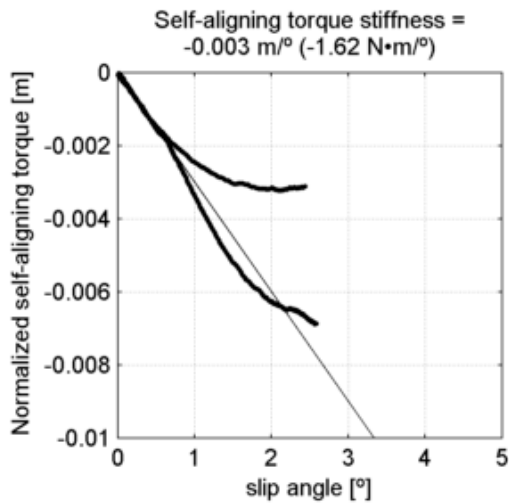Mean pneumatic trail = 9.06 mm
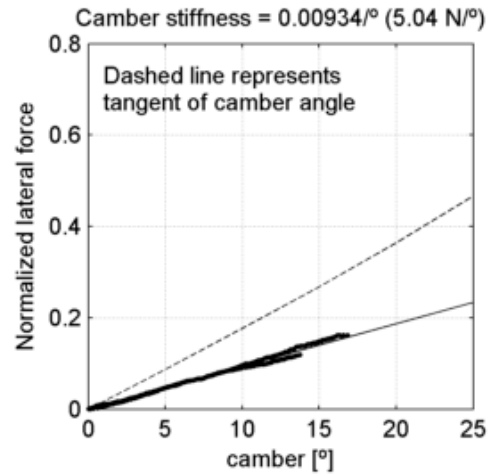
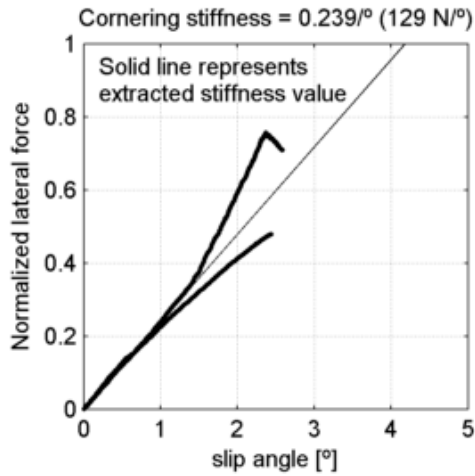Pneumatic trail [mm]

slip angle [°]
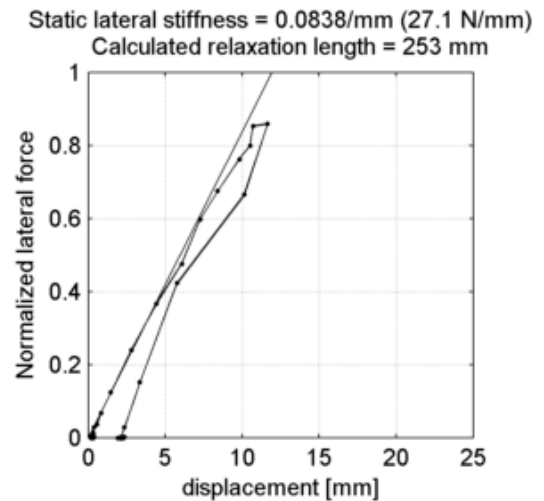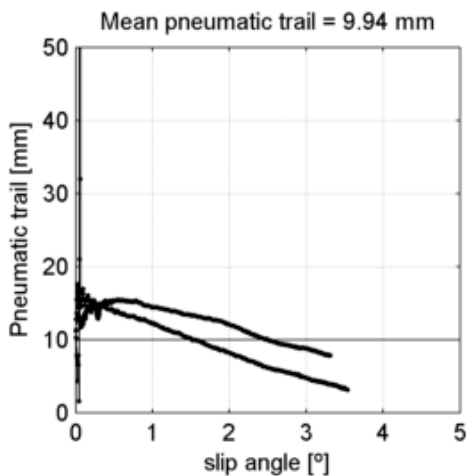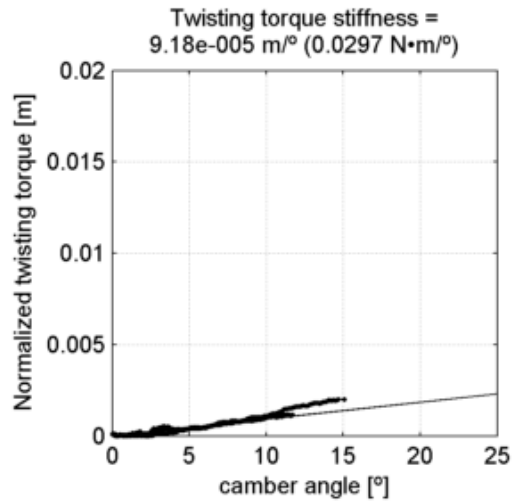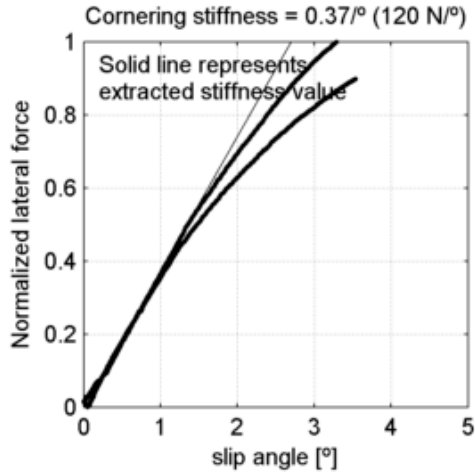
Schwalbe Big Apple 55-622
tire radius = 23.5 mm, rim width = 18.7 mm

at 2 bar (29 psi) and
under 324 N (33 kg, 72.8 lb)

Schwalbe Big Apple 55-622
tire radius = 23.5 mm, rim width = 18.7 mm

at 2 bar (29 psi) and
under 540 N (55 kg, 121 lb)

Cornering stiffness = 0.27/° (146 N/°)

Solid line represents
extracted stiffness value

x-axis: slip angle [°] — *y-axis:* Normalized lateral force

Camber stiffness = 0.0127/° (6.86 N/°)

Dashed line represents
tangent of camber angle

x-axis: camber [°] — *y-axis:* Normalized lateral force

Self-aligning torque stiffness = −0.00649 m/° (−3.5 N·m/°)

x-axis: slip angle [°] — *y-axis:* Normalized self-aligning torque [m]

Twisting torque stiffness = 0.000144 m/° (0.0779 N·m/°)

x-axis: camber angle [°] — *y-axis:* Normalized twisting torque [m]

Mean pneumatic trail = 19.7 mm

x-axis: slip angle [°] — *y-axis:* Pneumatic trail [mm]

Static lateral stiffness = 0.0469/mm (25.3 N/mm)
Calculated relaxation length = 330 mm

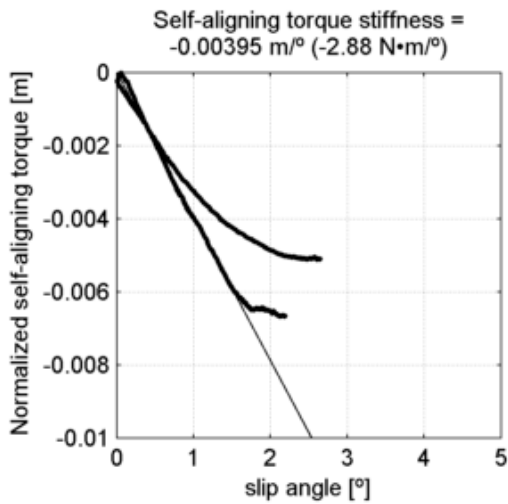*x-axis:* displacement [mm] — *y-axis:* Normalized lateral force

Schwalbe Big Apple 55-622
tire radius = 23.5 mm, rim width = 18.7 mm

at 2 bar (29 psi) and
under 731 N (74.5 kg, 164 lb)

Cornering stiffness = 0.21/° (153 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.0129/° (9.4 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00696 m/° (-5.09 N•m/°)

Twisting torque stiffness =
0.00015 m/° (0.11 N•m/°)

Mean pneumatic trail = 28.5 mm

Static lateral stiffness = 0.0436/mm (31.9 N/mm)
Calculated relaxation length = 275 mm

Schwalbe Big Apple 55-622
tire radius = 23.5 mm, rim width = 18.7 mm

at 3.03 bar (44 psi) and
under 324 N (33 kg, 72.8 lb)

Cornering stiffness = 0.335/° (109 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.0088/° (2.85 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00406 m/° (-1.32 N•m/°)

Twisting torque stiffness =
9.59e-005 m/° (0.0311 N•m/°)

Mean pneumatic trail = 7.91 mm

Static lateral stiffness = 0.105/mm (34 N/mm)
Calculated relaxation length = 183 mm

Schwalbe Big Apple 55-622
tire radius = 23.5 mm, rim width = 18.7 mm

at 3.03 bar (44 psi) and
under 540 N (55 kg, 121 lb)

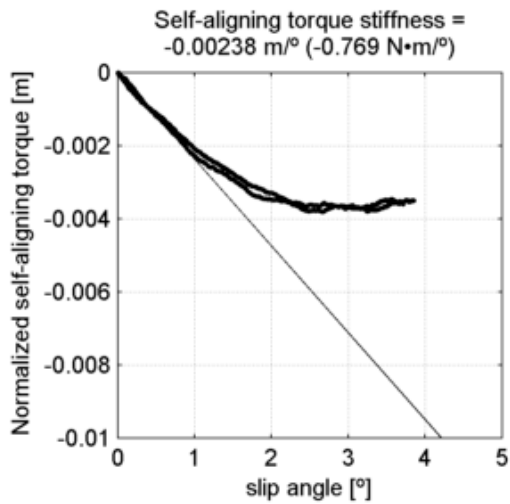Cornering stiffness = 0.288/° (156 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.0108/° (5.84 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00563 m/° (-3.04 N•m/°)

Twisting torque stiffness =
9.44e-005 m/° (0.0509 N•m/°)

Mean pneumatic trail = 15.2 mm

Static lateral stiffness = 0.0629/mm (34 N/mm)
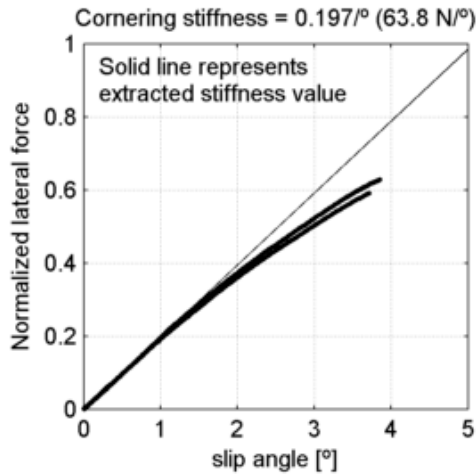Calculated relaxation length = 262 mm

Schwalbe Big Apple 55-622
tire radius = 23.5 mm, rim width = 18.7 mm

at 3.03 bar (44 psi) and
under 731 N (74.5 kg, 164 lb)

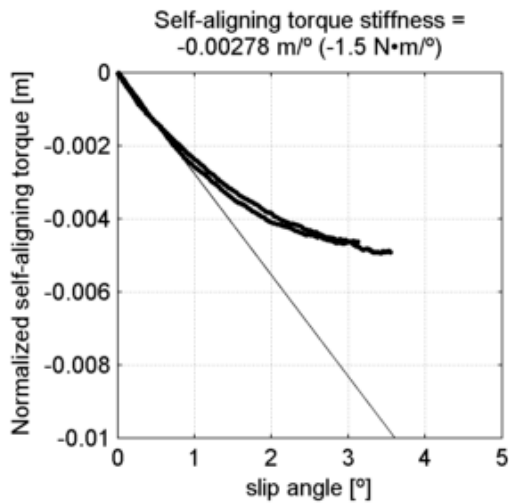--

Cornering stiffness = 0.24/° (175 N/°)

Solid line represents extracted stiffness value

Camber stiffness = 0.0125/° (9.1 N/°)

Dashed line represents tangent of camber angle

Self-aligning torque stiffness = -0.00614 m/° (-4.49 N•m/°)

Twisting torque stiffness = 0.000115 m/° (0.084 N•m/°)

Mean pneumatic trail = 20.3 mm

**Static lateral stiffness = 0.0378/mm (27.6 N/mm)
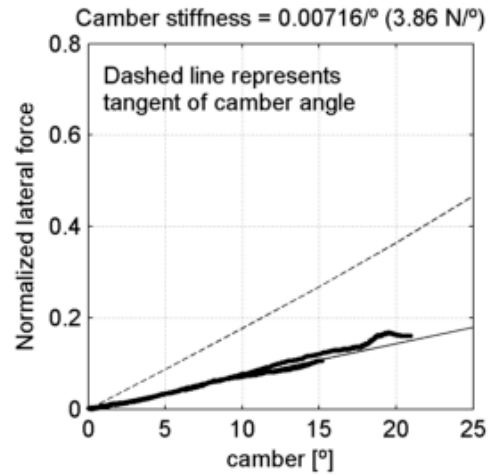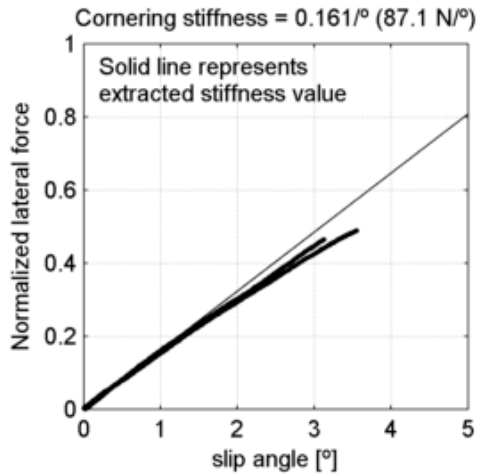Calculated relaxation length = 364 mm**

Schwalbe Big Apple 55-622
tire radius = 23.5 mm, rim width = 18.7 mm

at 4 bar (58 psi) and
under 324 N (33 kg, 72.8 lb)

Cornering stiffness = 0.322/° (104 N/°)

Solid line represents
extracted stiffness value

Normalized lateral force vs slip angle [°]

Camber stiffness = 0.0079/° (2.56 N/°)

Dashed line represents
tangent of camber angle

Normalized lateral force vs camber [°]

Self-aligning torque stiffness =
-0.00317 m/° (-1.03 N•m/°)

Normalized self-aligning torque [m] vs slip angle [°]

Twisting torque stiffness =
0.000102 m/° (0.033 N•m/°)

Normalized twisting torque [m] vs camber angle [°]

Mean pneumatic trail = 6.82 mm

Pneumatic trail [mm] vs slip angle [°]

Static lateral stiffness = 0.105/mm (33.9 N/mm)
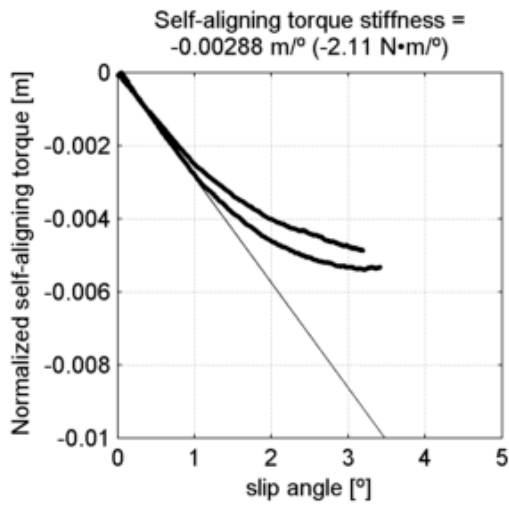Calculated relaxation length = 176 mm
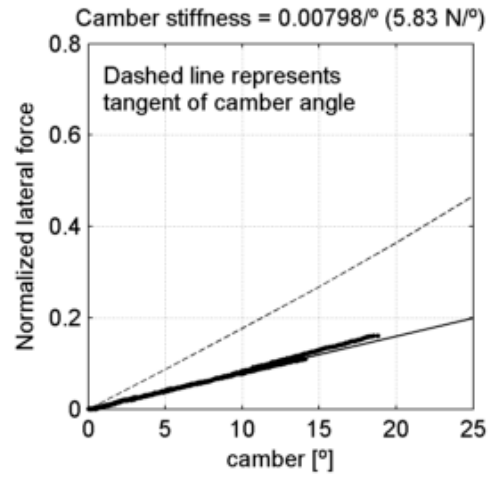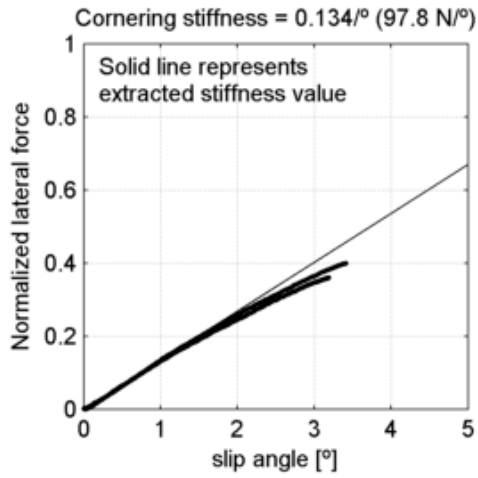
Normalized lateral force vs displacement [mm]

Schwalbe Big Apple 55-622
tire radius = 23.5 mm, rim width = 18.7 mm

at 4 bar (58 psi) and
under 540 N (55 kg, 121 lb)

Cornering stiffness = 0.266/° (143 N/°)

Solid line represents
extracted stiffness value

Normalized lateral force

slip angle [°]

Camber stiffness = 0.00968/° (5.22 N/°)

Dashed line represents
tangent of camber angle

Normalized lateral force

camber [°]

Self-aligning torque stiffness =
-0.00397 m/° (-2.14 N•m/°)

Normalized self-aligning torque [m]

slip angle [°]

Twisting torque stiffness =
8.3e-005 m/° (0.0448 N•m/°)

Normalized twisting torque [m]

camber angle [°]

Mean pneumatic trail = 12.1 mm

Pneumatic trail [mm]

slip angle [°]

Static lateral stiffness = 0.074/mm (39.9 N/mm)
Calculated relaxation length = 206 mm

Normalized lateral force

displacement [mm]
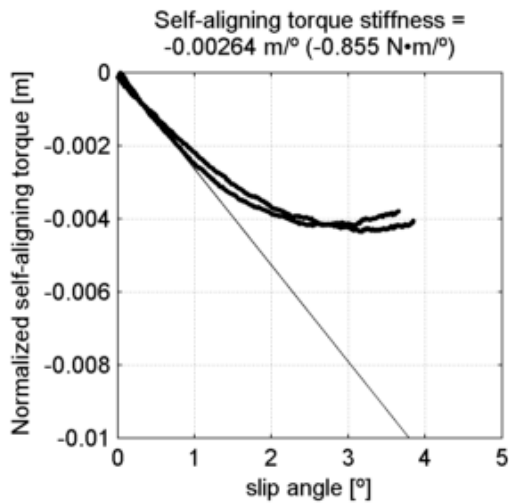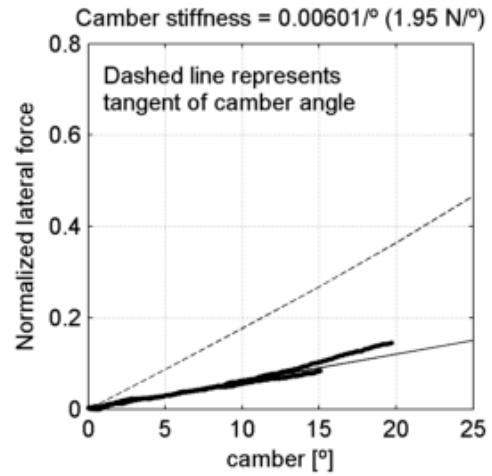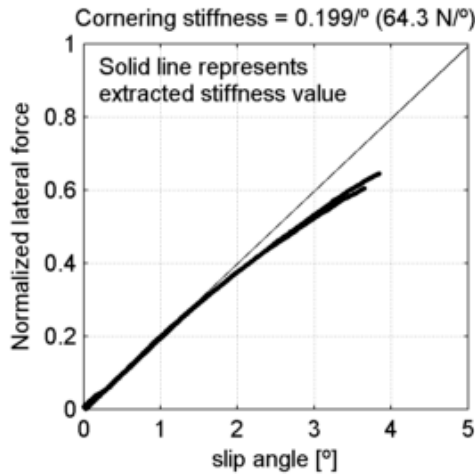
Schwalbe Big Apple 55-622
tire radius = 23.5 mm, rim width = 18.7 mm

at 4 bar (58 psi) and
under 731 N (74.5 kg, 164 lb)

Cornering stiffness = 0.235/° (171 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.0113/° (8.23 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.0047 m/° (-3.44 N•m/°)

Twisting torque stiffness =
8.39e-005 m/° (0.0613 N•m/°)

Mean pneumatic trail = 16.4 mm

Static lateral stiffness = 0.0485/mm (35.5 N/mm)
Calculated relaxation length = 277 mm

Schwalbe Kojak 35-622
tire radius = 15.7 mm, rim width = 18.7 mm

at 4 bar (58 psi) and
under 324 N (33 kg, 72.8 lb)

Cornering stiffness = 0.315/° (102 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.00845/° (2.74 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.0036 m/° (-1.16 N•m/°)

Twisting torque stiffness =
9.07e-005 m/° (0.0294 N•m/°)

Mean pneumatic trail = 8.47 mm

Static lateral stiffness = 0.132/mm (42.8 N/mm)
Calculated relaxation length = 136 mm

Schwalbe Kojak 35-622
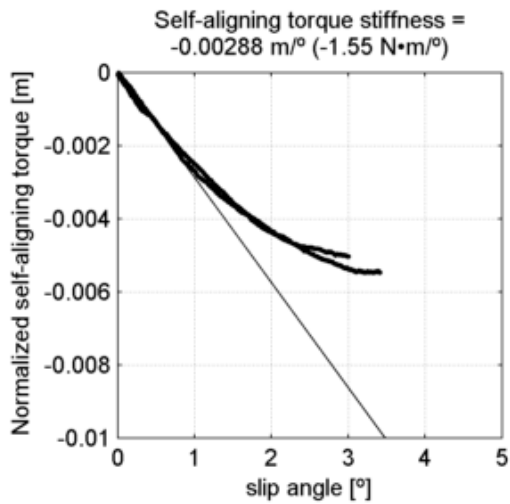tire radius = 15.7 mm, rim width = 18.7 mm

at 4 bar (58 psi) and
under 540 N (55 kg, 121 lb)

Cornering stiffness = 0.224/° (121 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.0091/° (4.91 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.0041 m/° (-2.21 N•m/°)

Twisting torque stiffness =
7.98e-005 m/° (0.043 N•m/°)

Mean pneumatic trail = 14.4 mm

Static lateral stiffness = 0.0758/mm (40.9 N/mm)
Calculated relaxation length = 169 mm

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Schwalbe Kojak 35-622
tire radius = 15.7 mm, rim width = 18.7 mm

at 4 bar (58 psi) and
under 731 N (74.5 kg, 164 lb)

Cornering stiffness = 0.185/° (136 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.0104/° (7.6 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00452 m/° (-3.3 N•m/°)

Twisting torque stiffness =
6.59e-005 m/° (0.0481 N•m/°)

Mean pneumatic trail = 21.5 mm

Static lateral stiffness = 0.0564/mm (41.2 N/mm)
Calculated relaxation length = 188 mm

Schwalbe Kojak 35-622
tire radius = 15.7 mm, rim width = 18.7 mm

at 3.45 bar (50 psi) and
under 324 N (33 kg, 72.8 lb)

Cornering stiffness = 0.318/° (103 N/°)
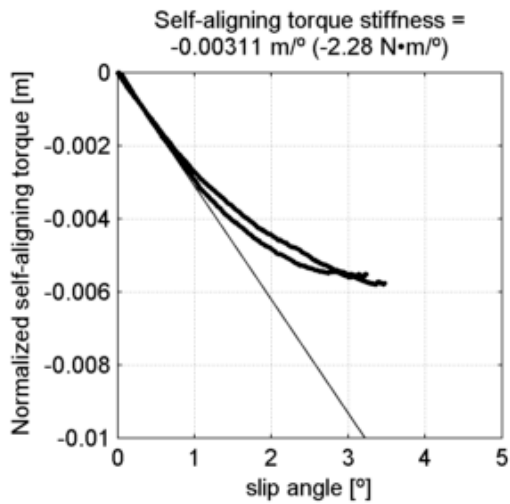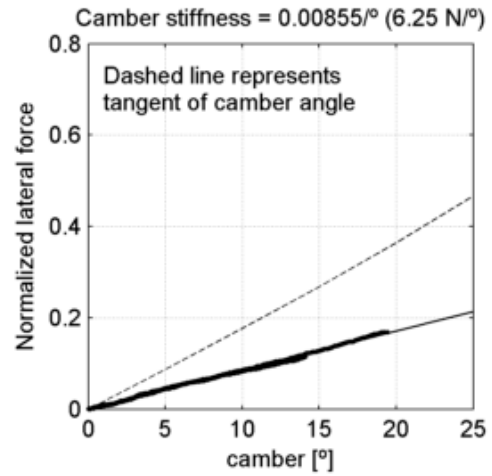
Solid line represents
extracted stiffness value

Camber stiffness = 0.00911/° (2.95 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00376 m/° (-1.22 N•m/°)

Twisting torque stiffness =
0.00011 m/° (0.0356 N•m/°)

Mean pneumatic trail = 8.65 mm

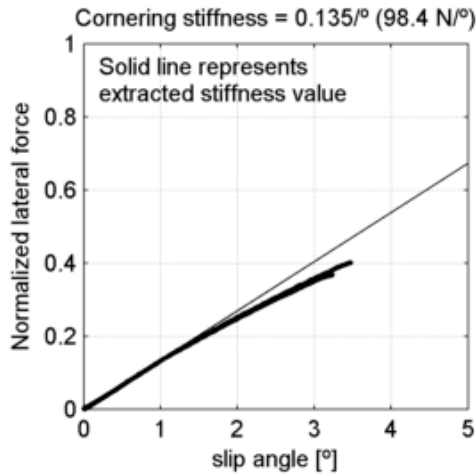Static lateral stiffness = 0.117/mm (37.7 N/mm)
Calculated relaxation length = 156 mm

Schwalbe Kojak 35-622
tire radius = 15.7 mm, rim width = 18.7 mm

at 3.45 bar (50 psi) and
under 540 N (55 kg, 121 lb)

Cornering stiffness = 0.249/° (134 N/°)
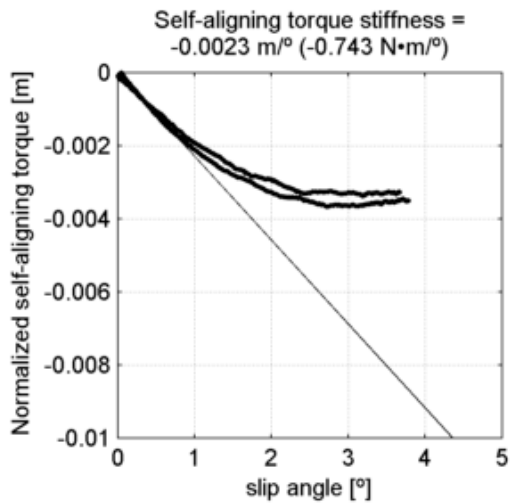
Solid line represents
extracted stiffness value

Normalized lateral force vs slip angle [°]

Camber stiffness = 0.0105/° (5.64 N/°)

Dashed line represents
tangent of camber angle

Normalized lateral force vs camber [°]

Self-aligning torque stiffness = -0.00517 m/° (-2.79 N•m/°)

Normalized self-aligning torque [m] vs slip angle [°]

Twisting torque stiffness = 8.45e-005 m/° (0.0456 N•m/°)

Normalized twisting torque [m] vs camber angle [°]

Mean pneumatic trail = 16 mm

Pneumatic trail [mm] vs slip angle [°]

Static lateral stiffness = 0.0693/mm (37.4 N/mm)
Calculated relaxation length = 206 mm
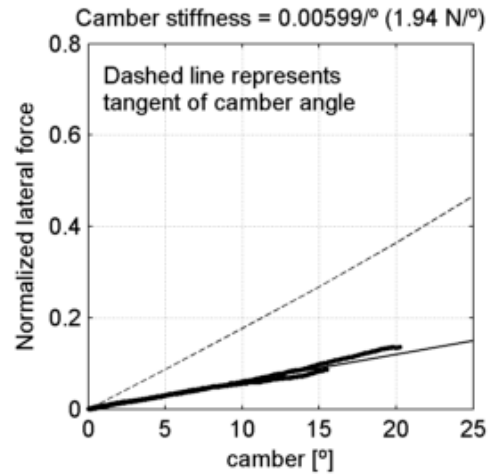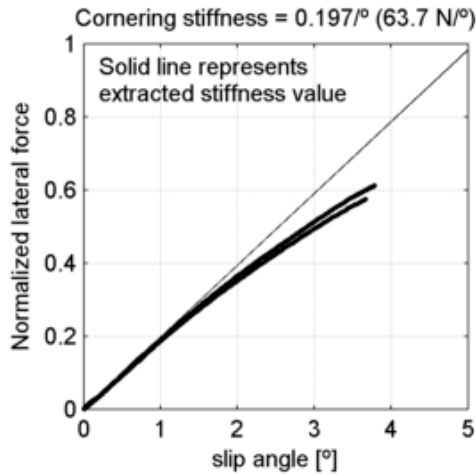
Normalized lateral force vs displacement [mm]

Schwalbe Kojak 35-622
tire radius = 15.7 mm, rim width = 18.7 mm

at 3.45 bar (50 psi) and
under 731 N (74.5 kg, 164 lb)

Cornering stiffness = 0.183/° (134 N/°)
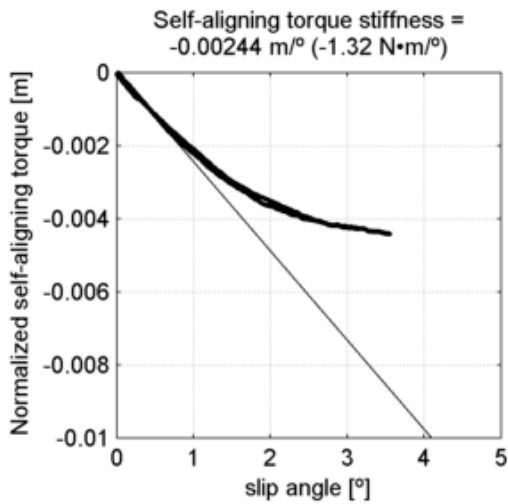
Camber stiffness = 0.0108/° (7.92 N/°)

Self-aligning torque stiffness = −0.00524 m/° (−3.83 N·m/°)

Twisting torque stiffness = 7.74e-005 m/° (0.0566 N·m/°)

Mean pneumatic trail = 25.1 mm

Static lateral stiffness = 0.0528/mm (38.6 N/mm)
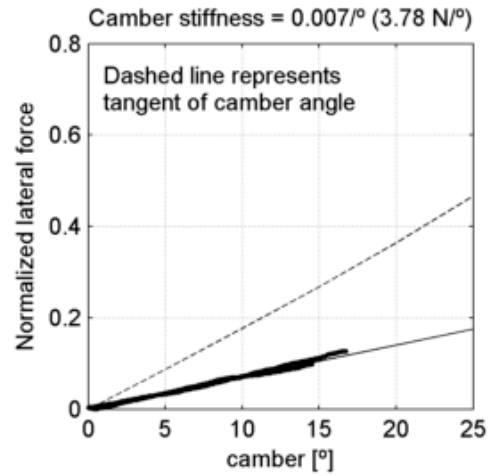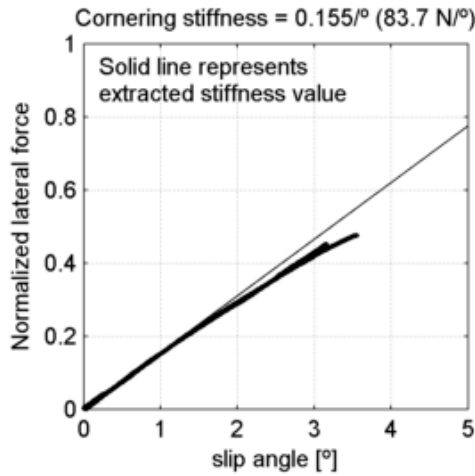Calculated relaxation length = 199 mm

Schwalbe Kojak 35-622
tire radius = 15.7 mm, rim width = 18.7 mm

at 4.83 bar (70 psi) and
under 324 N (33 kg, 72.8 lb)

Cornering stiffness = 0.301/° (97.5 N/°)

Solid line represents
extracted stiffness value

slip angle [°]

Normalized lateral force

Camber stiffness = 0.00826/° (2.67 N/°)

Dashed line represents
tangent of camber angle

camber [°]

Normalized lateral force

Self-aligning torque stiffness =
-0.00293 m/° (-0.947 N•m/°)

slip angle [°]

Normalized self-aligning torque [m]

Twisting torque stiffness =
7.66e-005 m/° (0.0248 N•m/°)

camber angle [°]

Normalized twisting torque [m]

Mean pneumatic trail = 7.53 mm

slip angle [°]

Pneumatic trail [mm]

Static lateral stiffness = 0.131/mm (42.4 N/mm)
Calculated relaxation length = 132 mm

displacement [mm]

Normalized lateral force

Schwalbe Kojak 35-622
tire radius = 15.7 mm, rim width = 18.7 mm
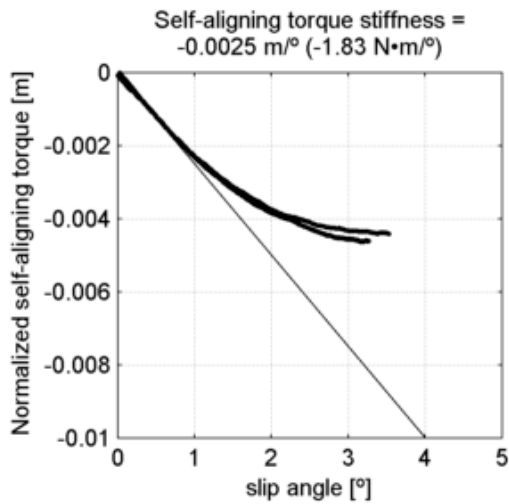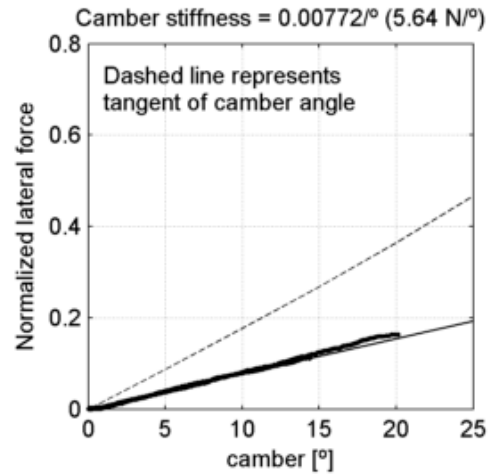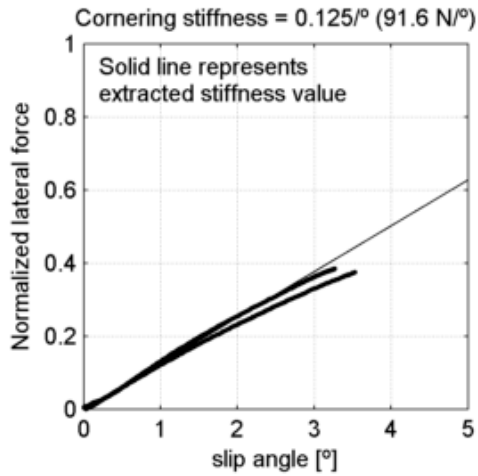
at 4.83 bar (70 psi) and
under 540 N (55 kg, 121 lb)

Cornering stiffness = 0.235/° (127 N/°)

Solid line represents
extracted stiffness value

slip angle [°]

Camber stiffness = 0.00947/° (5.11 N/°)

Dashed line represents
tangent of camber angle

camber [°]

Self-aligning torque stiffness = −0.00396 m/° (−2.14 N·m/°)

slip angle [°]

Twisting torque stiffness = 7.59e-005 m/° (0.0409 N·m/°)

camber angle [°]

Mean pneumatic trail = 12.7 mm

slip angle [°]

Static lateral stiffness = 0.0837/mm (45.1 N/mm)
Calculated relaxation length = 161 mm

displacement [mm]

Schwalbe Kojak 35-622
tire radius = 15.7 mm, rim width = 18.7 mm

at 4.83 bar (70 psi) and
under 731 N (74.5 kg, 164 lb)

Cornering stiffness = 0.199/° (146 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.0105/° (7.64 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00442 m/° (-3.23 N•m/°)

Twisting torque stiffness =
6.5e-005 m/° (0.0475 N•m/°)

Mean pneumatic trail = 18.5 mm

Static lateral stiffness = 0.0554/mm (40.5 N/mm)
Calculated relaxation length = 206 mm
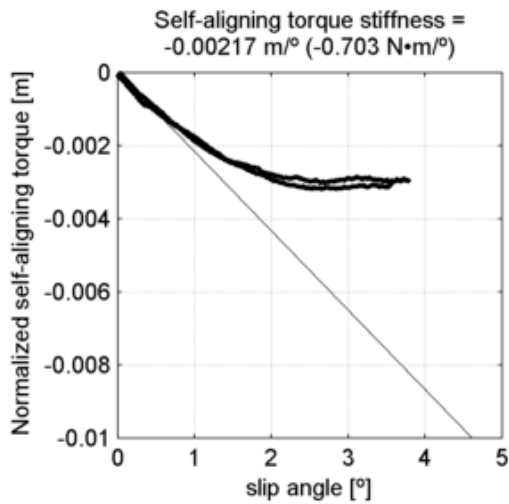
Schwalbe Kojak 35-622
tire radius = 15.7 mm, rim width = 18.7 mm

at 6.21 bar (90 psi) and
under 324 N (33 kg, 72.8 lb)

Cornering stiffness = 0.301/° (97.4 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.00809/° (2.62 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.0027 m/° (-0.875 N•m/°)

Twisting torque stiffness =
6.87e-005 m/° (0.0222 N•m/°)

Mean pneumatic trail = 6.5 mm

Static lateral stiffness = 0.15/mm (48.5 N/mm)
Calculated relaxation length = 115 mm

Schwalbe Kojak 35-622
tire radius = 15.7 mm, rim width = 18.7 mm

at 6.21 bar (90 psi) and
under 540 N (55 kg, 121 lb)

Cornering stiffness = 0.239/° (129 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.00934/° (5.04 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.003 m/° (-1.62 N•m/°)

Twisting torque stiffness =
7.33e-005 m/° (0.0395 N•m/°)

Mean pneumatic trail = 10.7 mm

Static lateral stiffness = 0.0935/mm (50.4 N/mm)
Calculated relaxation length = 146 mm

Schwalbe Kojak 35-622
tire radius = 15.7 mm, rim width = 18.7 mm
at 6.21 bar (90 psi) and
under 731 N (74.5 kg, 164 lb)

Schwalbe Marathon Plus 37-622
tire radius = 14.9 mm, rim width = 18.7 mm

at 4 bar (58 psi) and
under 324 N (33 kg, 72.8 lb)

Cornering stiffness = 0.197/° (63.8 N/°)
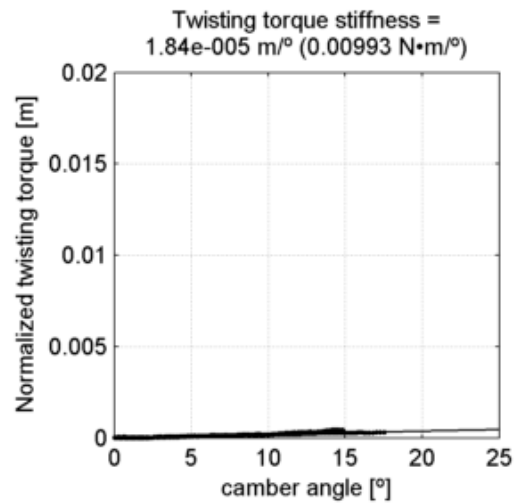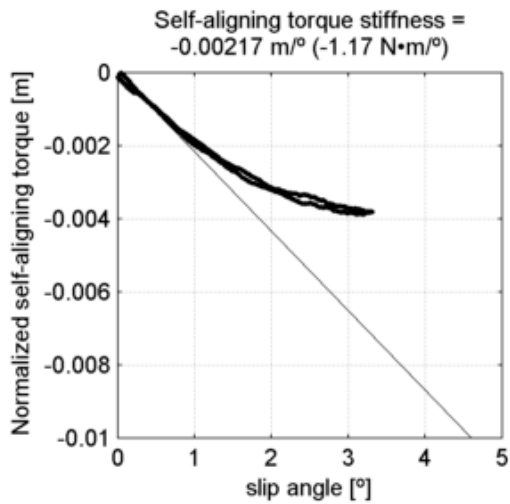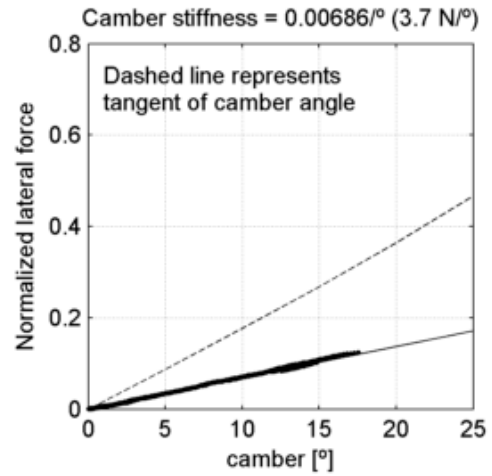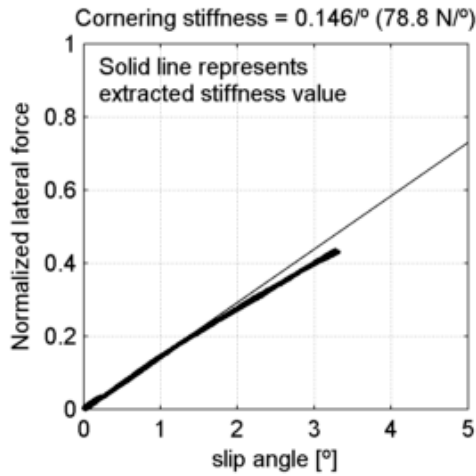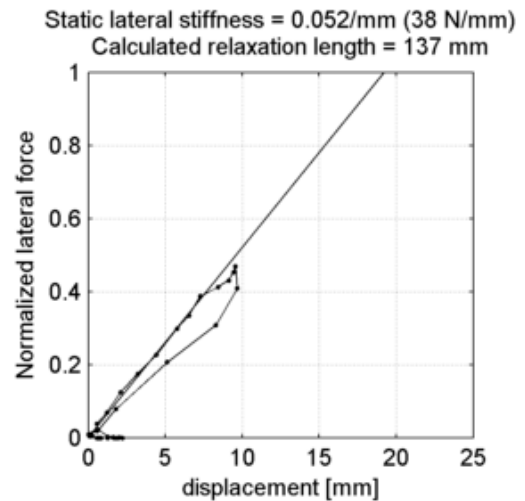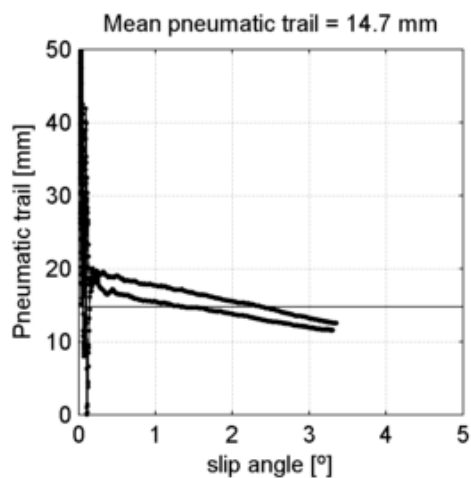
Solid line represents
extracted stiffness value

(x-axis: slip angle [°]; y-axis: Normalized lateral force)

Camber stiffness = 0.00563/° (1.82 N/°)

Dashed line represents
tangent of camber angle

(x-axis: camber [°]; y-axis: Normalized lateral force)

Self-aligning torque stiffness =
-0.00238 m/° (-0.769 N•m/°)

(x-axis: slip angle [°]; y-axis: Normalized self-aligning torque [m])

Twisting torque stiffness =
2.11e-005 m/° (0.00682 N•m/°)

(x-axis: camber angle [°]; y-axis: Normalized twisting torque [m])

Mean pneumatic trail = 8.72 mm

(x-axis: slip angle [°]; y-axis: Pneumatic trail [mm])

Static lateral stiffness = 0.0958/mm (31 N/mm)
Calculated relaxation length = 118 mm
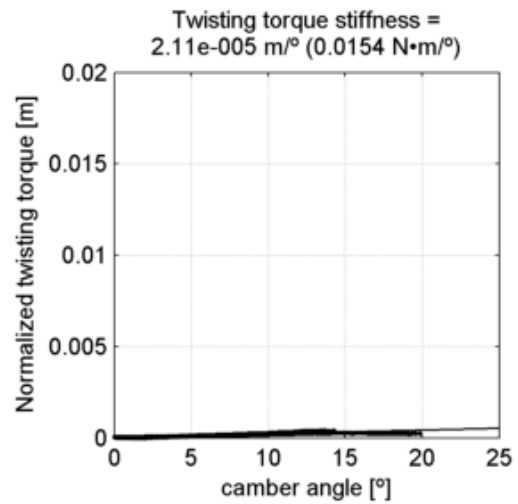
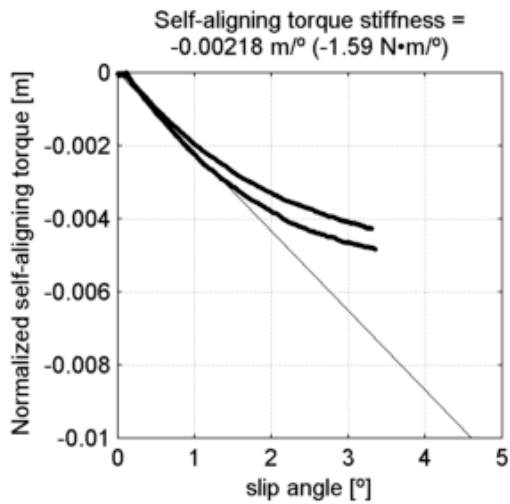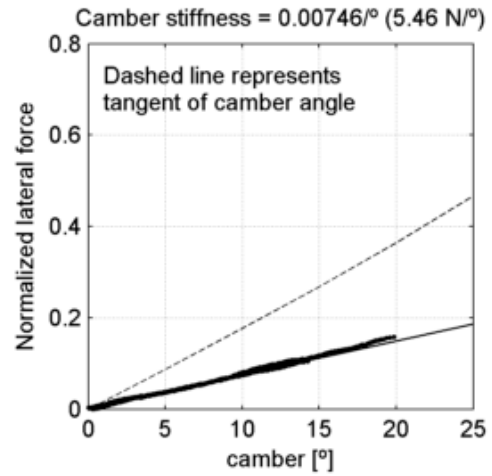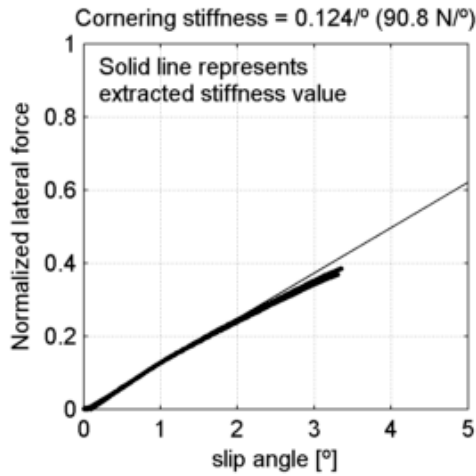*(x-axis: displacement [mm]; y-axis: Normalized lateral force)*

Schwalbe Marathon Plus 37-622
tire radius = 14.9 mm, rim width = 18.7 mm

at 4 bar (58 psi) and
under 540 N (55 kg, 121 lb)

Cornering stiffness = 0.161/° (87.1 N/°)
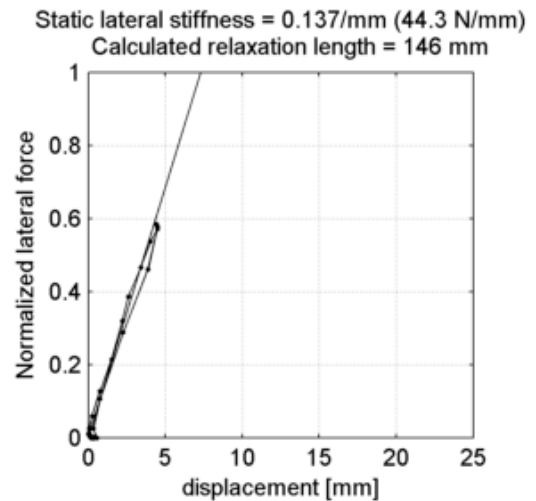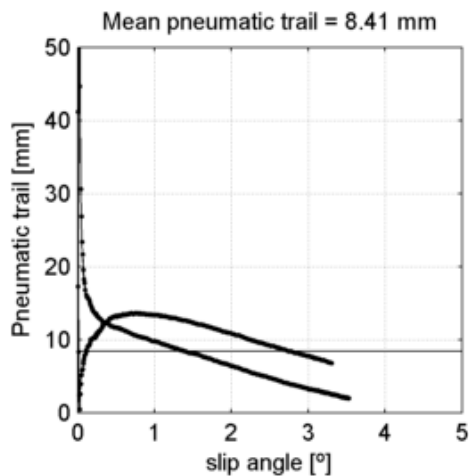
Solid line represents
extracted stiffness value

Camber stiffness = 0.00716/° (3.86 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00278 m/° (-1.5 N·m/°)

Twisting torque stiffness =
2.59e-005 m/° (0.014 N·m/°)

Mean pneumatic trail = 13.3 mm

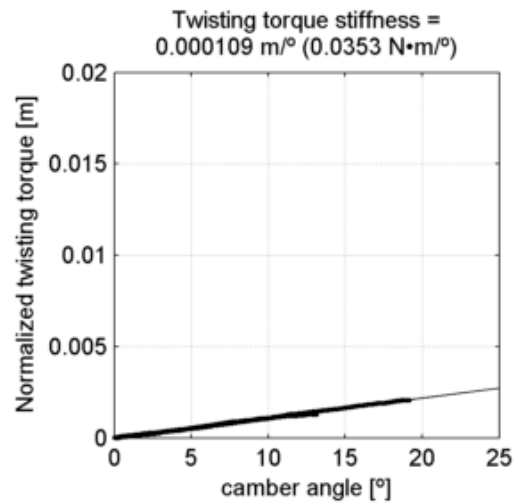Static lateral stiffness = 0.0598/mm (32.3 N/mm)
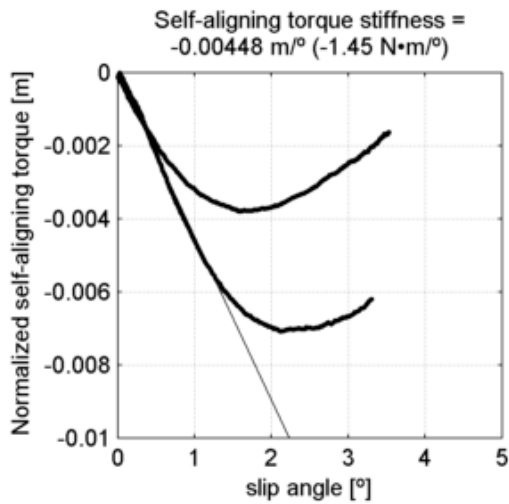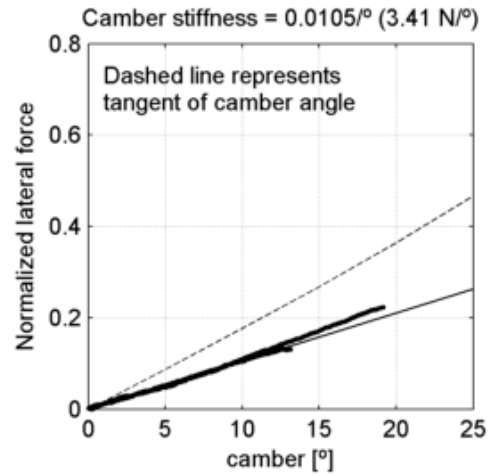Calculated relaxation length = 155 mm

Schwalbe Marathon Plus 37-622
tire radius = 14.9 mm, rim width = 18.7 mm

at 4 bar (58 psi) and
under 731 N (74.5 kg, 164 lb)

Cornering stiffness = 0.134/° (97.8 N/°)

Solid line represents extracted stiffness value

Camber stiffness = 0.00798/° (5.83 N/°)

Dashed line represents tangent of camber angle

Self-aligning torque stiffness = -0.00288 m/° (-2.11 N•m/°)

Twisting torque stiffness = 2.59e-005 m/° (0.0189 N•m/°)

Mean pneumatic trail = 17.1 mm

Static lateral stiffness = 0.0417/mm (30.4 N/mm)
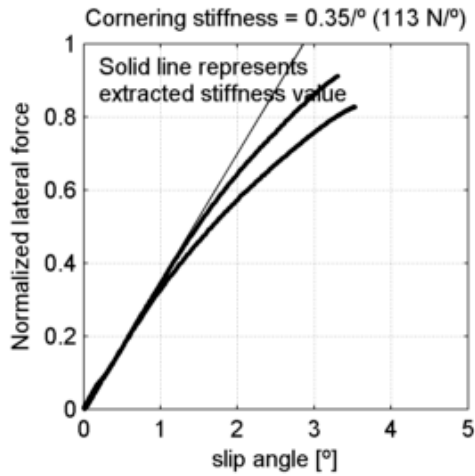Calculated relaxation length = 184 mm

Schwalbe Marathon Plus 37-622
tire radius = 14.9 mm, rim width = 18.7 mm

at 3.45 bar (50 psi) and
under 324 N (33 kg, 72.8 lb)

Cornering stiffness = 0.199/° (64.3 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.00601/° (1.95 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00264 m/° (-0.855 N•m/°)

Twisting torque stiffness =
3.57e-005 m/° (0.0116 N•m/°)

Mean pneumatic trail = 9.59 mm

Static lateral stiffness = 0.0924/mm (29.9 N/mm)
Calculated relaxation length = 123 mm

Schwalbe Marathon Plus 37-622
tire radius = 14.9 mm, rim width = 18.7 mm

at 3.45 bar (50 psi) and
under 540 N (55 kg, 121 lb)

Cornering stiffness = 0.157/° (84.9 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.00721/° (3.89 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00288 m/° (-1.55 N•m/°)

Twisting torque stiffness =
2.32e-005 m/° (0.0125 N•m/°)

Mean pneumatic trail = 14.3 mm

Static lateral stiffness = 0.0618/mm (33.4 N/mm)
Calculated relaxation length = 146 mm

Schwalbe Marathon Plus 37-622
tire radius = 14.9 mm, rim width = 18.7 mm
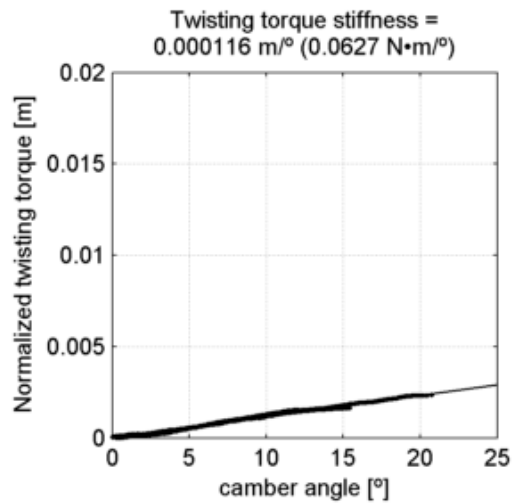
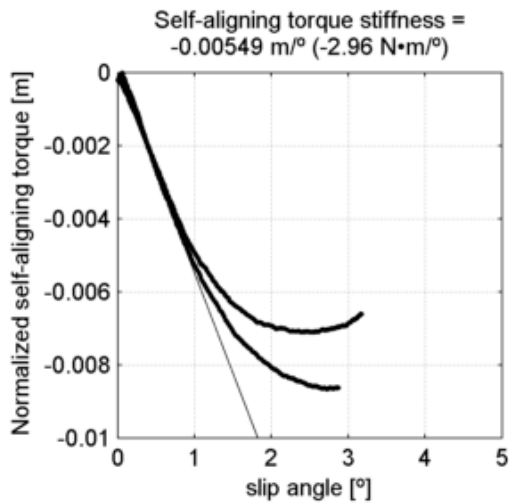at 3.45 bar (50 psi) and
under 731 N (74.5 kg, 164 lb)

Cornering stiffness = 0.135/° (98.4 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.00855/° (6.25 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00311 m/° (-2.28 N•m/°)

Twisting torque stiffness =
2.97e-005 m/° (0.0217 N•m/°)

Mean pneumatic trail = 18.5 mm

Static lateral stiffness = 0.0435/mm (31.8 N/mm)
Calculated relaxation length = 177 mm

Schwalbe Marathon Plus 37-622
tire radius = 14.9 mm, rim width = 18.7 mm
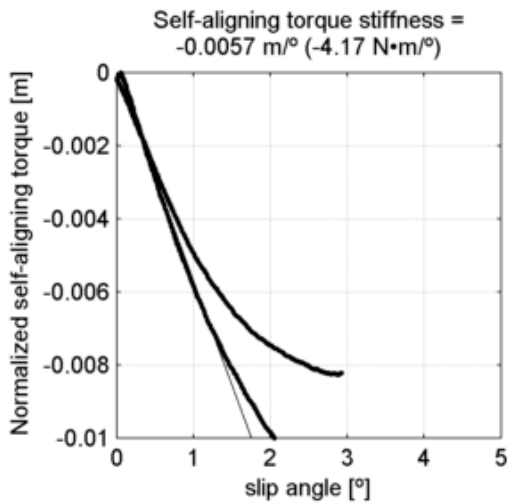
at 4.83 bar (70 psi) and
under 324 N (33 kg, 72.8 lb)

Cornering stiffness = 0.197/° (63.7 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.00599/° (1.94 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.0023 m/° (-0.743 N•m/°)

Twisting torque stiffness =
2.5e-005 m/° (0.00809 N•m/°)

Mean pneumatic trail = 8.5 mm

Static lateral stiffness = 0.0978/mm (31.7 N/mm)
Calculated relaxation length = 115 mm

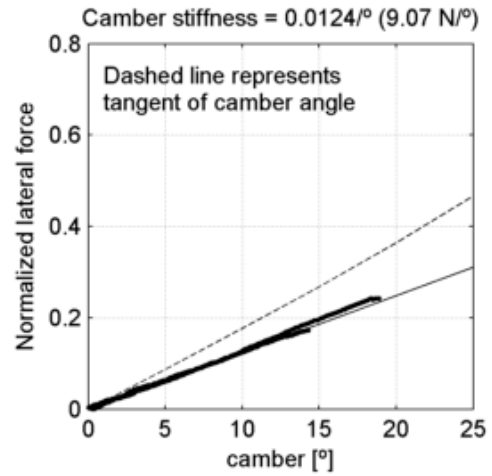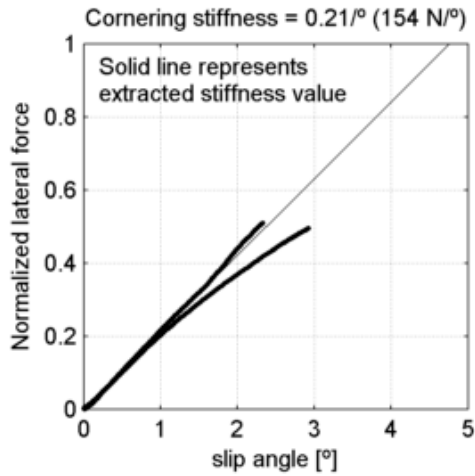Schwalbe Marathon Plus 37-622
tire radius = 14.9 mm, rim width = 18.7 mm

at 4.83 bar (70 psi) and
under 540 N (55 kg, 121 lb)

Cornering stiffness = 0.155/° (83.7 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.007/° (3.78 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00244 m/° (-1.32 N•m/°)

Twisting torque stiffness =
2.07e-005 m/° (0.0112 N•m/°)

Mean pneumatic trail = 12.2 mm

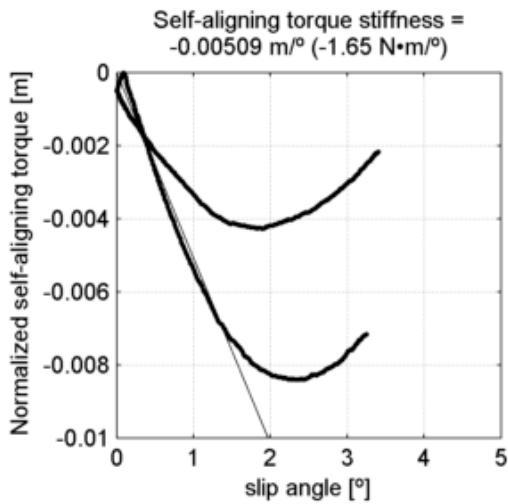Static lateral stiffness = 0.0575/mm (31 N/mm)
Calculated relaxation length = 155 mm

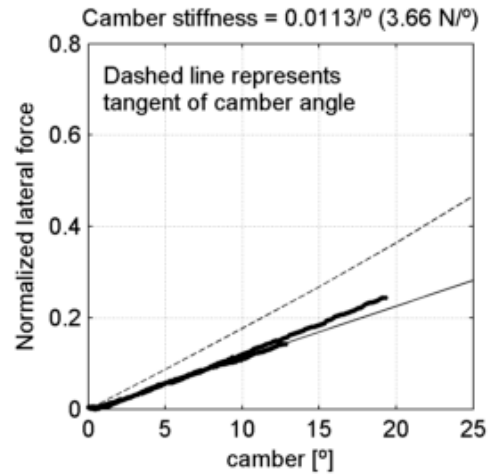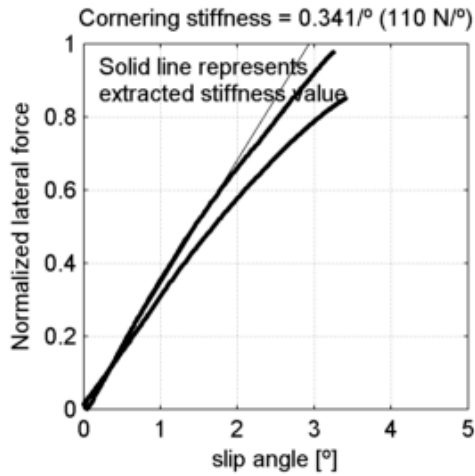Schwalbe Marathon Plus 37-622
tire radius = 14.9 mm, rim width = 18.7 mm

at 4.83 bar (70 psi) and
under 731 N (74.5 kg, 164 lb)

Cornering stiffness = 0.125/° (91.6 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.00772/° (5.64 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.0025 m/° (-1.83 N•m/°)

Twisting torque stiffness =
2.02e-005 m/° (0.0148 N•m/°)

Mean pneumatic trail = 15.6 mm

Static lateral stiffness = 0.0472/mm (34.5 N/mm)
Calculated relaxation length = 152 mm

Schwalbe Marathon Plus 37-622
tire radius = 14.9 mm, rim width = 18.7 mm

at 6.21 bar (90 psi) and
under 324 N (33 kg, 72.8 lb)

Cornering stiffness = 0.184/° (59.6 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.00525/° (1.7 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00217 m/° (-0.703 N•m/°)

Twisting torque stiffness =
1.61e-005 m/° (0.00521 N•m/°)

Mean pneumatic trail = 8.13 mm

Static lateral stiffness = 0.107/mm (34.7 N/mm)
Calculated relaxation length = 98.4 mm

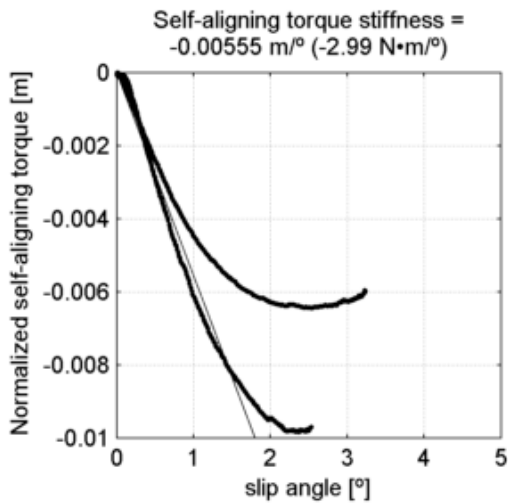Schwalbe Marathon Plus 37-622
tire radius = 14.9 mm, rim width = 18.7 mm

at 6.21 bar (90 psi) and
under 540 N (55 kg, 121 lb)

Cornering stiffness = 0.146/° (78.8 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.00686/° (3.7 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00217 m/° (-1.17 N•m/°)

Twisting torque stiffness =
1.84e-005 m/° (0.00993 N•m/°)

Mean pneumatic trail = 11.5 mm

Static lateral stiffness = 0.0603/mm (32.5 N/mm)
Calculated relaxation length = 139 mm

Schwalbe Marathon Plus 37-622
tire radius = 14.9 mm, rim width = 18.7 mm

at 6.21 bar (90 psi) and
under 731 N (74.5 kg, 164 lb)

Cornering stiffness = 0.124/° (90.8 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.00746/° (5.46 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00218 m/° (-1.59 N•m/°)

Twisting torque stiffness =
2.11e-005 m/° (0.0154 N•m/°)

Mean pneumatic trail = 14.7 mm

Static lateral stiffness = 0.052/mm (38 N/mm)
Calculated relaxation length = 137 mm

Vittoria Randonneur Hyper 37-622
tire radius = 17.5 mm, rim width = 18.7 mm
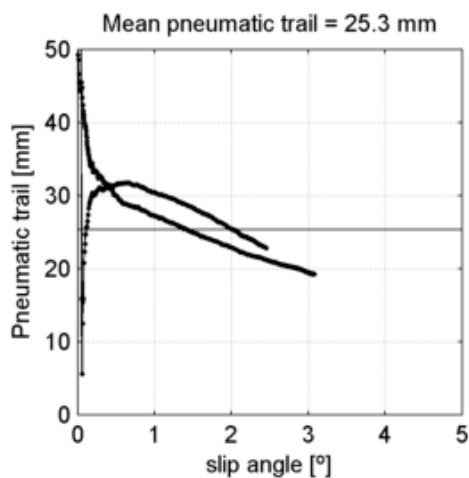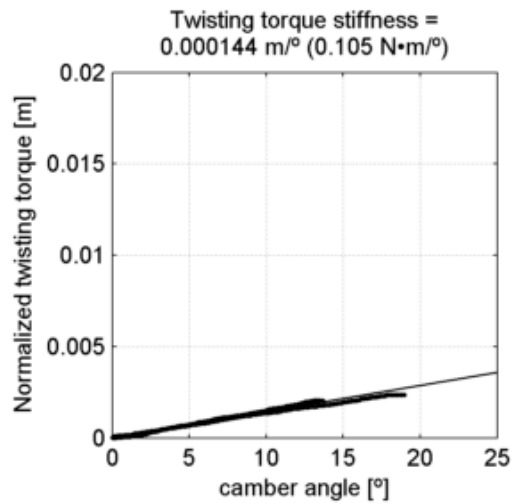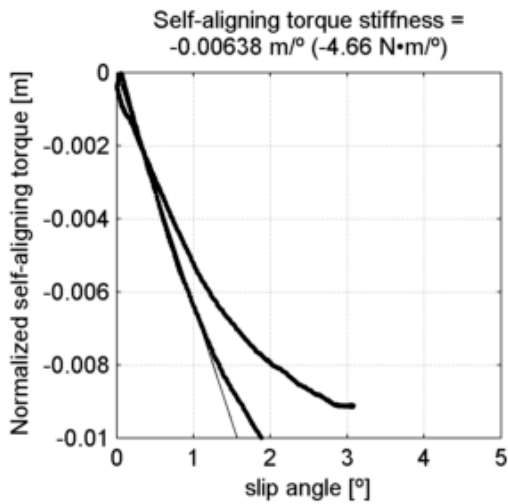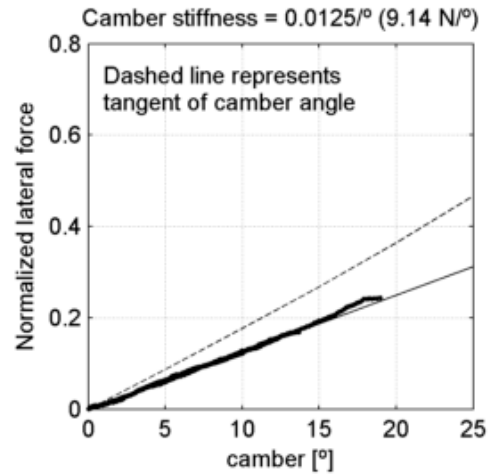
at 4 bar (58 psi) and
under 324 N (33 kg, 72.8 lb)

Cornering stiffness = 0.35/° (113 N/°)

Solid line represents
extracted stiffness value

slip angle [°]
Normalized lateral force

Camber stiffness = 0.0105/° (3.41 N/°)

Dashed line represents
tangent of camber angle

camber [°]
Normalized lateral force

Self-aligning torque stiffness =
-0.00448 m/° (-1.45 N•m/°)

slip angle [°]
Normalized self-aligning torque [m]

Twisting torque stiffness =
0.000109 m/° (0.0353 N•m/°)

camber angle [°]
Normalized twisting torque [m]

Mean pneumatic trail = 8.41 mm

slip angle [°]
Pneumatic trail [mm]

Static lateral stiffness = 0.137/mm (44.3 N/mm)
Calculated relaxation length = 146 mm
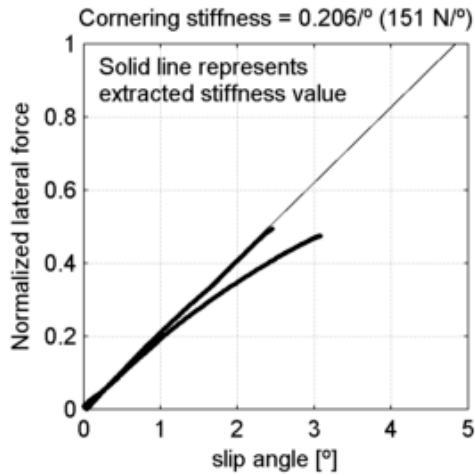
displacement [mm]
Normalized lateral force

Vittoria Randonneur Hyper 37-622
tire radius = 17.5 mm, rim width = 18.7 mm

at 4 bar (58 psi) and
under 540 N (55 kg, 121 lb)

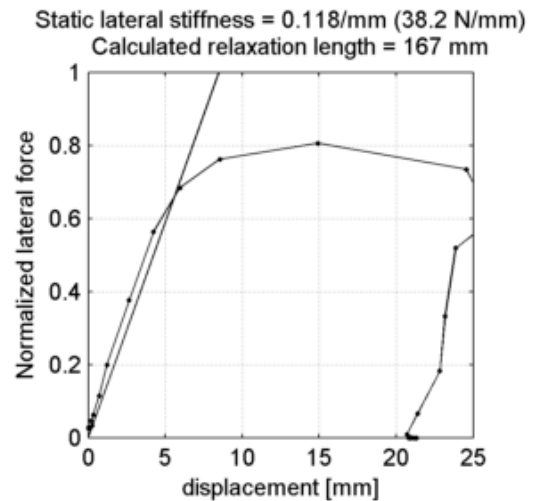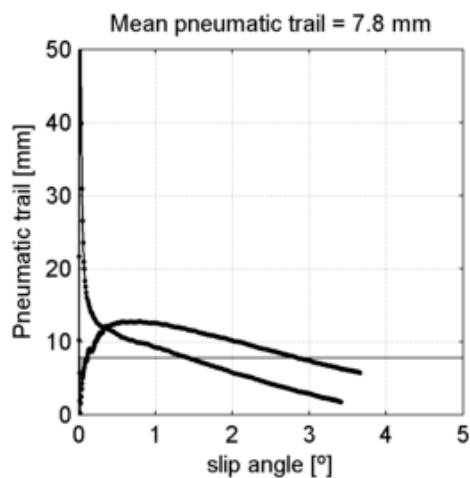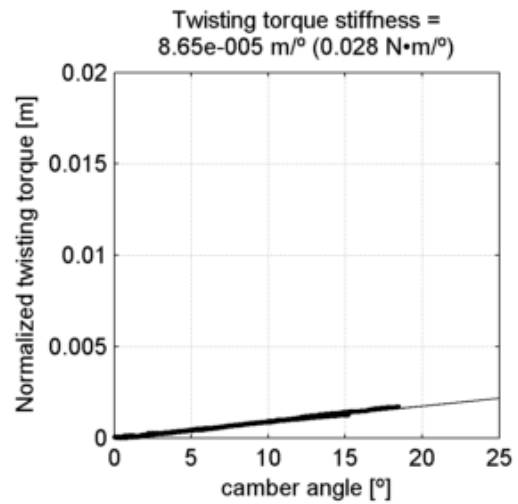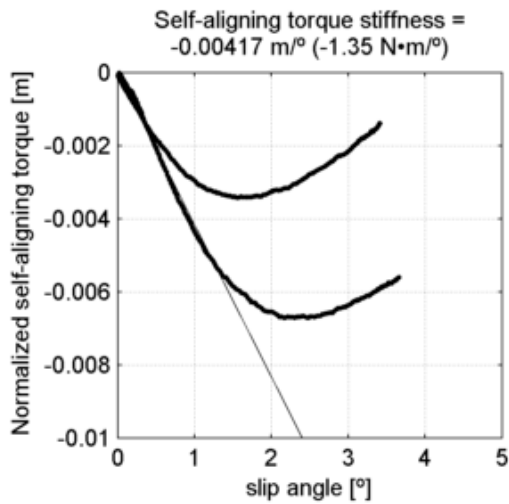Cornering stiffness = 0.26/° (140 N/°)

Solid line represents
extracted stiffness value

Normalized lateral force vs *slip angle [°]*

Camber stiffness = 0.0123/° (6.66 N/°)

Dashed line represents
tangent of camber angle

Normalized lateral force vs *camber [°]*

Self-aligning torque stiffness =
-0.00549 m/° (-2.96 N•m/°)

Normalized self-aligning torque [m] vs *slip angle [°]*

Twisting torque stiffness =
0.000116 m/° (0.0627 N•m/°)

Normalized twisting torque [m] vs *camber angle [°]*

Mean pneumatic trail = 16.1 mm

Pneumatic trail [mm] vs *slip angle [°]*

Static lateral stiffness = 0.0777/mm (41.9 N/mm)
Calculated relaxation length = 192 mm
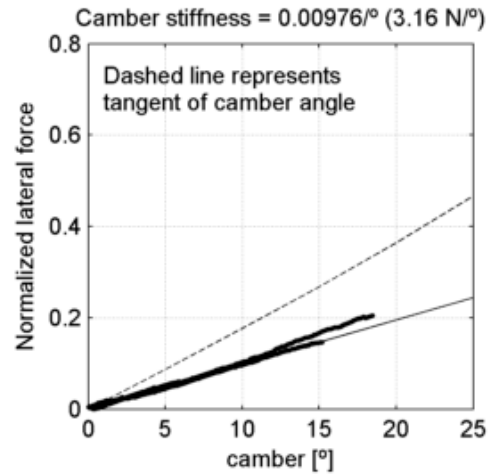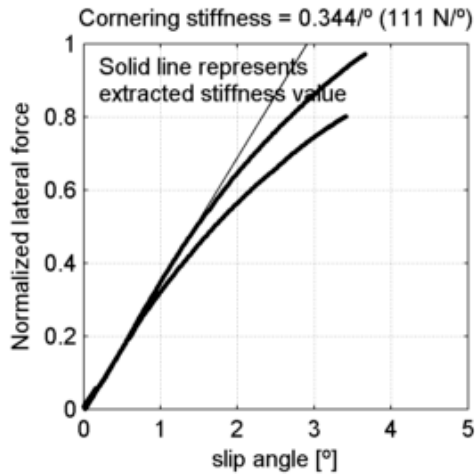
Normalized lateral force vs *displacement [mm]*

Vittoria Randonneur Hyper 37-622
tire radius = 17.5 mm, rim width = 18.7 mm
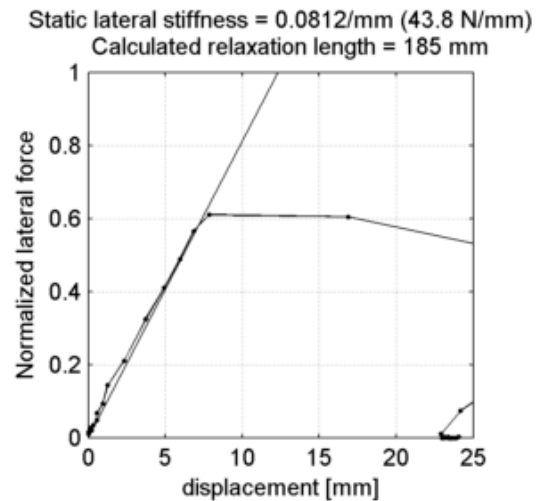
at 4 bar (58 psi) and
under 731 N (74.5 kg, 164 lb)

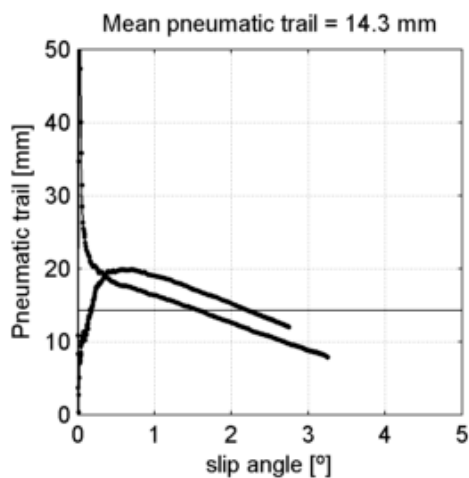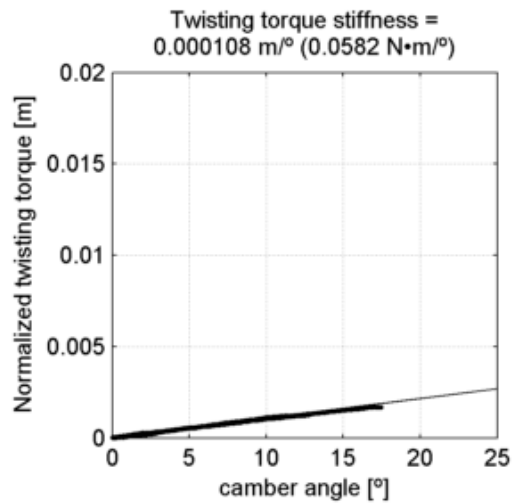--

Cornering stiffness = 0.21/° (154 N/°)

Solid line represents
extracted stiffness value

Normalized lateral force vs slip angle [°]

Camber stiffness = 0.0124/° (9.07 N/°)

Dashed line represents
tangent of camber angle

Normalized lateral force vs camber [°]

**Self-aligning torque stiffness =
-0.0057 m/° (-4.17 N•m/°)**

Normalized self-aligning torque [m] vs slip angle [°]

**Twisting torque stiffness =
0.000113 m/° (0.0826 N•m/°)**

Normalized twisting torque [m] vs camber angle [°]

Mean pneumatic trail = 22.9 mm

Pneumatic trail [mm] vs slip angle [°]

**Static lateral stiffness = 0.0528/mm (38.6 N/mm)
Calculated relaxation length = 228 mm**

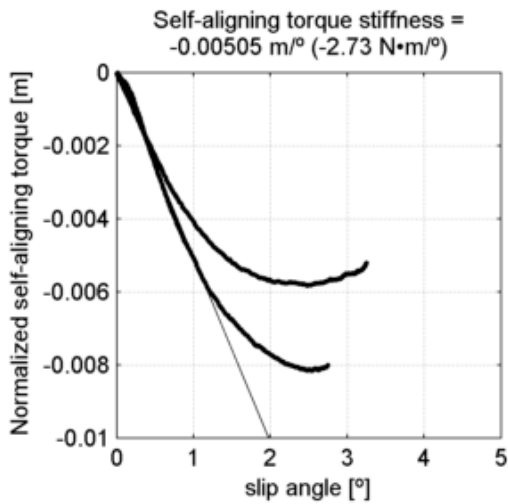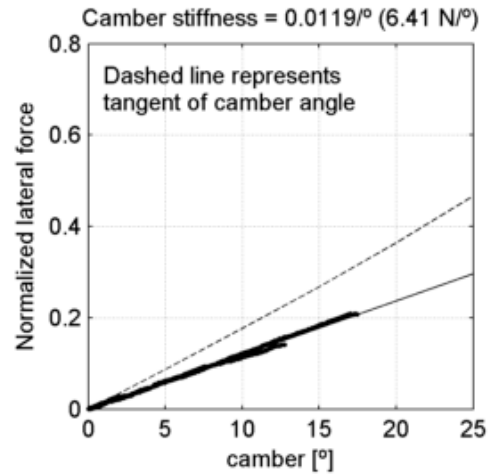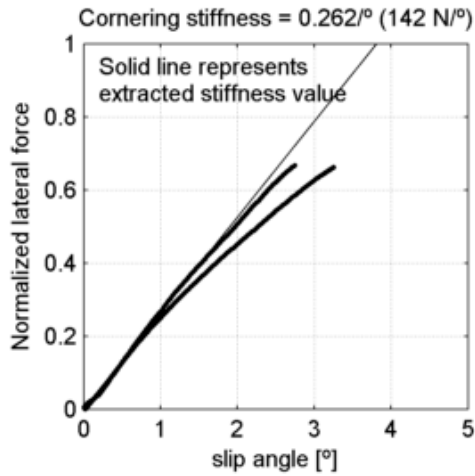Normalized lateral force vs displacement [mm]

Vittoria Randonneur Hyper 37-622
tire radius = 17.5 mm, rim width = 18.7 mm
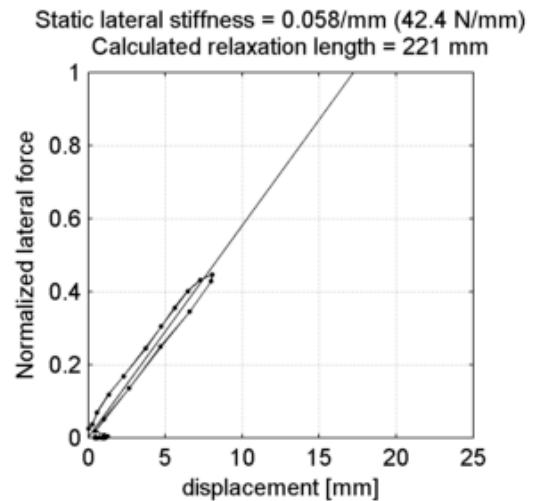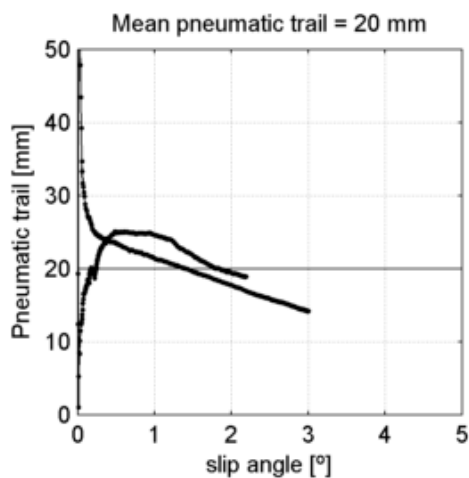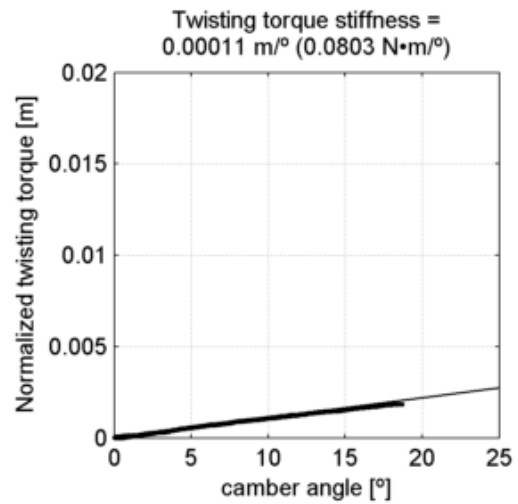
at 3.45 bar (50 psi) and
under 324 N (33 kg, 72.8 lb)

Cornering stiffness = 0.341/° (110 N/°)

Solid line represents
extracted stiffness value

slip angle [°]

Normalized lateral force

Camber stiffness = 0.0113/° (3.66 N/°)

Dashed line represents
tangent of camber angle

camber [°]

Normalized lateral force

Self-aligning torque stiffness =
-0.00509 m/° (-1.65 N•m/°)

slip angle [°]

Normalized self-aligning torque [m]

Twisting torque stiffness =
0.000123 m/° (0.0398 N•m/°)

camber angle [°]

Normalized twisting torque [m]

Mean pneumatic trail = 9.51 mm

slip angle [°]

Pneumatic trail [mm]

Static lateral stiffness = 0.114/mm (36.9 N/mm)
Calculated relaxation length = 171 mm

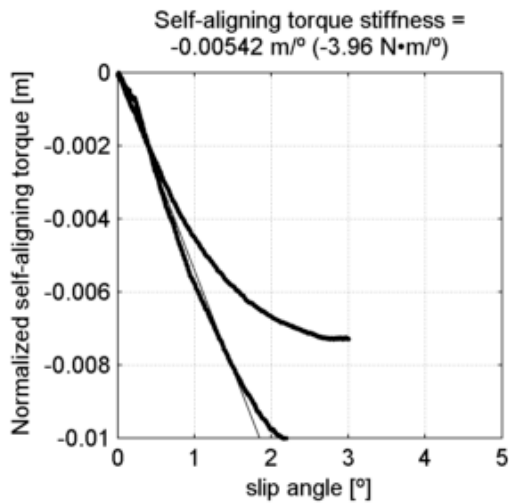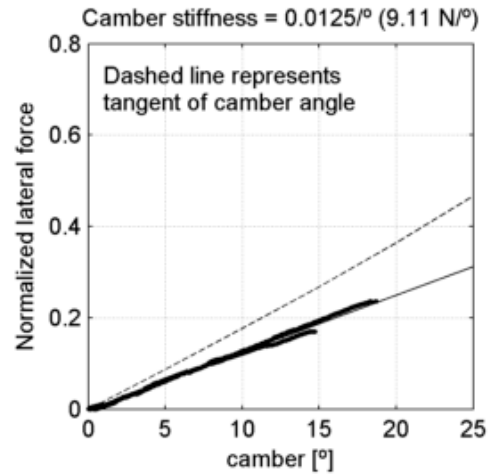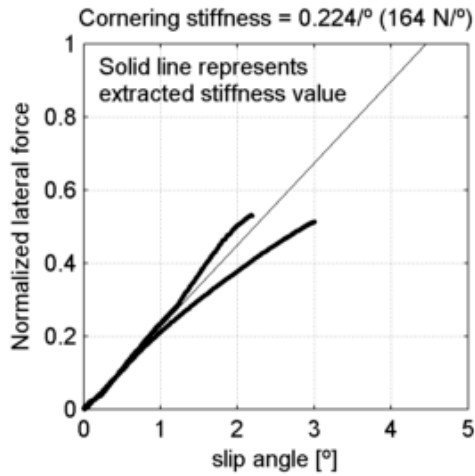displacement [mm]

Normalized lateral force

Vittoria Randonneur Hyper 37-622
tire radius = 17.5 mm, rim width = 18.7 mm

at 3.45 bar (50 psi) and
under 540 N (55 kg, 121 lb)

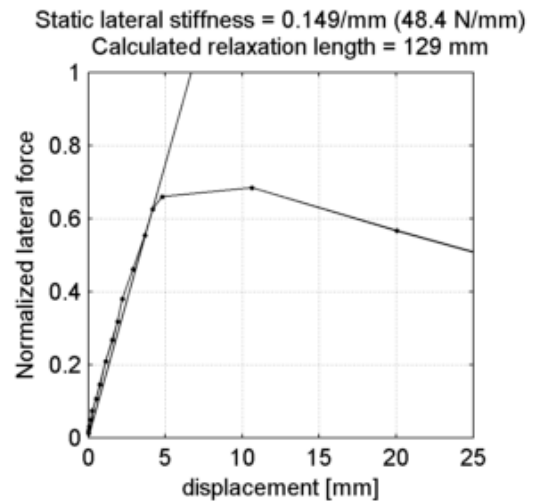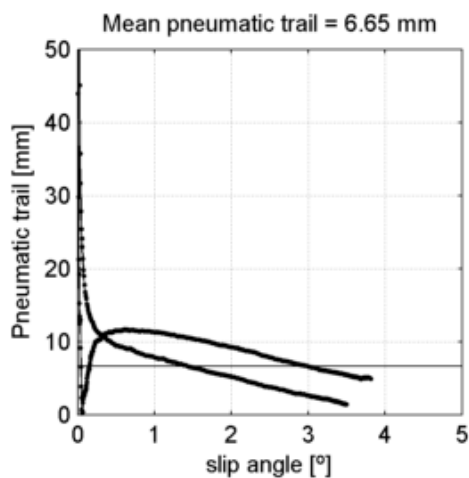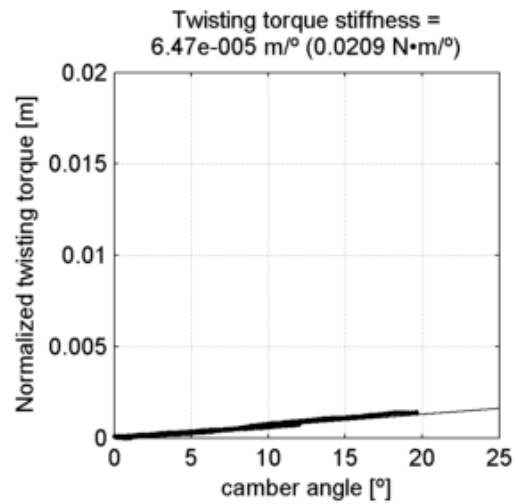Cornering stiffness = 0.25/° (135 N/°)

Solid line represents
extracted stiffness value

Normalized lateral force

slip angle [°]

Camber stiffness = 0.0119/° (6.42 N/°)

Dashed line represents
tangent of camber angle

Normalized lateral force

camber [°]

Self-aligning torque stiffness =
−0.00555 m/° (−2.99 N•m/°)

Normalized self-aligning torque [m]

slip angle [°]

Twisting torque stiffness =
0.000122 m/° (0.0657 N•m/°)

Normalized twisting torque [m]

camber angle [°]

Mean pneumatic trail = 17 mm

Pneumatic trail [mm]

slip angle [°]

Static lateral stiffness = 0.0735/mm (39.7 N/mm)
Calculated relaxation length = 195 mm

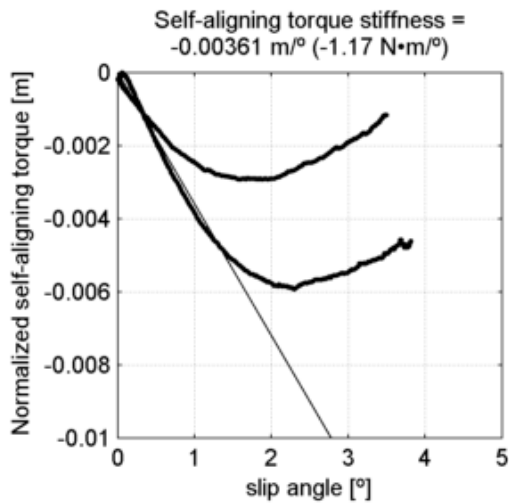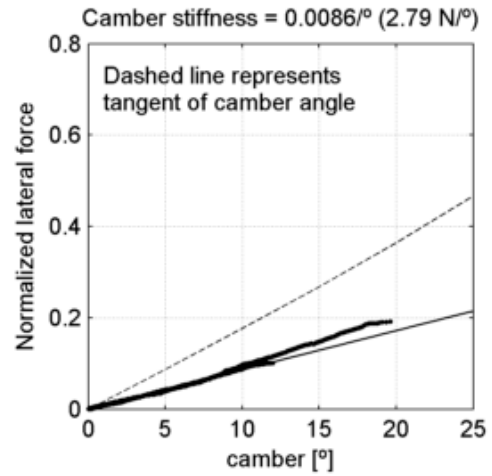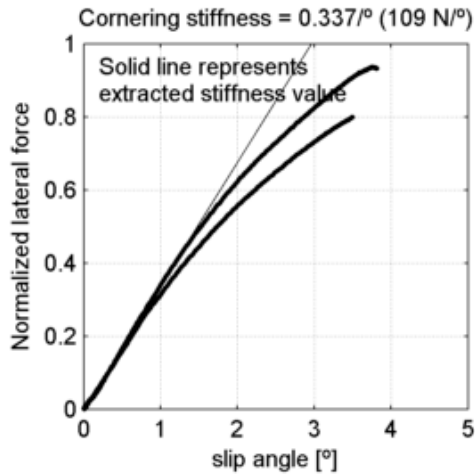Normalized lateral force

displacement [mm]

Vittoria Randonneur Hyper 37-622
tire radius = 17.5 mm, rim width = 18.7 mm

at 3.45 bar (50 psi) and
under 731 N (74.5 kg, 164 lb)

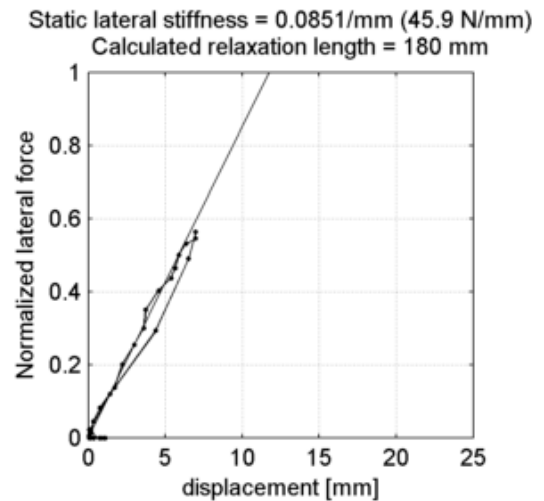Cornering stiffness = 0.206/° (151 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.0125/° (9.14 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00638 m/° (-4.66 N•m/°)

Twisting torque stiffness =
0.000144 m/° (0.105 N•m/°)

Mean pneumatic trail = 25.3 mm

Static lateral stiffness = 0.0517/mm (37.8 N/mm)
Calculated relaxation length = 229 mm

Vittoria Randonneur Hyper 37-622
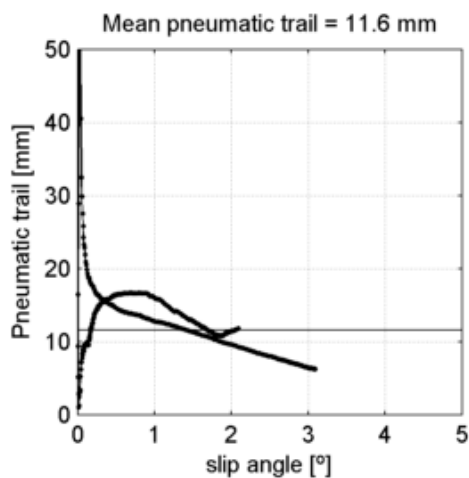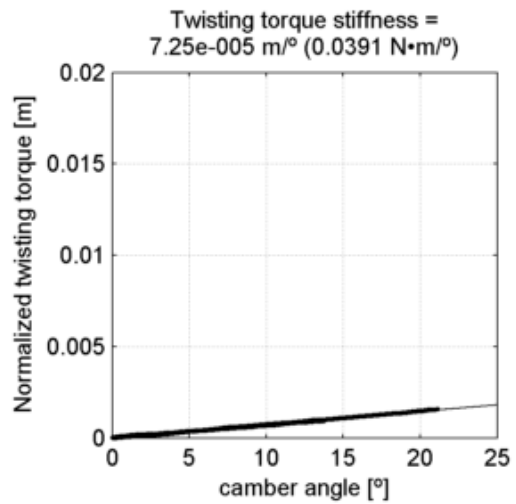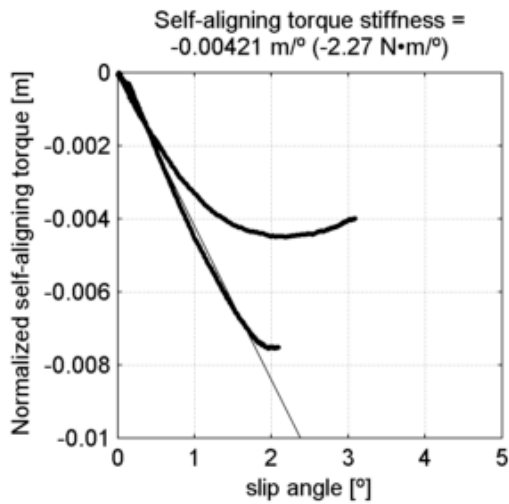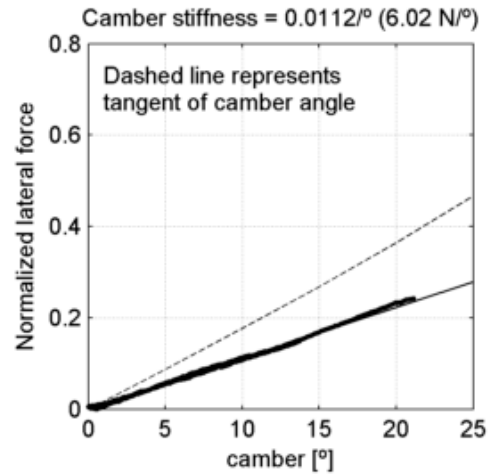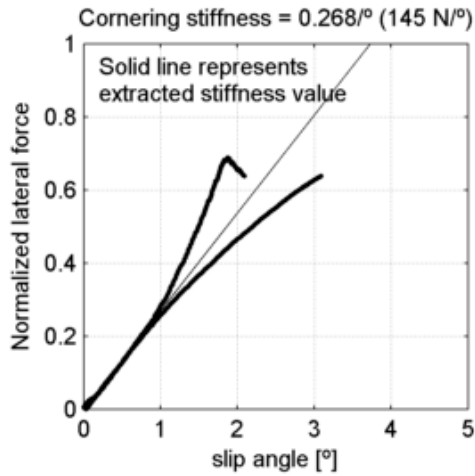tire radius = 17.5 mm, rim width = 18.7 mm — at 4.83 bar (70 psi) and under 324 N (33 kg, 72.8 lb)

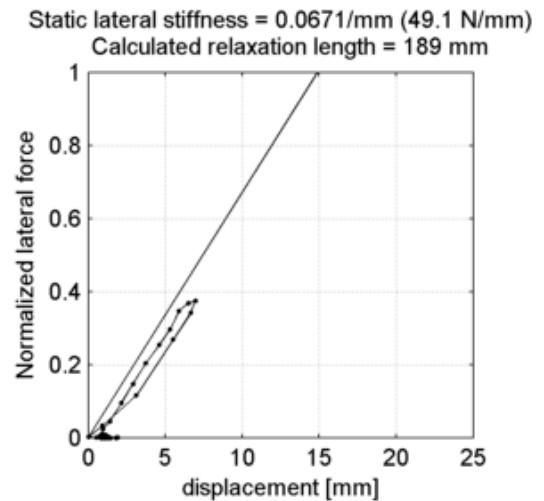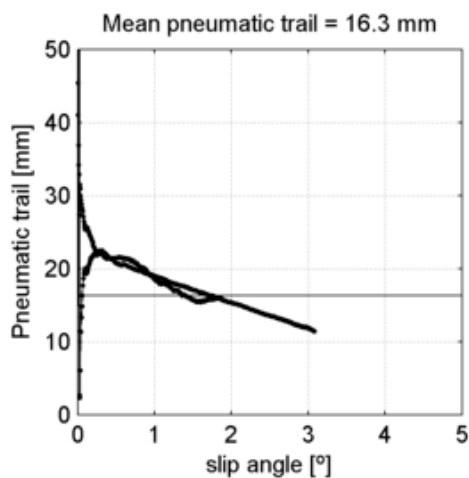Vittoria Randonneur Hyper 37-622
tire radius = 17.5 mm, rim width = 18.7 mm

at 4.83 bar (70 psi) and
under 540 N (55 kg, 121 lb)

Cornering stiffness = 0.262/° (142 N/°)

Solid line represents
extracted stiffness value

Normalized lateral force vs. slip angle [°]

Camber stiffness = 0.0119/° (6.41 N/°)

Dashed line represents
tangent of camber angle

Normalized lateral force vs. camber [°]

Self-aligning torque stiffness =
-0.00505 m/° (-2.73 N·m/°)

Normalized self-aligning torque [m] vs. slip angle [°]

Twisting torque stiffness =
0.000108 m/° (0.0582 N·m/°)

Normalized twisting torque [m] vs. camber angle [°]

Mean pneumatic trail = 14.3 mm

Pneumatic trail [mm] vs. slip angle [°]

Static lateral stiffness = 0.0812/mm (43.8 N/mm)
Calculated relaxation length = 185 mm

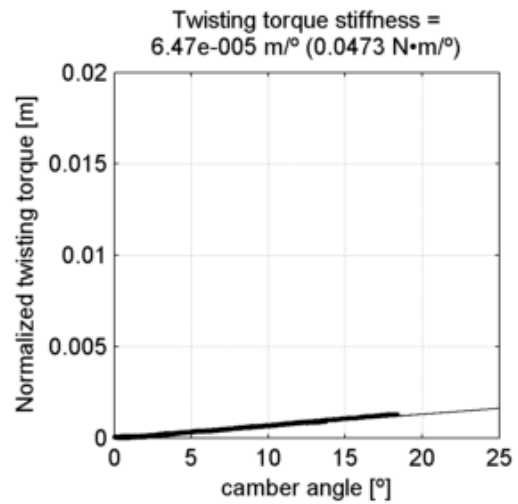*Normalized lateral force vs. displacement [mm]*

Vittoria Randonneur Hyper 37-622
tire radius = 17.5 mm, rim width = 18.7 mm

at 4.83 bar (70 psi) and
under 731 N (74.5 kg, 164 lb)

Cornering stiffness = 0.224/° (164 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.0125/° (9.11 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00542 m/° (-3.96 N•m/°)

Twisting torque stiffness =
0.00011 m/° (0.0803 N•m/°)

Mean pneumatic trail = 20 mm

Static lateral stiffness = 0.058/mm (42.4 N/mm)
Calculated relaxation length = 221 mm

Vittoria Randonneur Hyper 37-622
tire radius = 17.5 mm, rim width = 18.7 mm

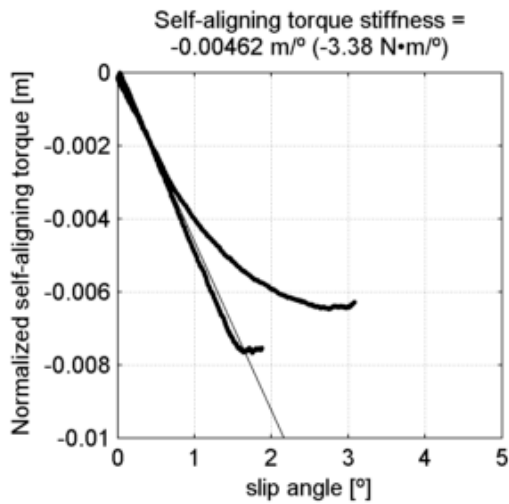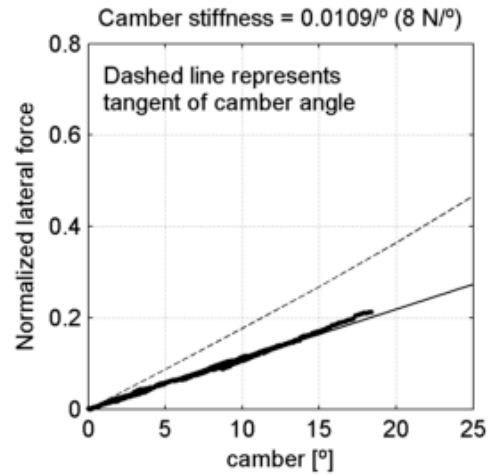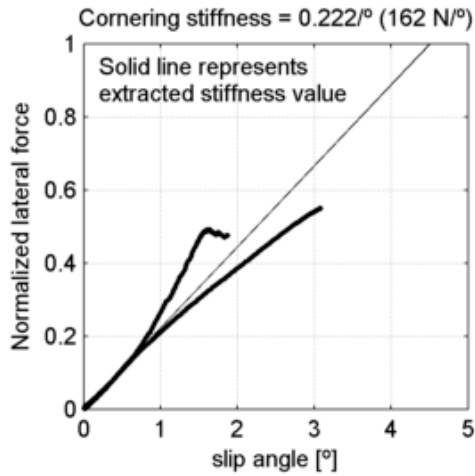at 6.21 bar (90 psi) and
under 324 N (33 kg, 72.8 lb)

Cornering stiffness = 0.337/° (109 N/°)

Solid line represents
extracted stiffness value

Normalized lateral force

slip angle [°]

Camber stiffness = 0.0086/° (2.79 N/°)

Dashed line represents
tangent of camber angle

Normalized lateral force

camber [°]

Self-aligning torque stiffness =
-0.00361 m/° (-1.17 N•m/°)

Normalized self-aligning torque [m]

slip angle [°]

Twisting torque stiffness =
6.47e-005 m/° (0.0209 N•m/°)

Normalized twisting torque [m]

camber angle [°]

Mean pneumatic trail = 6.65 mm

Pneumatic trail [mm]

slip angle [°]

Static lateral stiffness = 0.149/mm (48.4 N/mm)
Calculated relaxation length = 129 mm

Normalized lateral force

displacement [mm]

Vittoria Randonneur Hyper 37-622
tire radius = 17.5 mm, rim width = 18.7 mm
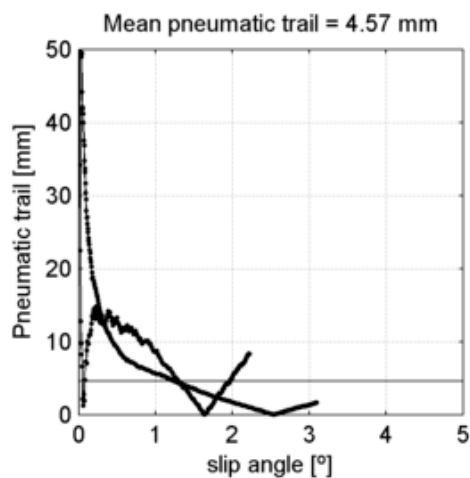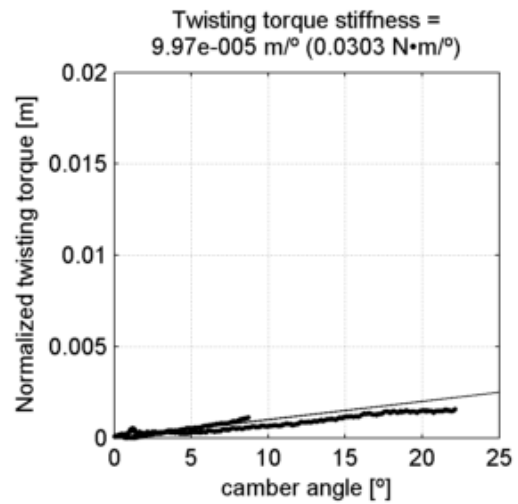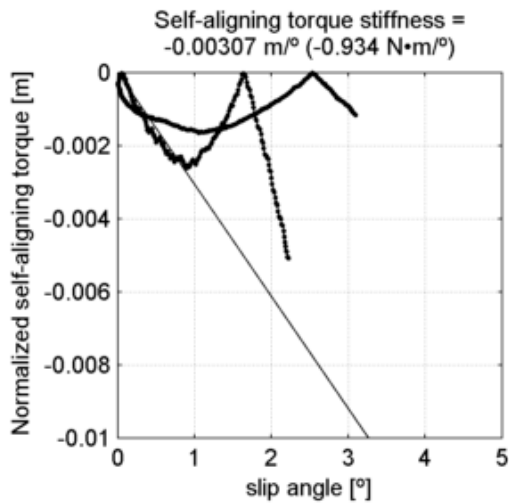
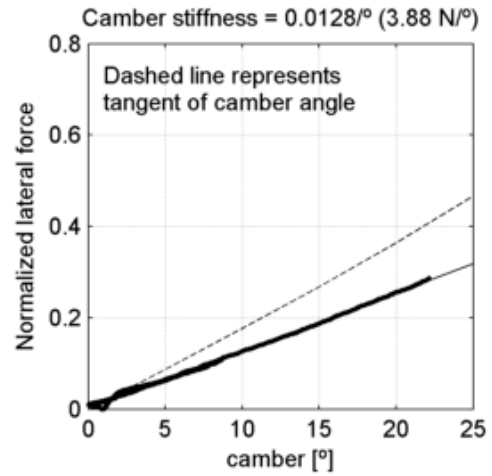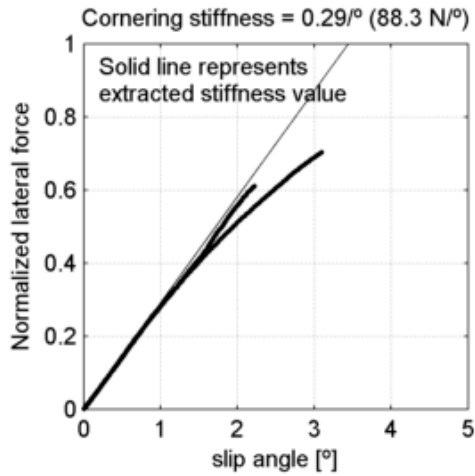at 6.21 bar (90 psi) and
under 540 N (55 kg, 121 lb)

Cornering stiffness = 0.268/° (145 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.0112/° (6.02 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness = −0.00421 m/° (−2.27 N•m/°)

Twisting torque stiffness = 7.25e-005 m/° (0.0391 N•m/°)

Mean pneumatic trail = 11.6 mm

Static lateral stiffness = 0.0851/mm (45.9 N/mm)
Calculated relaxation length = 180 mm

Vittoria Randonneur Hyper 37-622
tire radius = 17.5 mm, rim width = 18.7 mm
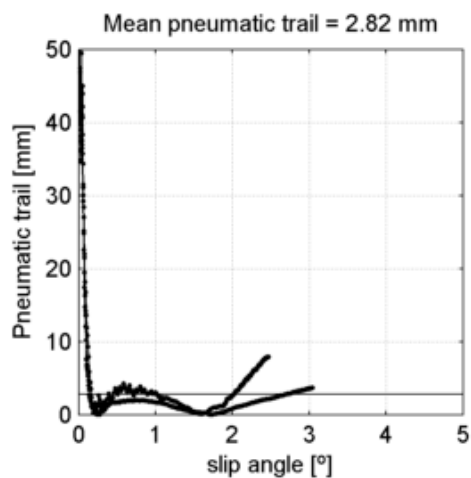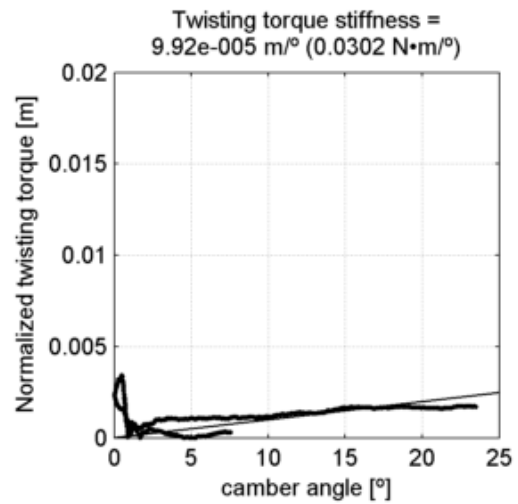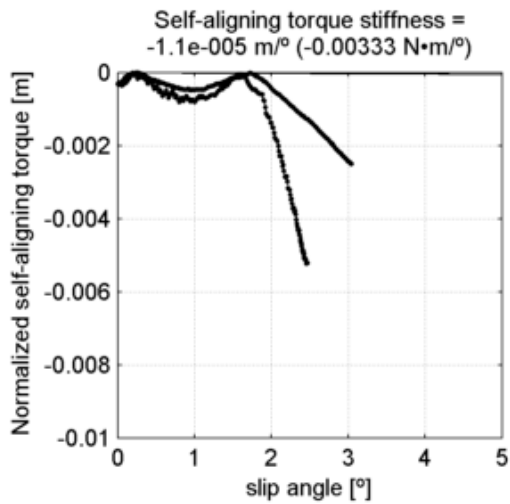
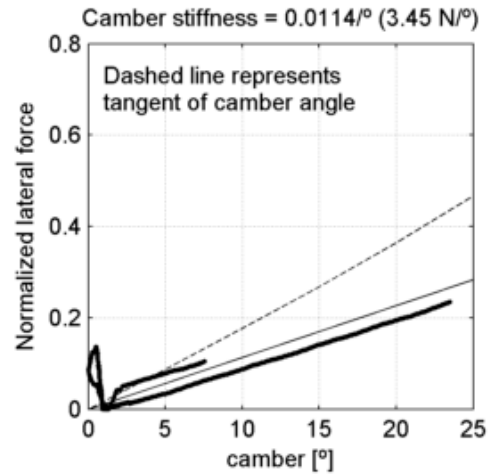at 6.21 bar (90 psi) and
under 731 N (74.5 kg, 164 lb)

Cornering stiffness = 0.222/º (162 N/º)

Solid line represents
extracted stiffness value

Normalized lateral force vs *slip angle [º]*

Camber stiffness = 0.0109/º (8 N/º)

Dashed line represents
tangent of camber angle

Normalized lateral force vs *camber [º]*

Self-aligning torque stiffness =
-0.00462 m/º (-3.38 N•m/º)

Normalized self-aligning torque [m] vs *slip angle [º]*

Twisting torque stiffness =
6.47e-005 m/º (0.0473 N•m/º)

Normalized twisting torque [m] vs *camber angle [º]*

Mean pneumatic trail = 16.3 mm

Pneumatic trail [mm] vs *slip angle [º]*

Static lateral stiffness = 0.0671/mm (49.1 N/mm)
Calculated relaxation length = 189 mm

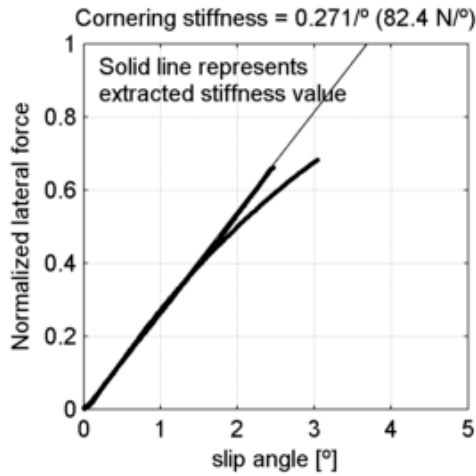*Normalized lateral force* vs *displacement [mm]*

Vredestein Fortezza DuoComp 23-622
tire radius = 11.5 mm, rim width = 13.4 mm
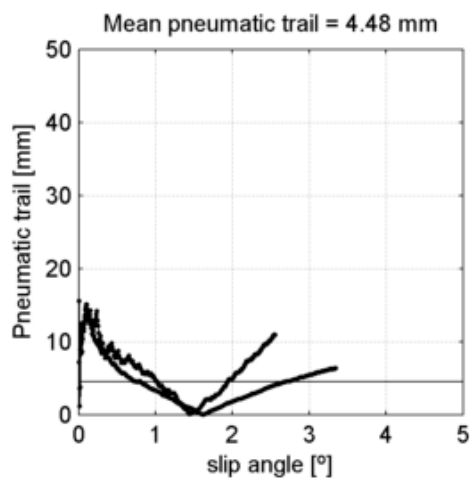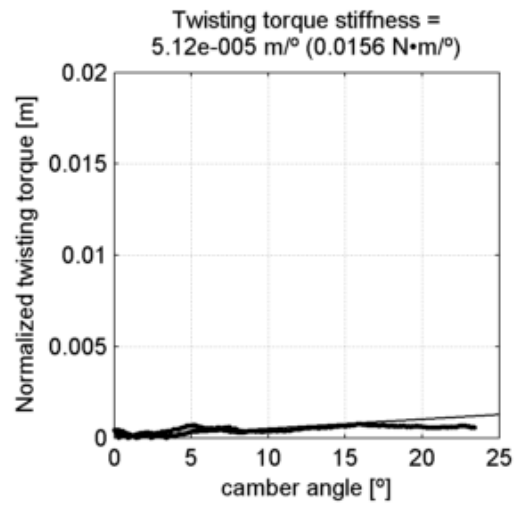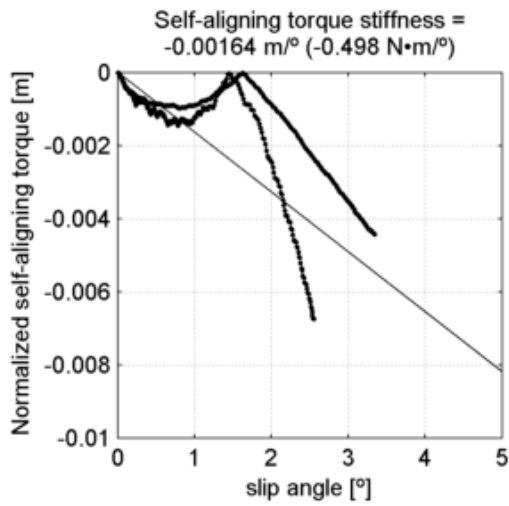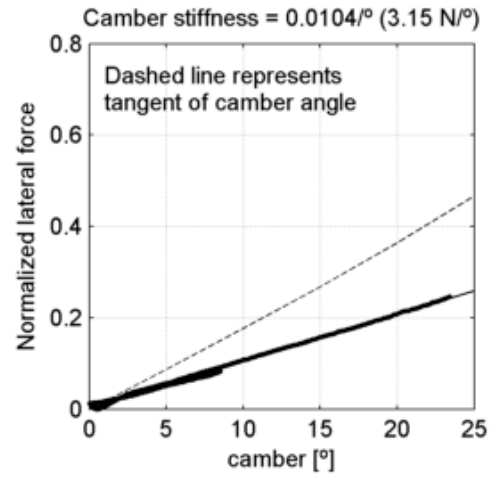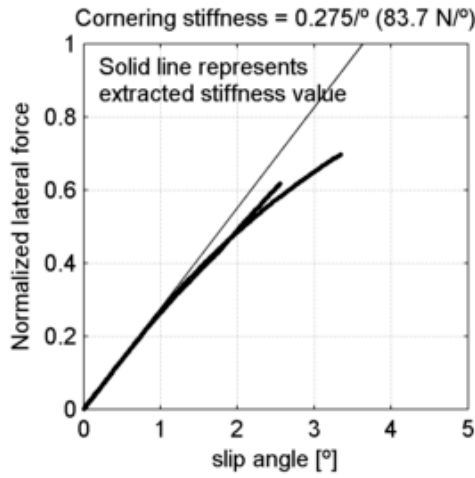
at 5.52 bar (80 psi) and
under 304 N (31 kg, 68.3 lb)

Cornering stiffness = 0.29/° (88.3 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.0128/° (3.88 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00307 m/° (-0.934 N•m/°)

Twisting torque stiffness =
9.97e-005 m/° (0.0303 N•m/°)

Mean pneumatic trail = 4.57 mm

Vredestein Fortezza DuoComp 23-622
tire radius = 11.5 mm, rim width = 13.4 mm
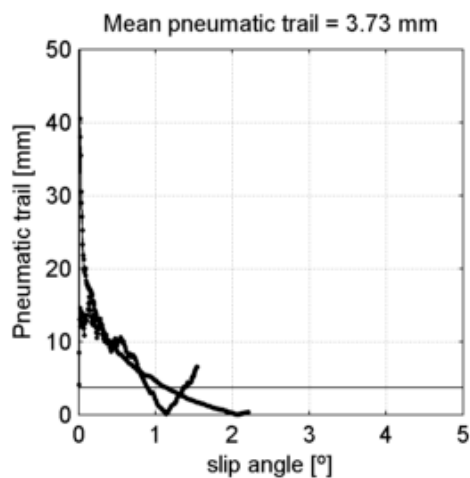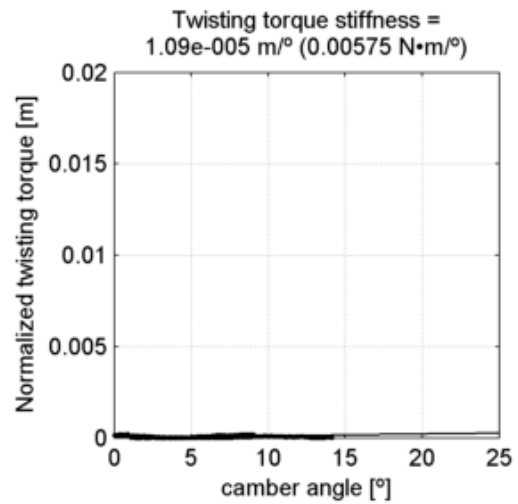
at 8.27 bar (120 psi) and
under 304 N (31 kg, 68.3 lb)

Cornering stiffness = 0.271/° (82.4 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.0114/° (3.45 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
−1.1e-005 m/° (−0.00333 N•m/°)

Twisting torque stiffness =
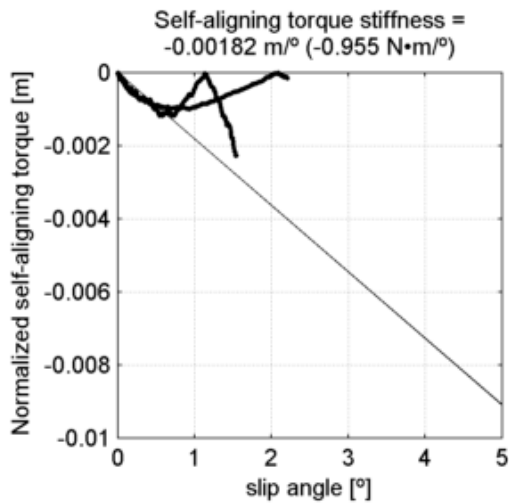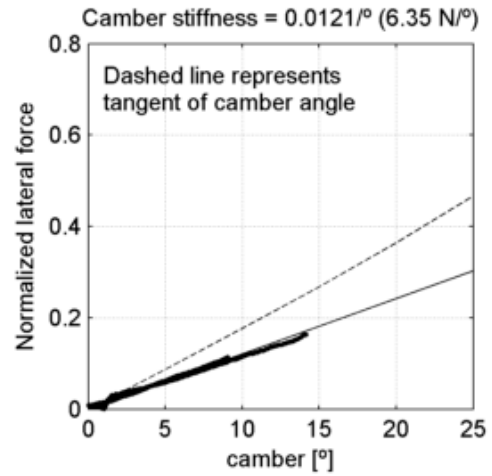9.92e-005 m/° (0.0302 N•m/°)
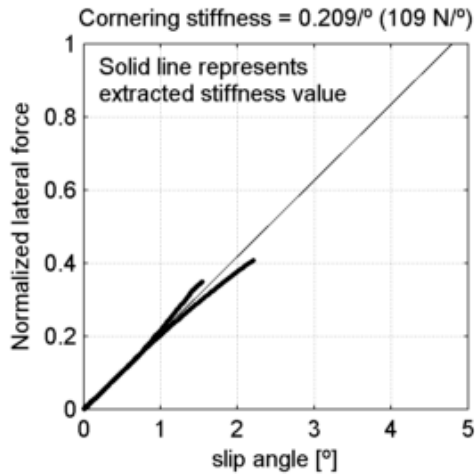
Mean pneumatic trail = 2.82 mm

Vredestein Fortezza DuoComp 23-622
tire radius = 11.5 mm, rim width = 13.4 mm

at 11 bar (160 psi) and
under 304 N (31 kg, 68.3 lb)

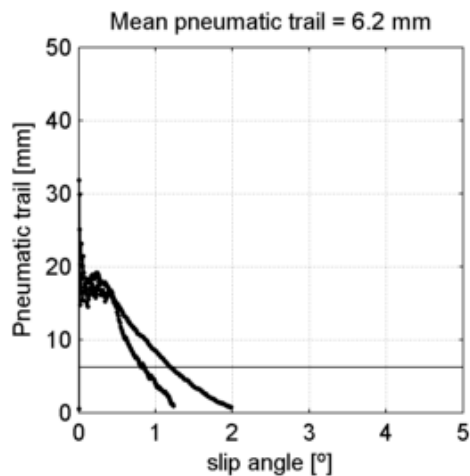Cornering stiffness = 0.275/° (83.7 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.0104/° (3.15 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00164 m/° (-0.498 N•m/°)

Twisting torque stiffness =
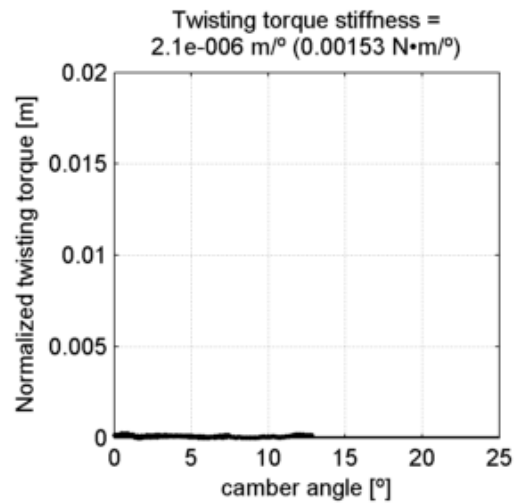5.12e-005 m/° (0.0156 N•m/°)
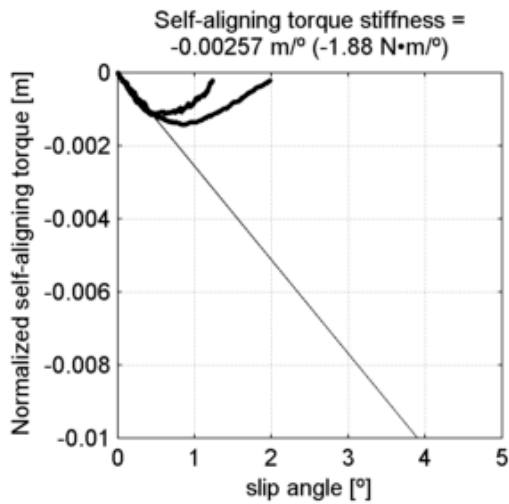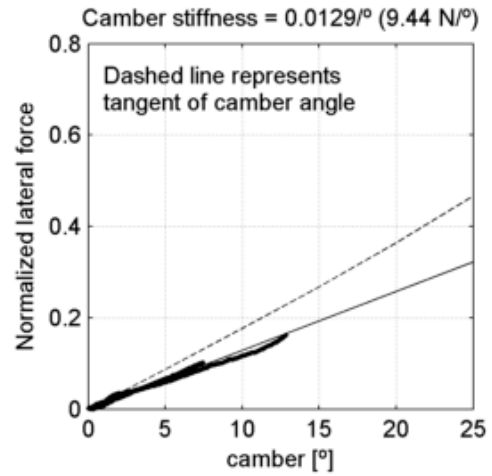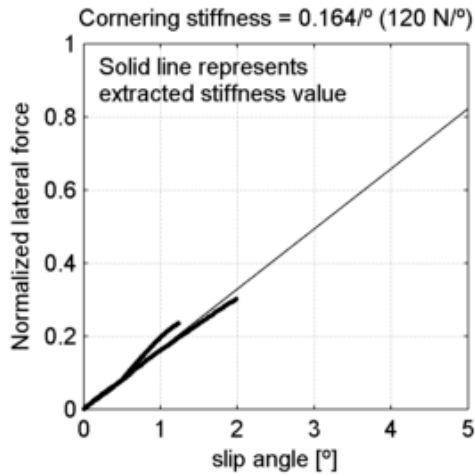
Mean pneumatic trail = 4.48 mm

Vredestein Fortezza DuoComp 23-622
tire radius = 11.5 mm, rim width = 13.4 mm

at 11 bar (160 psi) and
under 525 N (53.5 kg, 118 lb)

Cornering stiffness = 0.209/° (109 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.0121/° (6.35 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00182 m/° (-0.955 N•m/°)

Twisting torque stiffness =
1.09e-005 m/° (0.00575 N•m/°)

Mean pneumatic trail = 3.73 mm

Vredestein Fortezza DuoComp 23-622
tire radius = 11.5 mm, rim width = 13.4 mm

at 11 bar (160 psi) and
under 731 N (74.5 kg, 164 lb)

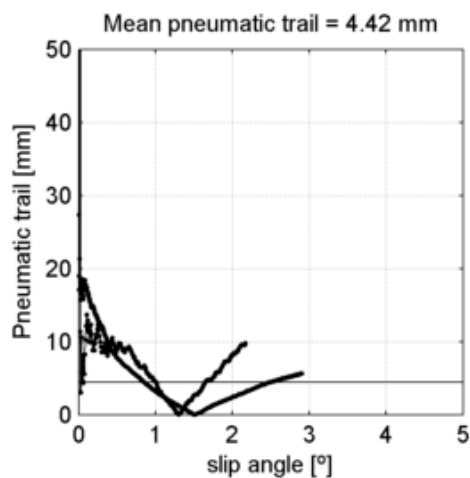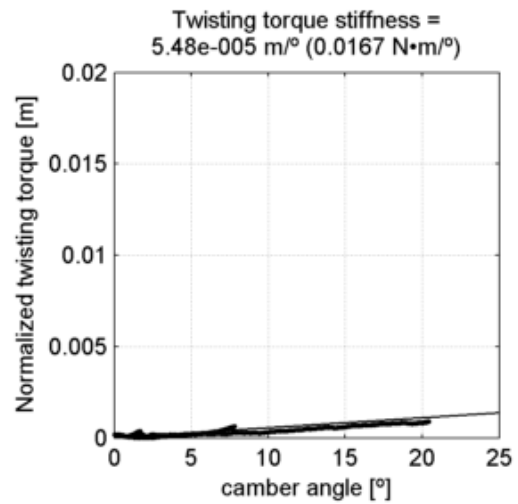Cornering stiffness = 0.164/° (120 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.0129/° (9.44 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00257 m/° (-1.88 N•m/°)

Twisting torque stiffness =
2.1e-006 m/° (0.00153 N•m/°)
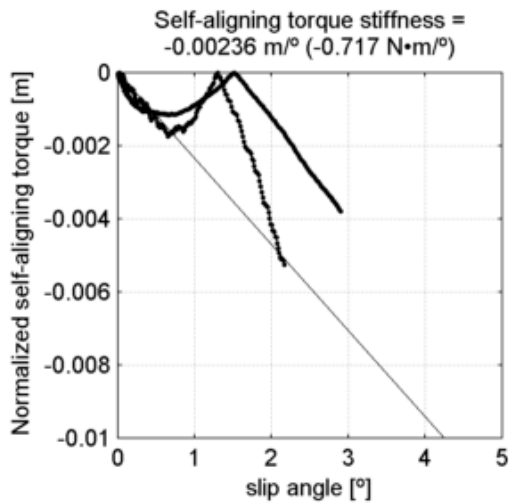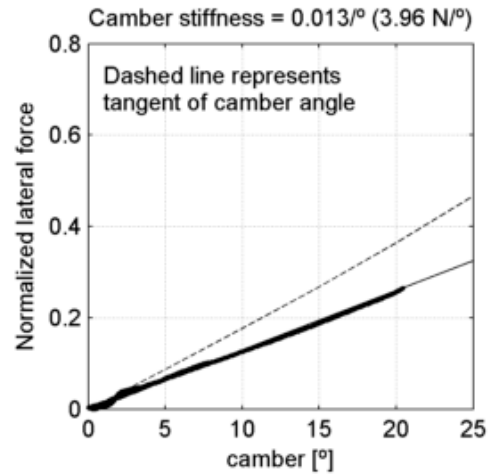
Mean pneumatic trail = 6.2 mm

Vredestein Fortezza TriComp 23-622
tire radius = 11.5 mm, rim width = 13.4 mm

at 5.52 bar (80 psi) and
under 304 N (31 kg, 68.3 lb)

Cornering stiffness = 0.29/° (88.1 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.013/° (3.96 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00236 m/° (-0.717 N•m/°)

Twisting torque stiffness =
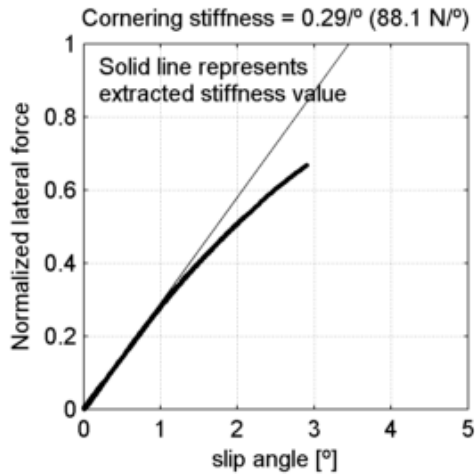5.48e-005 m/° (0.0167 N•m/°)

Mean pneumatic trail = 4.42 mm

Vredestein Fortezza TriComp 23-622
tire radius = 11.5 mm, rim width = 13.4 mm

at 8.27 bar (120 psi) and
under 304 N (31 kg, 68.3 lb)

Cornering stiffness = 0.278/° (84.6 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.0126/° (3.82 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00124 m/° (-0.376 N•m/°)

Twisting torque stiffness =
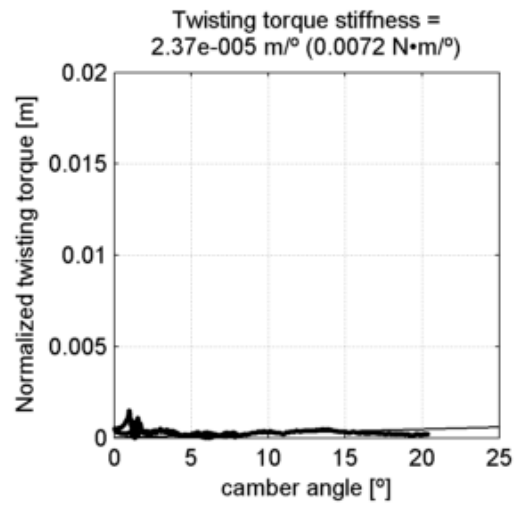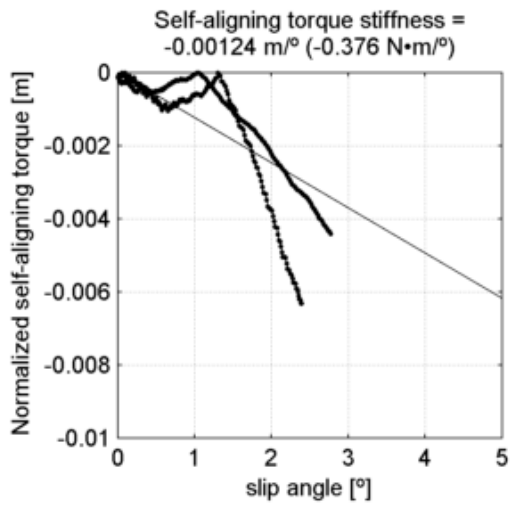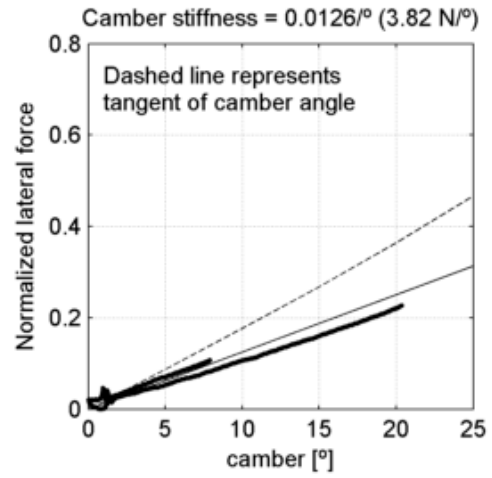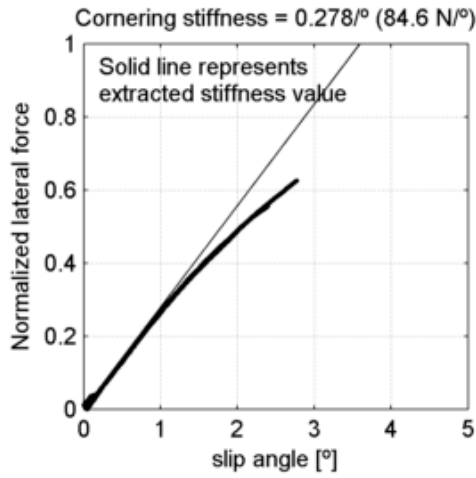2.37e-005 m/° (0.0072 N•m/°)

Mean pneumatic trail = 4.4 mm

Vredestein Fortezza TriComp 23-622
tire radius = 11.5 mm, rim width = 13.4 mm

at 11 bar (160 psi) and
under 304 N (31 kg, 68.3 lb)

Cornering stiffness = 0.267/° (81.3 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.0115/° (3.49 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00195 m/° (-0.594 N•m/°)

Twisting torque stiffness =
3.38e-006 m/° (0.00103 N•m/°)

Mean pneumatic trail = 5.05 mm

Static lateral stiffness = 0.151/mm (46 N/mm)
Calculated relaxation length = 101 mm

Vredestein Fortezza TriComp 23-622
tire radius = 11.5 mm, rim width = 13.4 mm

at 11 bar (160 psi) and
under 525 N (53.5 kg, 118 lb)

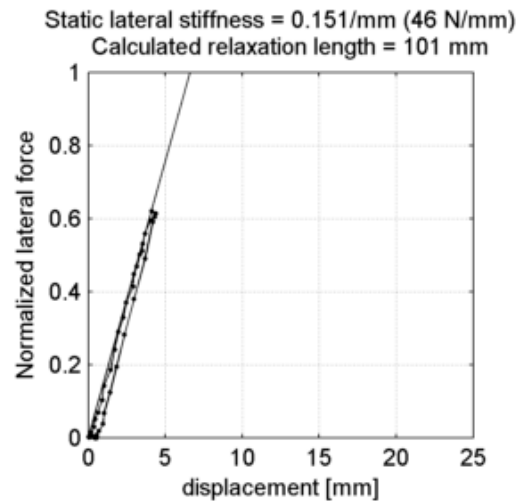Cornering stiffness = 0.205/° (108 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.0132/° (6.95 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00219 m/° (-1.15 N•m/°)

Twisting torque stiffness =
2.18e-005 m/° (0.0115 N•m/°)

Mean pneumatic trail = 4.25 mm

Static lateral stiffness = 0.103/mm (54.2 N/mm)
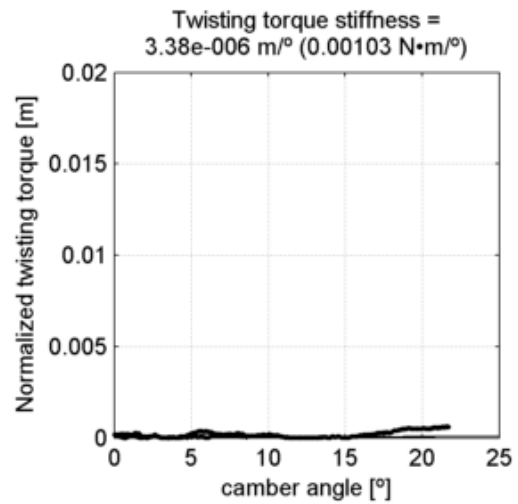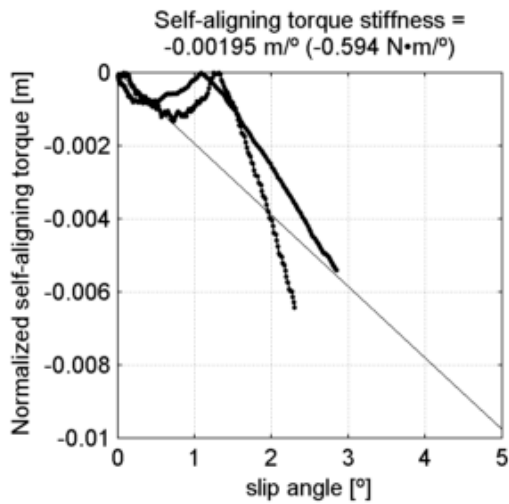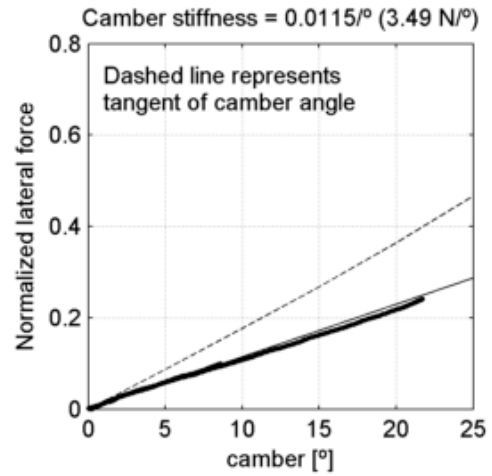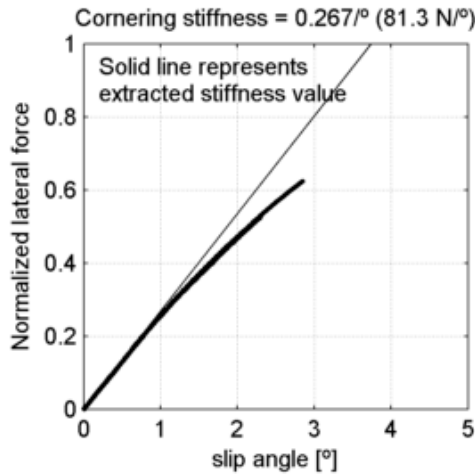Calculated relaxation length = 114 mm

Vredestein Fortezza TriComp 23-622
tire radius = 11.5 mm, rim width = 13.4 mm

at 11 bar (160 psi) and
under 731 N (74.5 kg, 164 lb)

Cornering stiffness = 0.168/° (123 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.0141/° (10.3 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00198 m/° (-1.45 N•m/°)

Twisting torque stiffness =
2.56e-005 m/° (0.0187 N•m/°)

Mean pneumatic trail = 4.33 mm

Static lateral stiffness = 0.0717/mm (52.4 N/mm)
Calculated relaxation length = 134 mm

Vredestein Perfect Tour 37-622
tire radius = 16.8 mm, rim width = 18.7 mm

at 3.45 bar (50 psi) and
under 304 N (31 kg, 68.3 lb)

Cornering stiffness = 0.297/° (90.3 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.00716/° (2.18 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00565 m/° (-1.72 N•m/°)

Twisting torque stiffness =
0.000145 m/° (0.0441 N•m/°)

Mean pneumatic trail = 12.8 mm

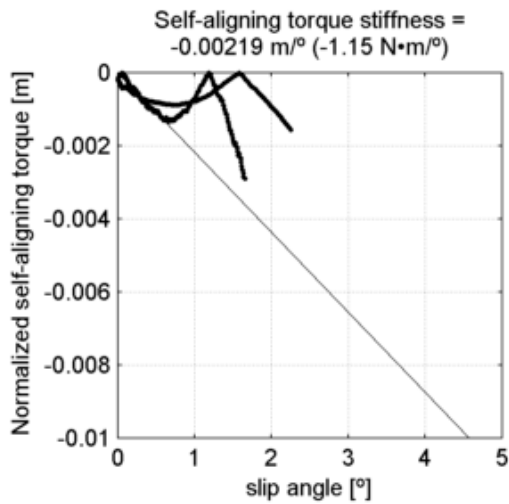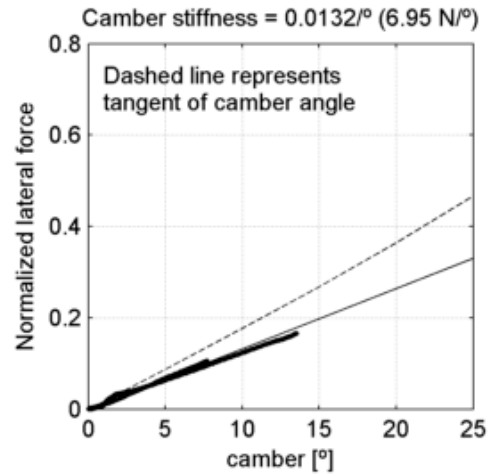Static lateral stiffness = 0.129/mm (39.1 N/mm)
Calculated relaxation length = 132 mm

Vredestein Perfect Tour 37-622
tire radius = 16.8 mm, rim width = 18.7 mm

at 3.45 bar (50 psi) and
under 525 N (53.5 kg, 118 lb)

Cornering stiffness = 0.228/º (120 N/º)

Solid line represents
extracted stiffness value

Camber stiffness = 0.00957/º (5.02 N/º)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00538 m/º (-2.82 N•m/º)

Twisting torque stiffness =
0.000105 m/º (0.0554 N•m/º)

Mean pneumatic trail = 16.3 mm

Static lateral stiffness = 0.0713/mm (37.4 N/mm)
Calculated relaxation length = 183 mm

Vredestein Perfect Tour 37-622
tire radius = 16.8 mm, rim width = 18.7 mm

at 3.45 bar (50 psi) and
under 731 N (74.5 kg, 164 lb)

Cornering stiffness = 0.171/° (125 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.00967/° (7.07 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.0053 m/° (-3.87 N•m/°)

Twisting torque stiffness =
9.8e-005 m/° (0.0716 N•m/°)

Mean pneumatic trail = 24.2 mm

Static lateral stiffness = 0.0492/mm (35.9 N/mm)
Calculated relaxation length = 199 mm

Vredestein Perfect Tour 37-622
tire radius = 16.8 mm, rim width = 18.7 mm

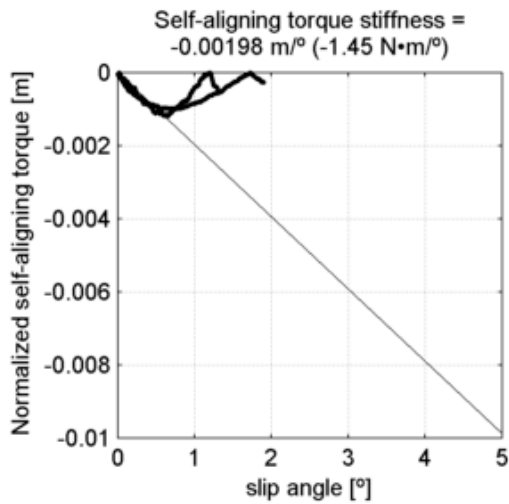at 4.14 bar (60 psi) and
under 304 N (31 kg, 68.3 lb)

Cornering stiffness = 0.292/° (88.9 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.0079/° (2.4 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00592 m/° (-1.8 N•m/°)

Twisting torque stiffness =
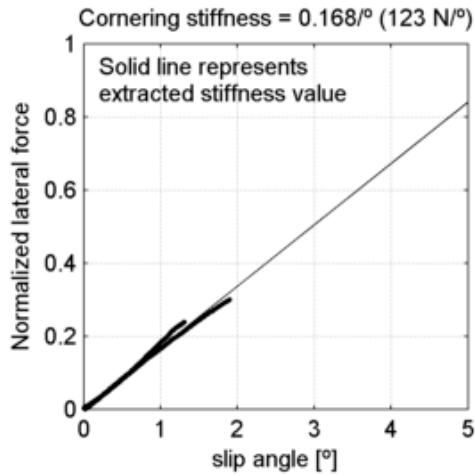0.000117 m/° (0.0355 N•m/°)

Mean pneumatic trail = 12.7 mm

Vredestein Perfect Tour 37-622
tire radius = 16.8 mm, rim width = 18.7 mm
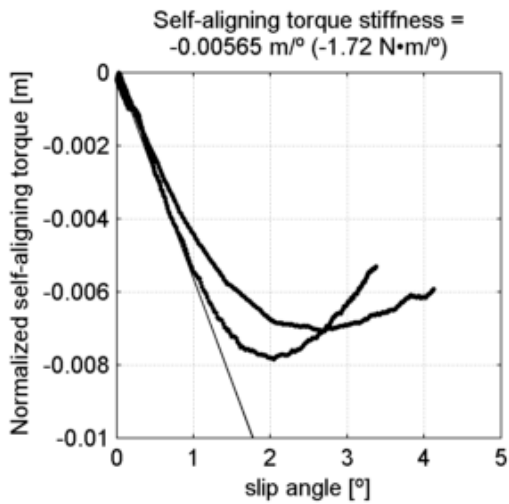
at 4.14 bar (60 psi) and
under 525 N (53.5 kg, 118 lb)

Cornering stiffness = 0.237/º (125 N/º)

Solid line represents
extracted stiffness value

Camber stiffness = 0.00941/º (4.94 N/º)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00572 m/º (-3 N•m/º)

Twisting torque stiffness =
9.46e-005 m/º (0.0497 N•m/º)

Mean pneumatic trail = 17.1 mm

Static lateral stiffness = 0.0785/mm (41.2 N/mm)
Calculated relaxation length = 173 mm
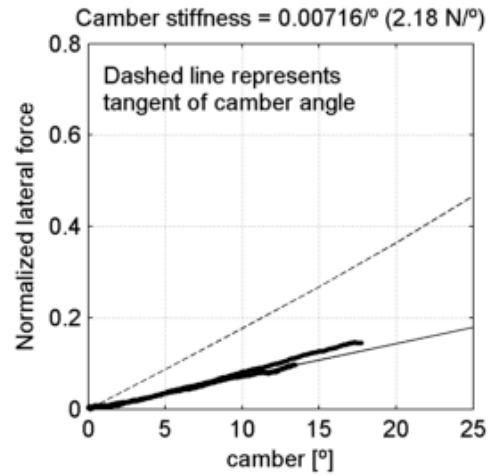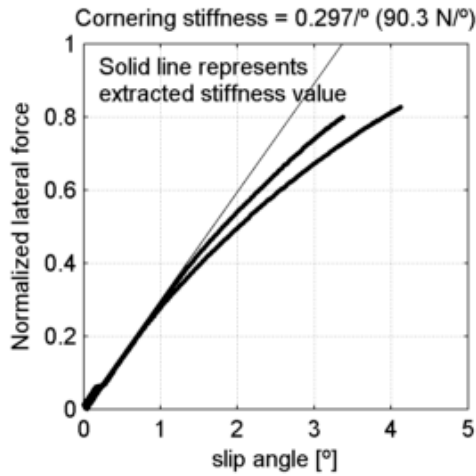
Vredestein Perfect Tour 37-622
tire radius = 16.8 mm, rim width = 18.7 mm

at 4.14 bar (60 psi) and
under 731 N (74.5 kg, 164 lb)

Cornering stiffness = 0.181/° (132 N/°)

Solid line represents
extracted stiffness value

Normalized lateral force

slip angle [°]

Camber stiffness = 0.01/° (7.33 N/°)

Dashed line represents
tangent of camber angle

Normalized lateral force

camber [°]

Self-aligning torque stiffness =
-0.00476 m/° (-3.48 N•m/°)

Normalized self-aligning torque [m]

slip angle [°]

Twisting torque stiffness =
8.74e-005 m/° (0.0639 N•m/°)

Normalized twisting torque [m]

camber angle [°]

Mean pneumatic trail = 20.1 mm

Pneumatic trail [mm]

slip angle [°]

Static lateral stiffness = 0.0571/mm (41.7 N/mm)
Calculated relaxation length = 181 mm

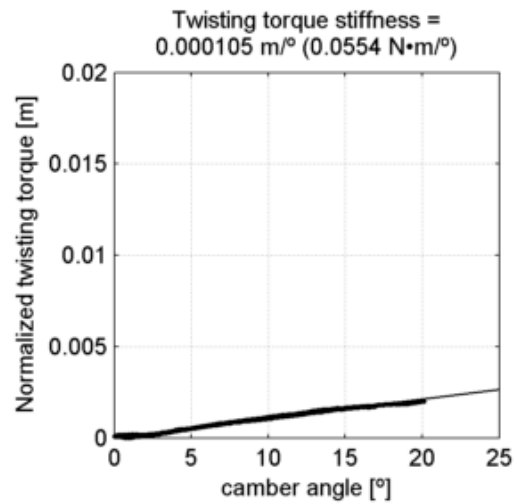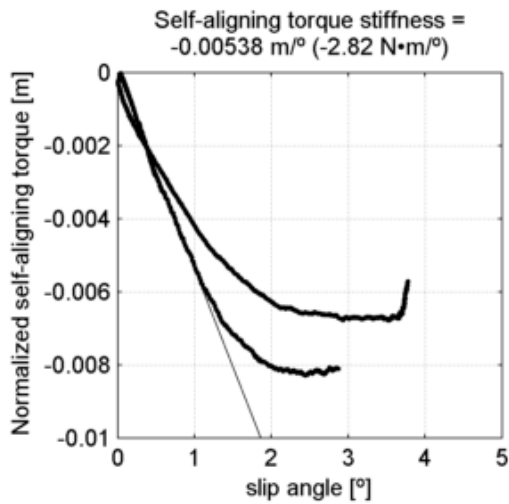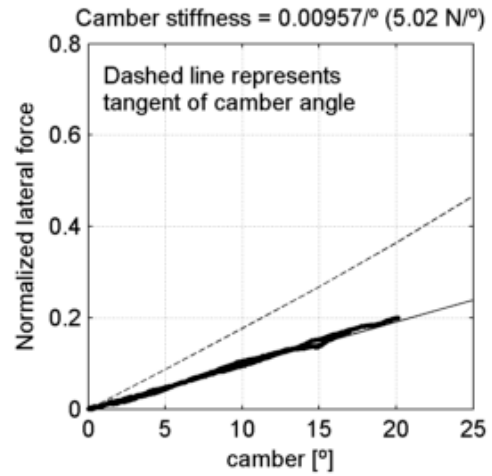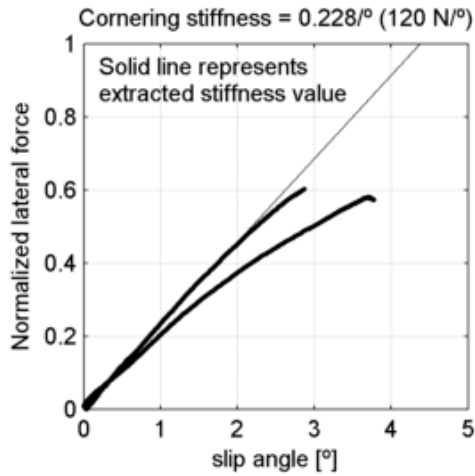Normalized lateral force

displacement [mm]

Vredestein Perfect Tour 37-622
tire radius = 16.8 mm, rim width = 18.7 mm

at 4.83 bar (70 psi) and
under 304 N (31 kg, 68.3 lb)

Cornering stiffness = 0.282/° (85.7 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.00717/° (2.18 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00473 m/° (-1.44 N•m/°)

Twisting torque stiffness =
0.000129 m/° (0.0392 N•m/°)

Mean pneumatic trail = 11.5 mm

Static lateral stiffness = 0.14/mm (42.7 N/mm)
Calculated relaxation length = 115 mm

Vredestein Perfect Tour 37-622
tire radius = 16.8 mm, rim width = 18.7 mm

at 4.83 bar (70 psi) and
under 525 N (53.5 kg, 118 lb)

Cornering stiffness = 0.222/° (116 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.00899/° (4.72 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00448 m/° (-2.35 N•m/°)

Twisting torque stiffness =
8.35e-005 m/° (0.0438 N•m/°)

Mean pneumatic trail = 13.2 mm

Static lateral stiffness = 0.0851/mm (44.7 N/mm)
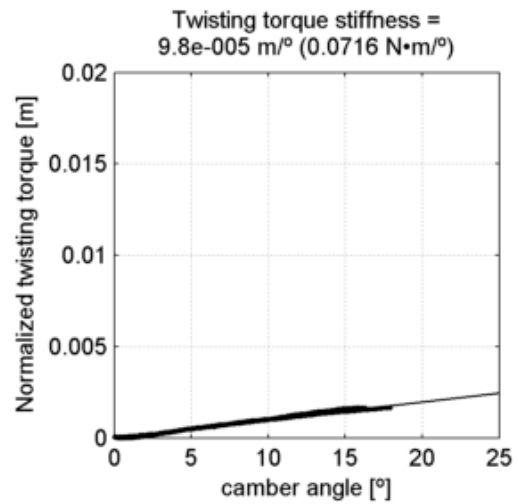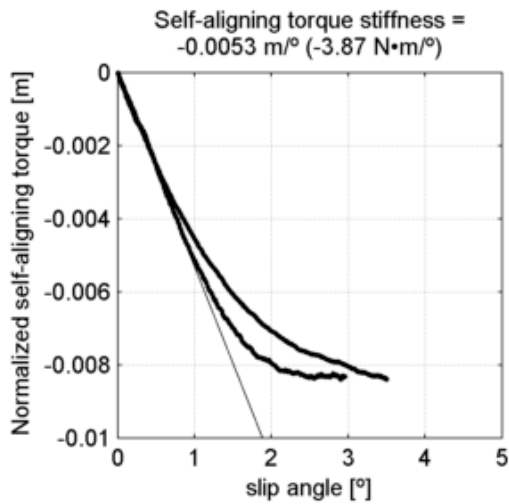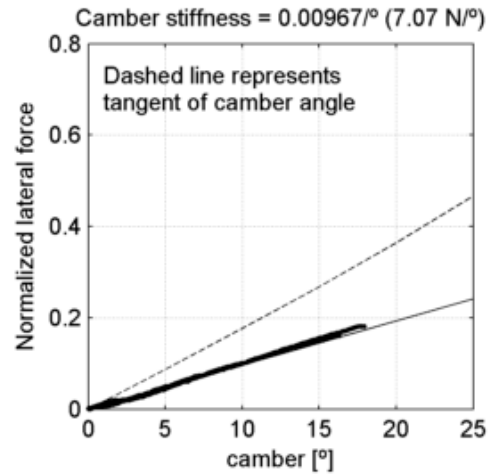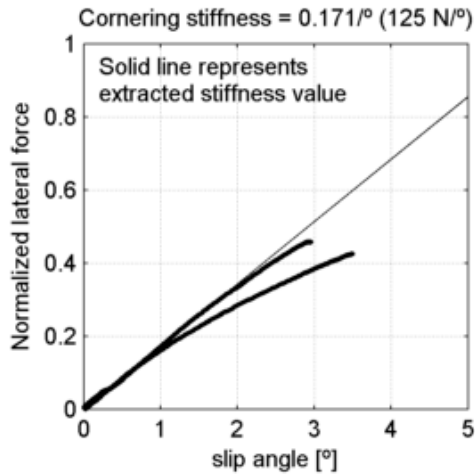Calculated relaxation length = 149 mm

Vredestein Perfect Tour 37-622
tire radius = 16.8 mm, rim width = 18.7 mm

at 4.83 bar (70 psi) and
under 731 N (74.5 kg, 164 lb)

Cornering stiffness = 0.19/° (139 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.0099/° (7.24 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.00452 m/° (-3.31 N•m/°)

Twisting torque stiffness =
8.94e-005 m/° (0.0653 N•m/°)

Mean pneumatic trail = 17.3 mm

Static lateral stiffness = 0.052/mm (38 N/mm)
Calculated relaxation length = 209 mm

Vredestein Perfect Tour 37-622
tire radius = 16.8 mm, rim width = 18.7 mm
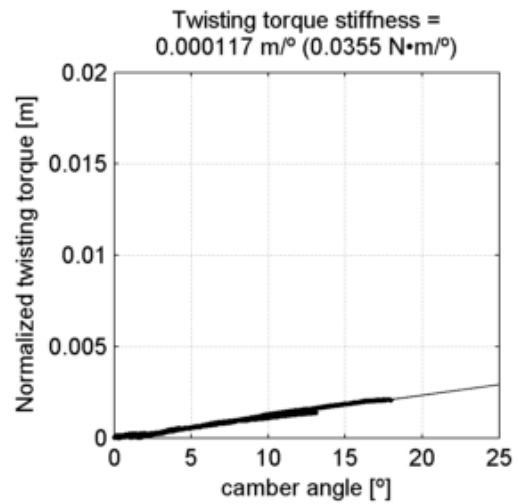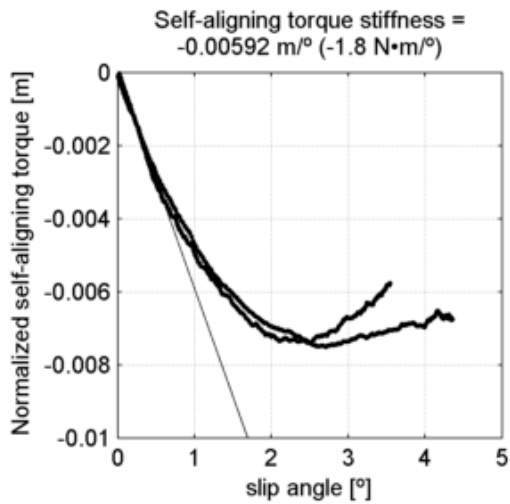
at 6.21 bar (90 psi) and
under 304 N (31 kg, 68.3 lb)

Cornering stiffness = 0.273/° (83.1 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.00638/° (1.94 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.0043 m/° (-1.31 N•m/°)

Twisting torque stiffness =
0.000104 m/° (0.0317 N•m/°)

Mean pneumatic trail = 10.8 mm

Static lateral stiffness = 0.149/mm (45.4 N/mm)
Calculated relaxation length = 105 mm

Vredestein Perfect Tour 37-622
tire radius = 16.8 mm, rim width = 18.7 mm

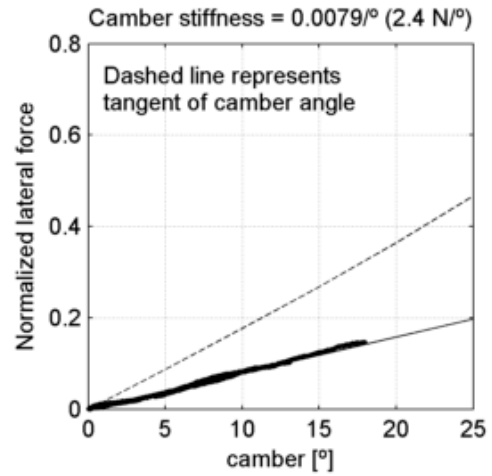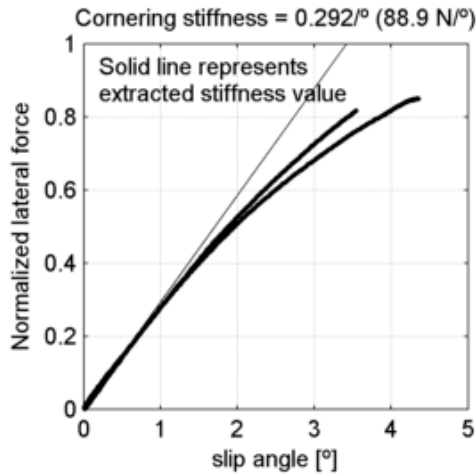at 6.21 bar (90 psi) and
under 525 N (53.5 kg, 118 lb)

Cornering stiffness = 0.226/° (119 N/°)

Solid line represents
extracted stiffness value

x-axis: slip angle [°], y-axis: Normalized lateral force

Camber stiffness = 0.00941/° (4.94 N/°)

Dashed line represents
tangent of camber angle

x-axis: camber [°], y-axis: Normalized lateral force

Self-aligning torque stiffness =
-0.00411 m/° (-2.16 N•m/°)

x-axis: slip angle [°], y-axis: Normalized self-aligning torque [m]

Twisting torque stiffness =
3.73e-005 m/° (0.0196 N•m/°)

x-axis: camber angle [°], y-axis: Normalized twisting torque [m]

Mean pneumatic trail = 12.6 mm

x-axis: slip angle [°], y-axis: Pneumatic trail [mm]

Static lateral stiffness = 0.0883/mm (46.3 N/mm)
Calculated relaxation length = 147 mm

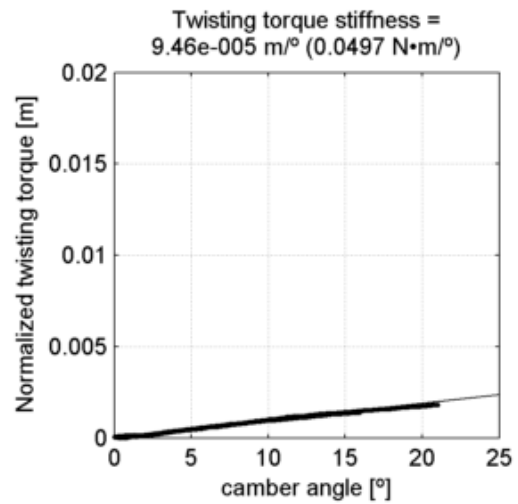*x-axis: displacement [mm], y-axis: Normalized lateral force*
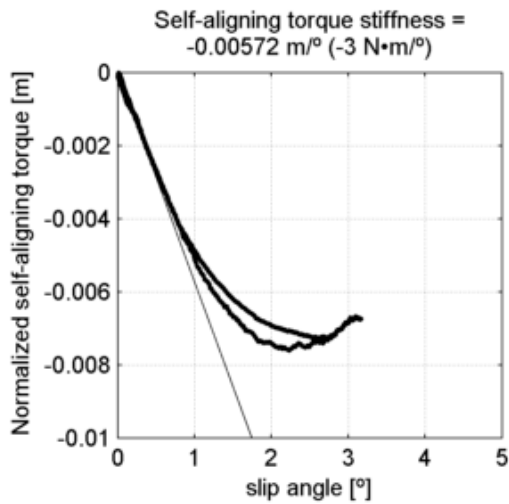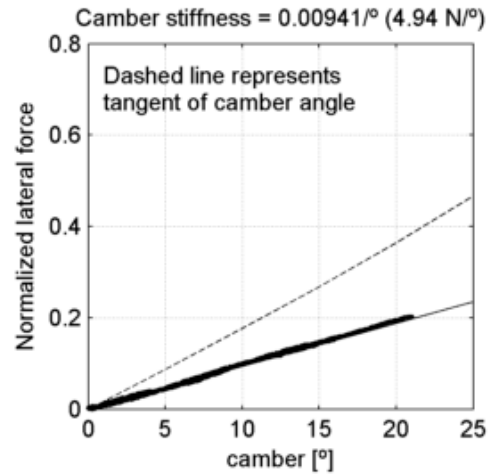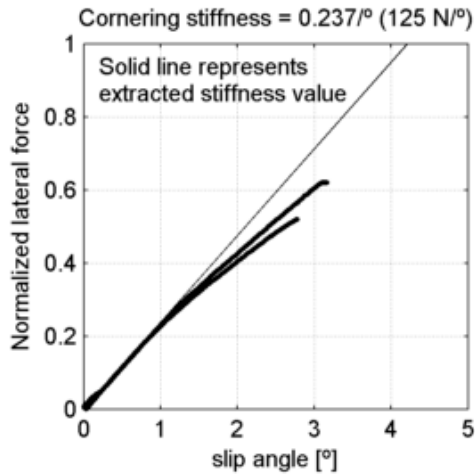
Vredestein Perfect Tour 37-622
tire radius = 16.8 mm, rim width = 18.7 mm

at 6.21 bar (90 psi) and
under 525 N (53.5 kg, 118 lb)

Cornering stiffness = 0.221/° (116 N/°)

Solid line represents
extracted stiffness value

x-axis: slip angle [°]
y-axis: Normalized lateral force

Camber stiffness = 0.00229/° (1.2 N/°)

Dashed line represents
tangent of camber angle

x-axis: camber [°]
y-axis: Normalized lateral force

Self-aligning torque stiffness =
-0.00403 m/° (-2.12 N•m/°)

x-axis: slip angle [°]
y-axis: Normalized self-aligning torque [m]

Twisting torque stiffness =
2.81e-007 m/° (0.000147 N•m/°)

x-axis: camber angle [°]
y-axis: Normalized twisting torque [m]

Mean pneumatic trail = 13 mm

x-axis: slip angle [°]
y-axis: Pneumatic trail [mm]

Static lateral stiffness = 0.0883/mm (46.3 N/mm)
Calculated relaxation length = 143 mm

x-axis: displacement [mm]
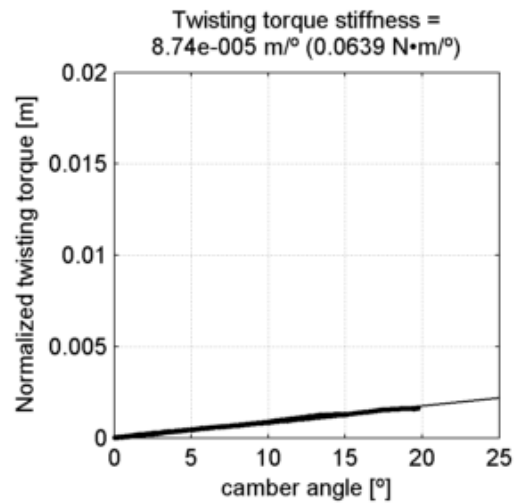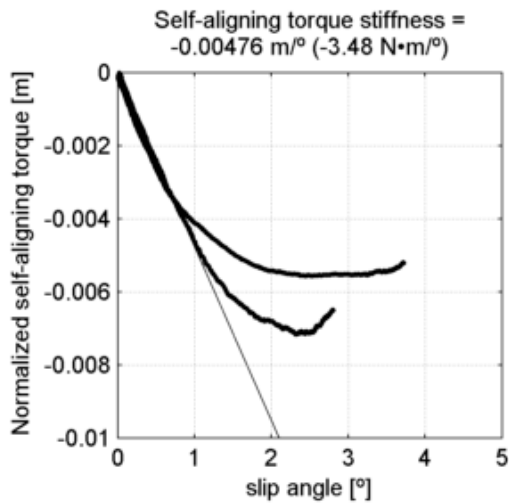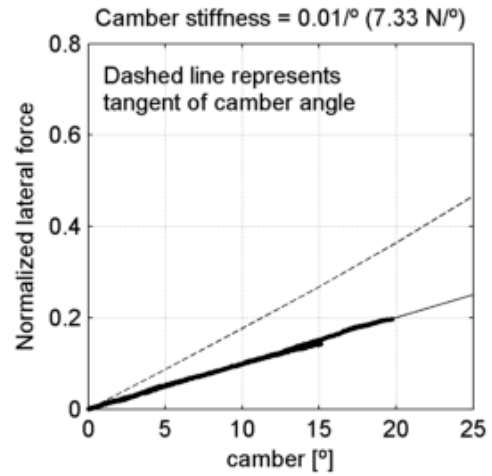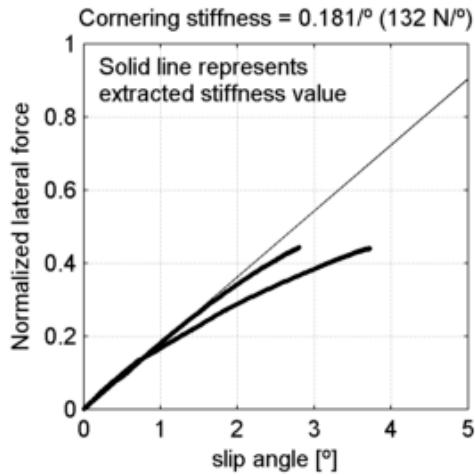*y-axis:* Normalized lateral force

Vredestein Perfect Tour 37-622
tire radius = 16.8 mm, rim width = 18.7 mm

at 6.21 bar (90 psi) and
under 731 N (74.5 kg, 164 lb)

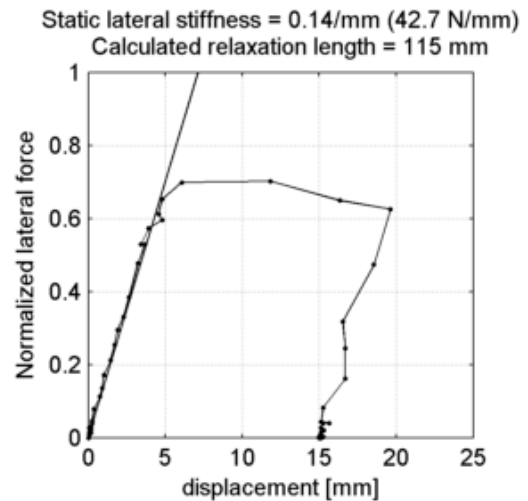Cornering stiffness = 0.178/° (130 N/°)

Solid line represents
extracted stiffness value

Normalized lateral force vs *slip angle [°]*

Camber stiffness = 0.00862/° (6.3 N/°)

Dashed line represents
tangent of camber angle

Normalized lateral force vs *camber [°]*

Self-aligning torque stiffness =
-0.00391 m/° (-2.86 N•m/°)

Normalized self-aligning torque [m] vs *slip angle [°]*

Twisting torque stiffness =
6.93e-005 m/° (0.0507 N•m/°)

Normalized twisting torque [m] vs *camber angle [°]*

Mean pneumatic trail = 16.1 mm

Pneumatic trail [mm] vs *slip angle [°]*

Static lateral stiffness = 0.0679/mm (49.6 N/mm)
Calculated relaxation length = 150 mm

Normalized lateral force vs *displacement [mm]*

Vredestein Perfect Tour 37-622
tire radius = 16.8 mm, rim width = 18.7 mm
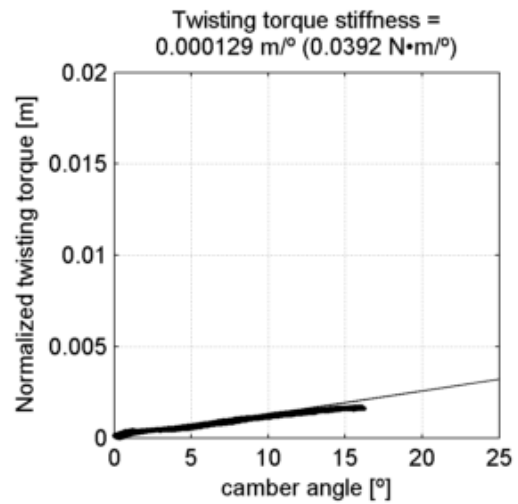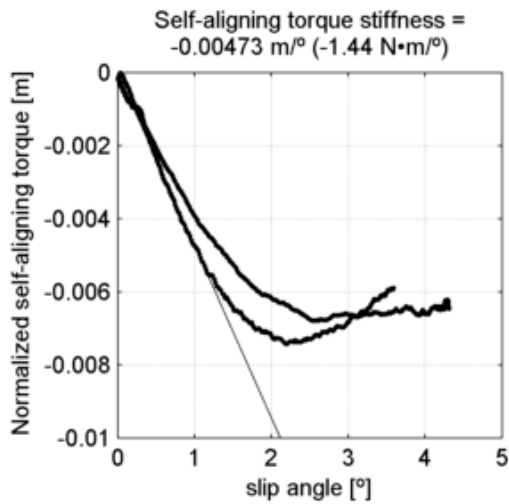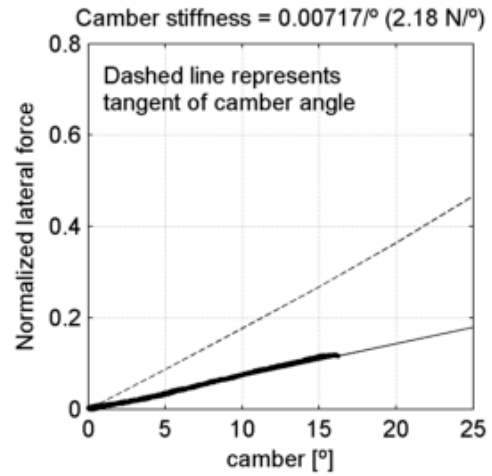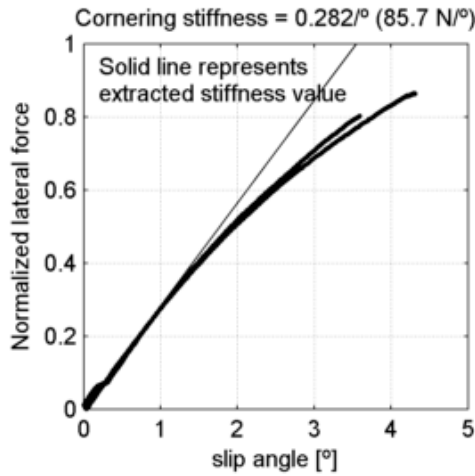
at 3.45 bar (50 psi) and
under 304 N (31 kg, 68.3 lb)

Cornering stiffness = 0.297/° (90.4 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.0102/° (3.1 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.000756 m/° (-0.23 N•m/°)

Twisting torque stiffness =
2.84e-005 m/° (0.00863 N•m/°)

Mean pneumatic trail = 6.63 mm

Static lateral stiffness = 0.123/mm (37.3 N/mm)
Calculated relaxation length = 139 mm

Vredestein Perfect Tour 37-622
tire radius = 16.8 mm, rim width = 18.7 mm

at 6.21 bar (90 psi) and
under 304 N (31 kg, 68.3 lb)

Cornering stiffness = 0.279/° (85 N/°)

Solid line represents
extracted stiffness value

Camber stiffness = 0.00886/° (2.69 N/°)

Dashed line represents
tangent of camber angle

Self-aligning torque stiffness =
-0.000962 m/° (-0.293 N•m/°)

Twisting torque stiffness =
2.12e-005 m/° (0.00645 N•m/°)

Mean pneumatic trail = 6.89 mm

Static lateral stiffness = 0.145/mm (44.1 N/mm)
Calculated relaxation length = 110 mm

Vredestein Perfect Tour 37-622
tire radius = 16.8 mm, rim width = 18.7 mm
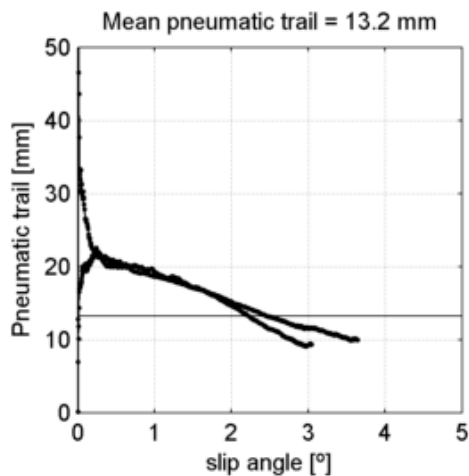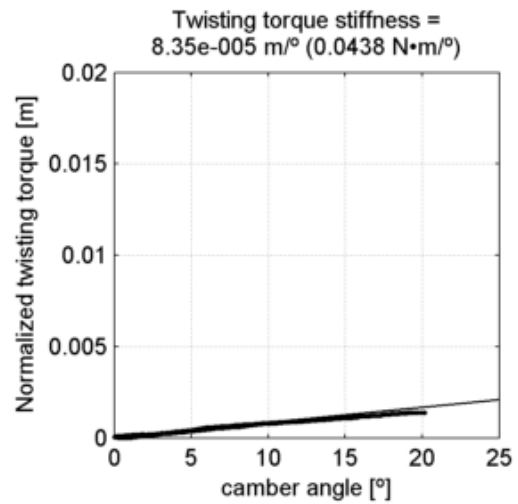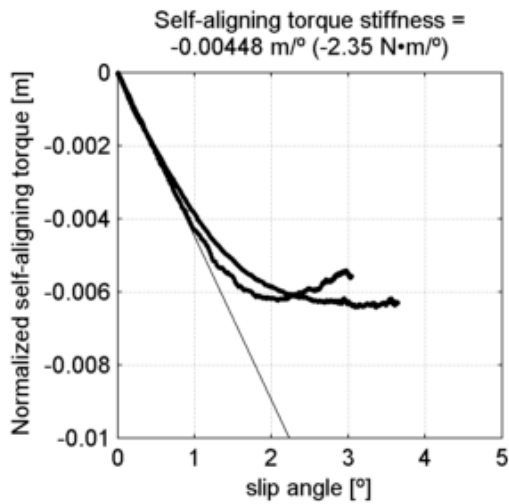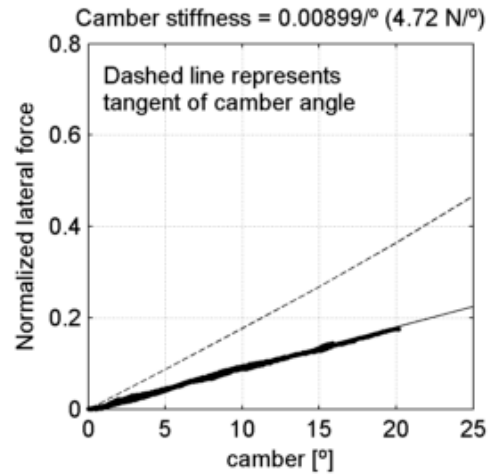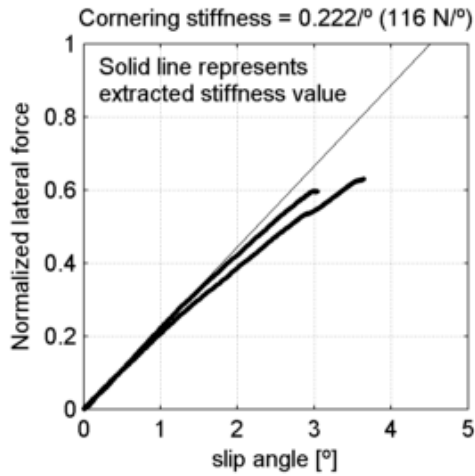
at 6.21 bar (90 psi) and
under 731 N (74.5 kg, 164 lb)

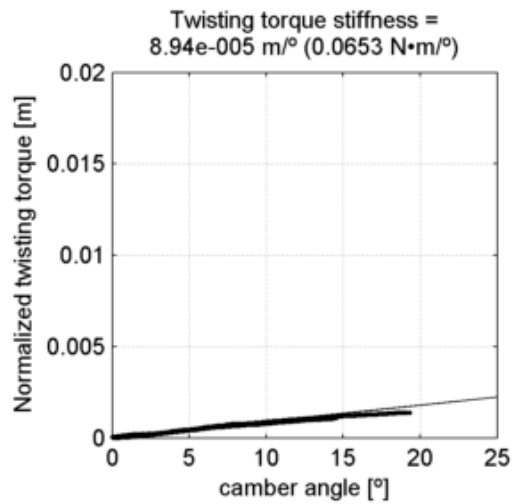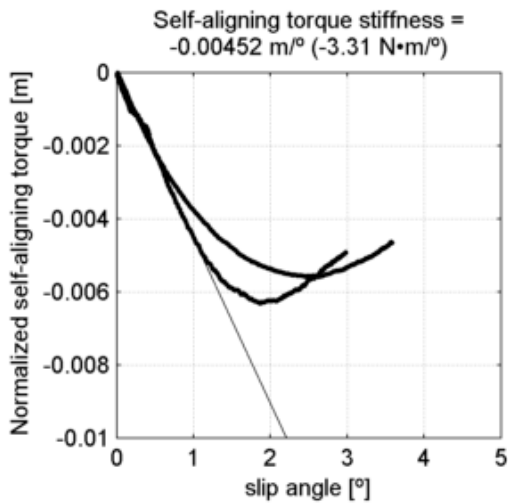Cornering stiffness = 0.21/° (153 N/°)

Solid line represents
extracted stiffness value

Normalized lateral force

slip angle [°]

Camber stiffness = 0.0107/° (7.82 N/°)

Dashed line represents
tangent of camber angle

Normalized lateral force

camber [°]

Self-aligning torque stiffness =
-0.000185 m/° (-0.135 N•m/°)

Normalized self-aligning torque [m]

slip angle [°]

Twisting torque stiffness =
4.09e-005 m/° (0.0299 N•m/°)

Normalized twisting torque [m]

camber angle [°]

Mean pneumatic trail = 2.58 mm

Pneumatic trail [mm]

slip angle [°]

Static lateral stiffness = 0.0594/mm (43.4 N/mm)
Calculated relaxation length = 202 mm

Normalized lateral force

displacement [mm]

Bontrager All Weather 23-622
tire radius = 11.5 mm, rim width = 13.4 mm

summary
part 1 of 2

Bontrager All Weather 23-622
tire radius = 11.5 mm, rim width = 13.4 mm

summary
part 2 of 2

Bontrager All Weather 25-622
tire radius = 12.5 mm, rim width = 13.4 mm

summary
part 1 of 2

Bontrager All Weather 25-622
tire radius = 12.5 mm, rim width = 13.4 mm

summary
part 2 of 2

Bontrager All Weather 28-622
tire radius = 14 mm, rim width = 13.4 mm

summary
part 1 of 2

Bontrager All Weather 28-622
tire radius = 14 mm, rim width = 13.4 mm

summary
part 2 of 2

Cheng Shin Classic Zeppelin 50-622
tire radius = 22.1 mm, rim width = 18.7 mm
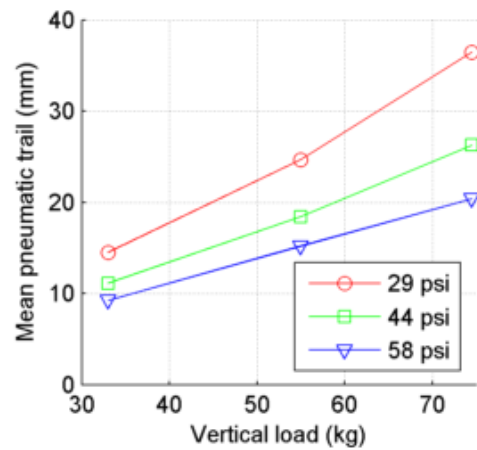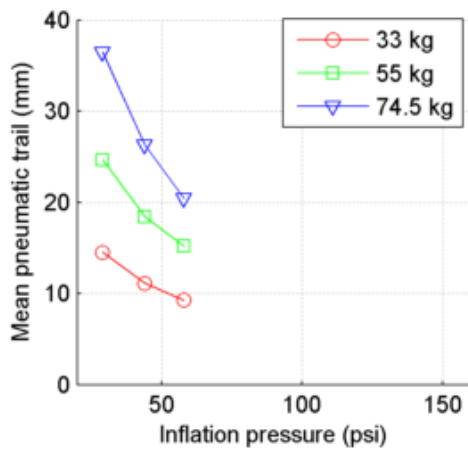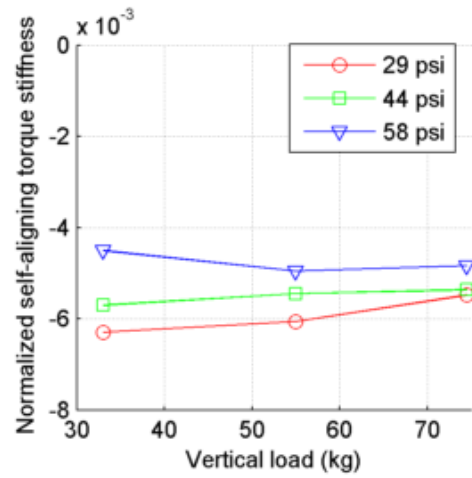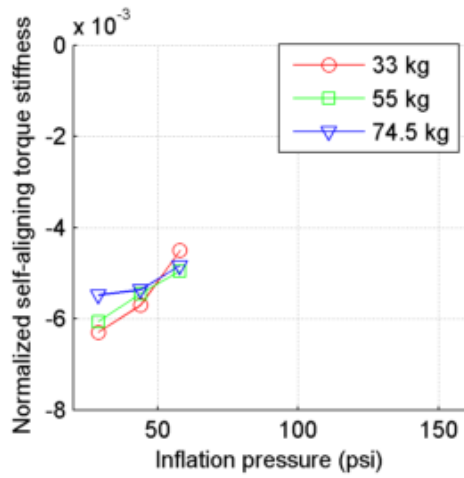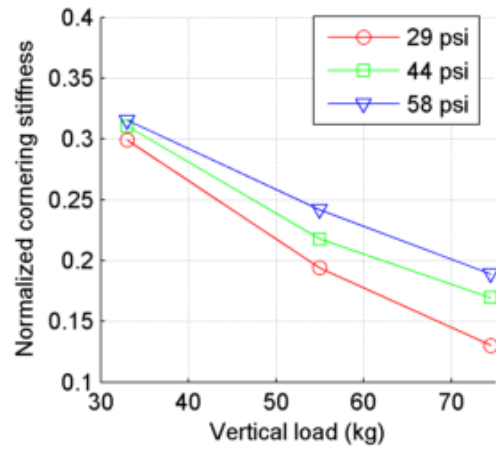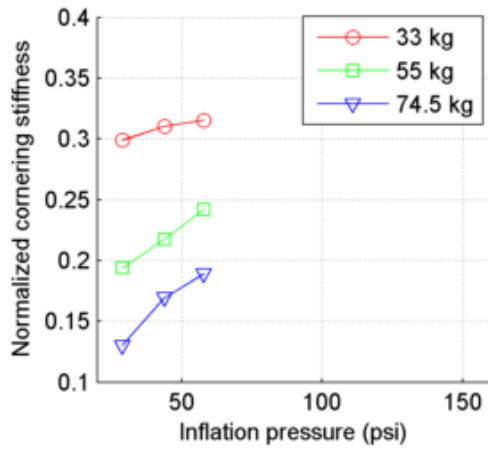
summary
part 1 of 2

Cheng Shin Classic Zeppelin 50-622
tire radius = 22.1 mm, rim width = 18.7 mm
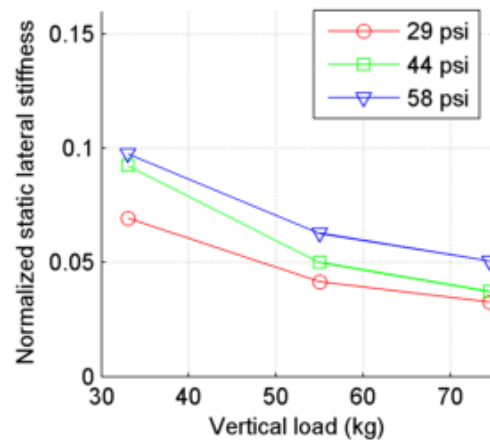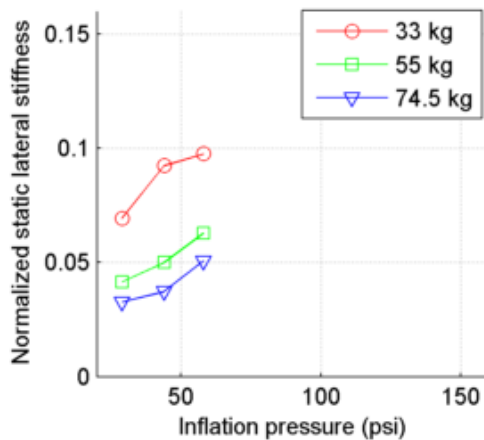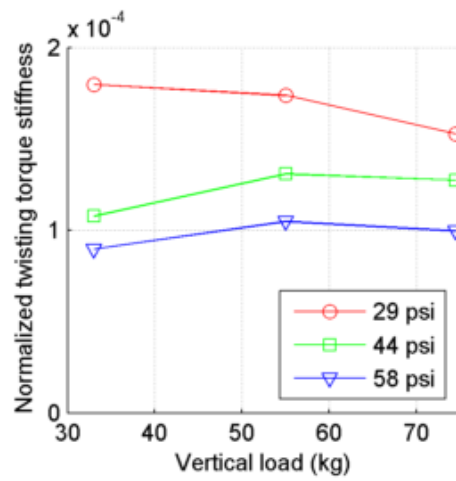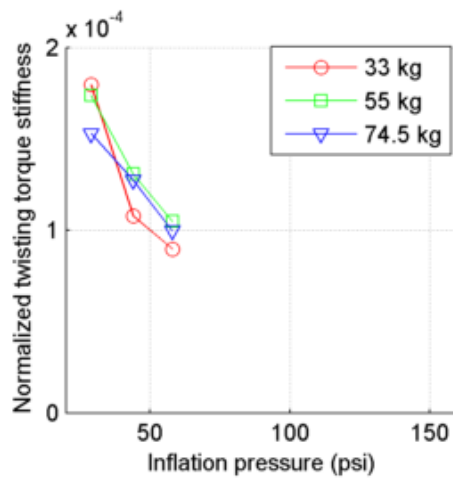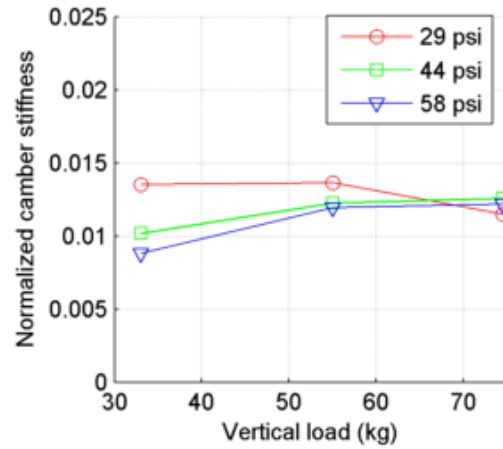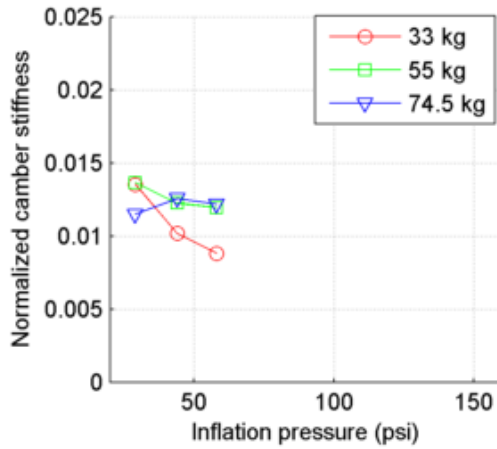
summary
part 2 of 2

Continental Top Contact Winter 37-622
tire radius = 15.6 mm, rim width = 18.7 mm
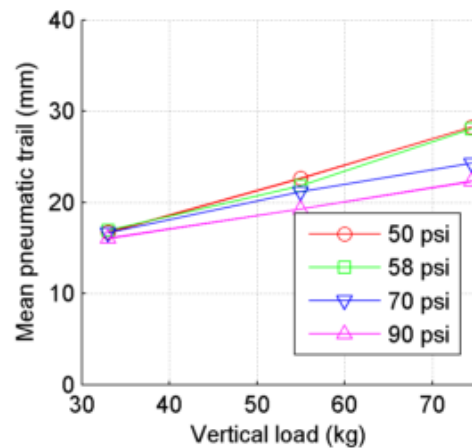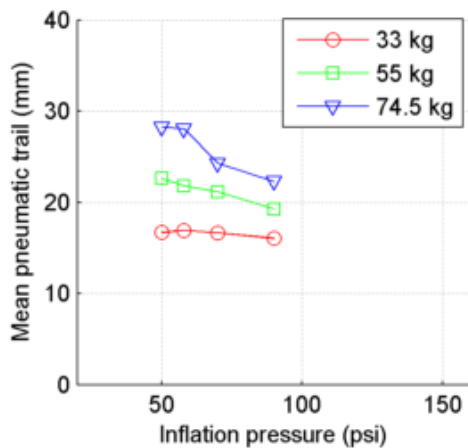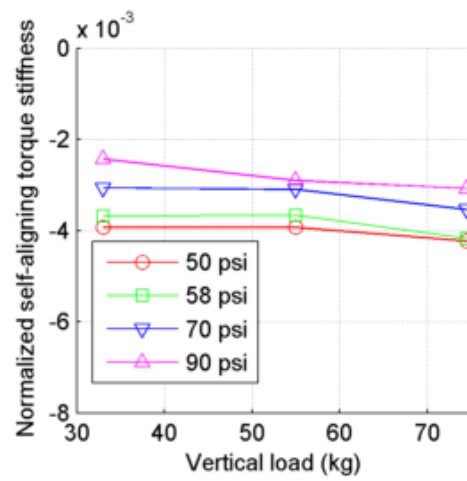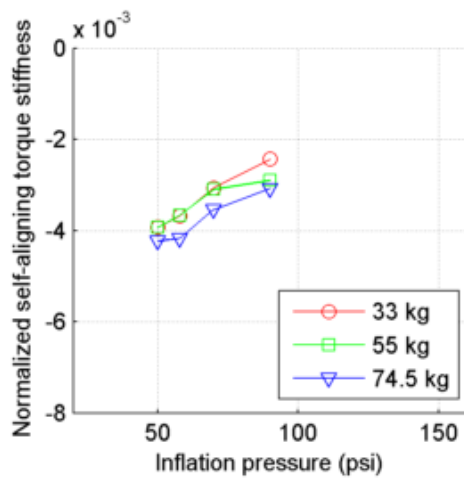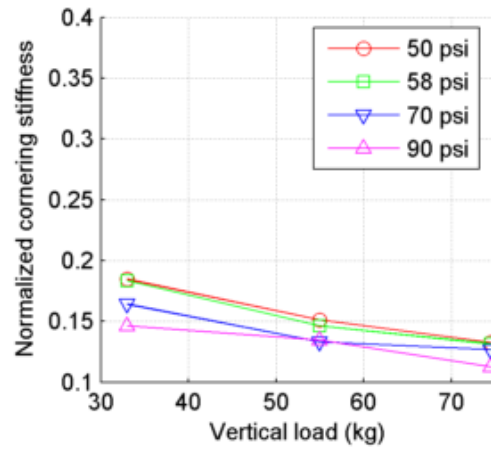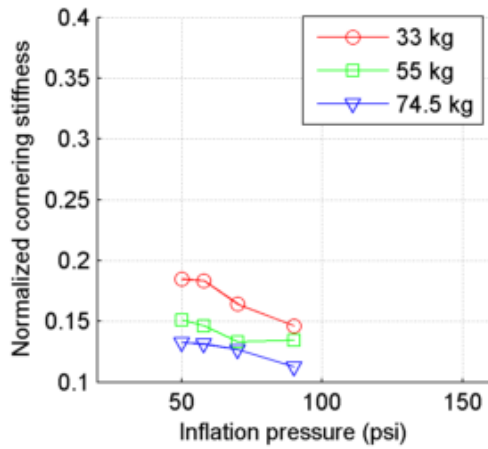
summary
part 1 of 2

Continental Top Contact Winter 37-622
tire radius = 15.6 mm, rim width = 18.7 mm
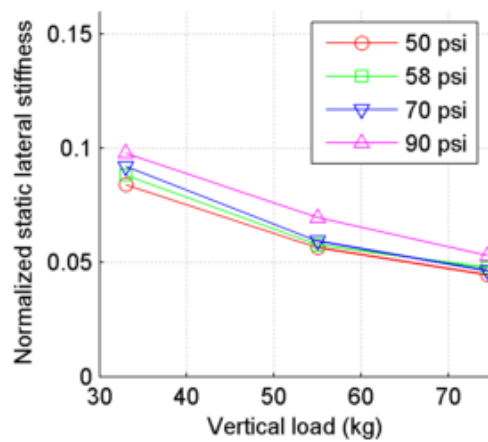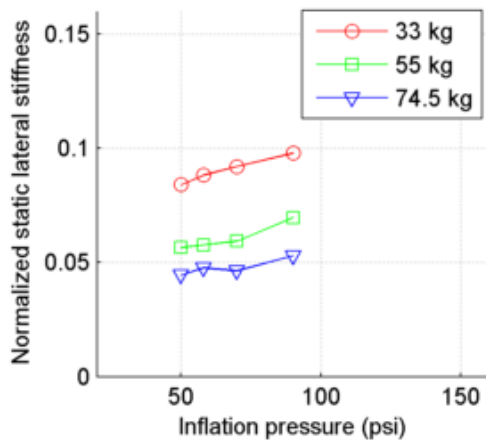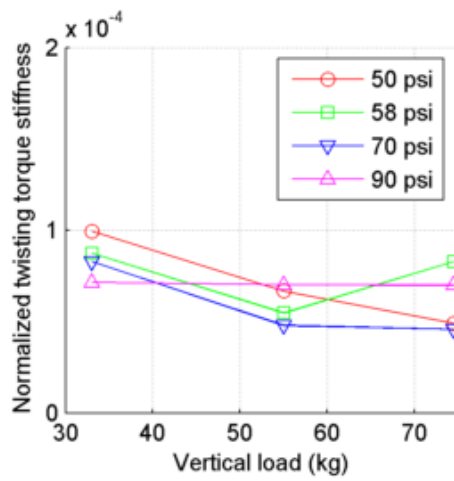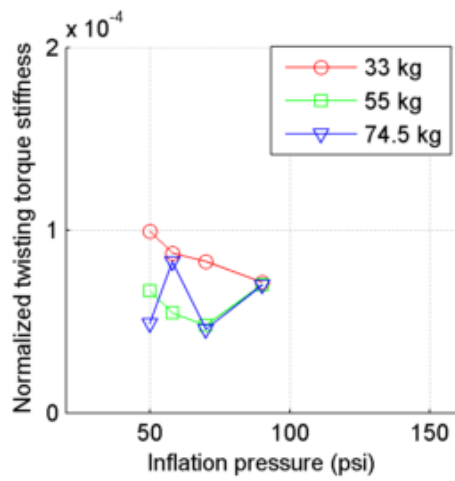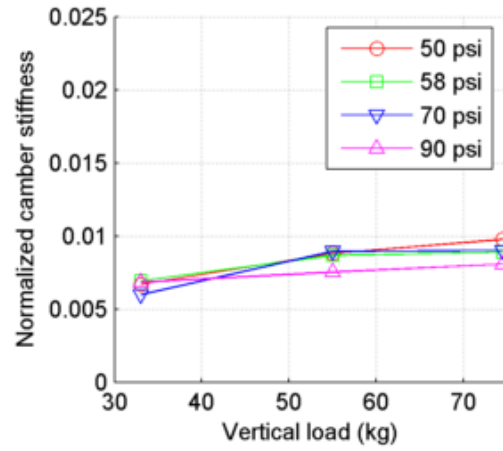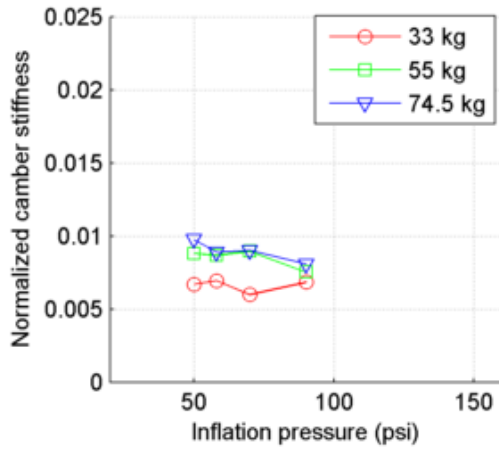
summary
part 2 of 2

Maxis Radial 22-622
tire radius = 11 mm, rim width = 13.4 mm
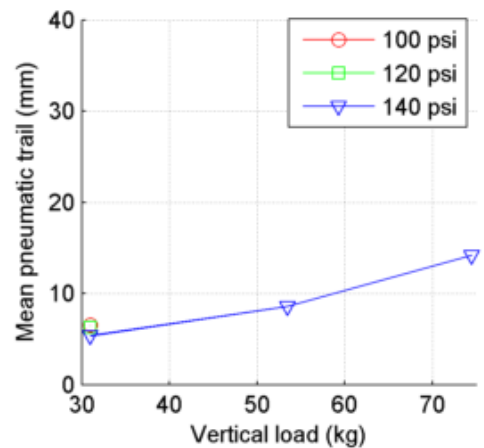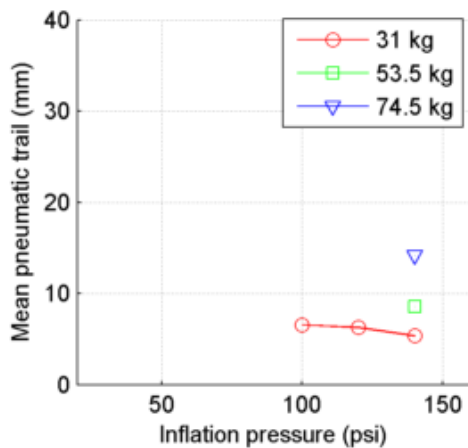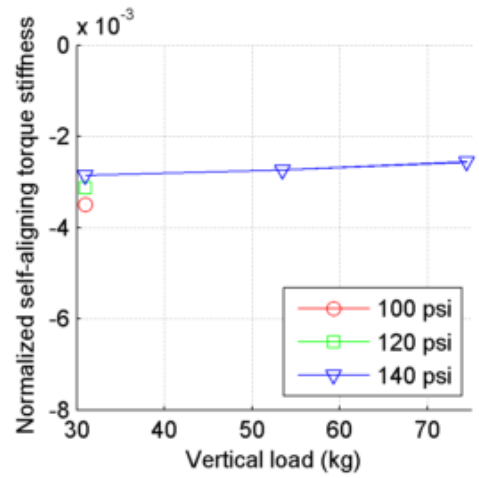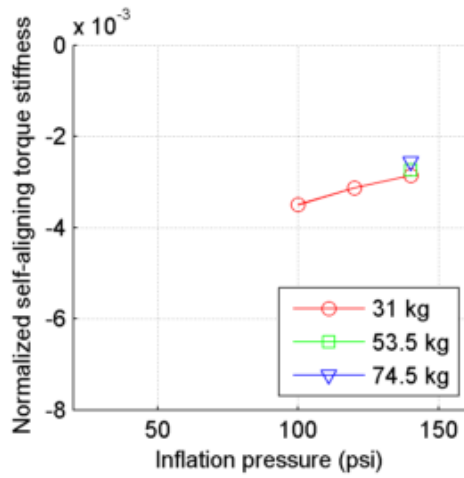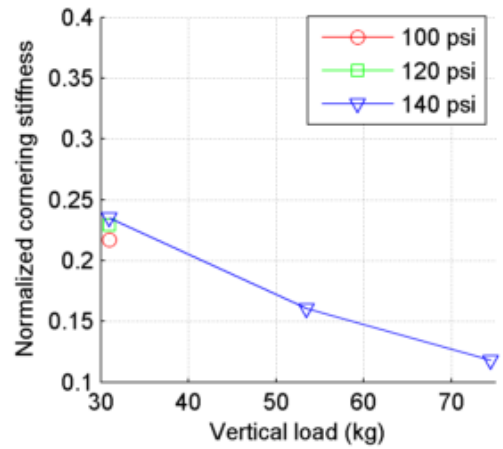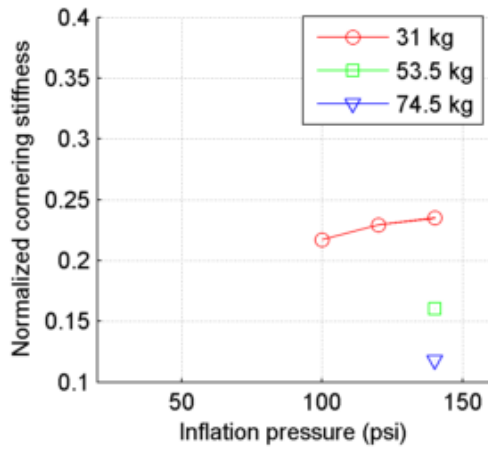
summary
part 1 of 2

Maxis Radial 22-622
tire radius = 11 mm, rim width = 13.4 mm
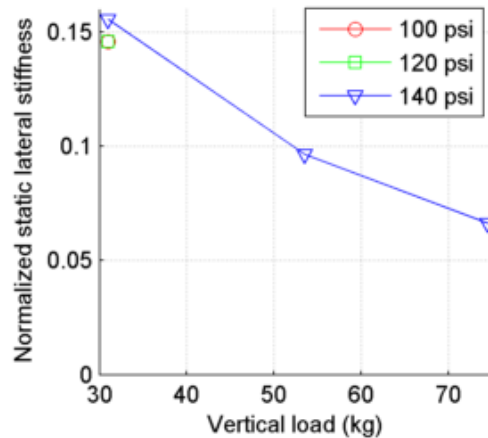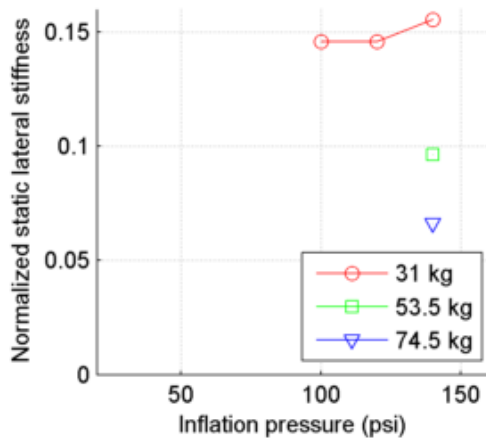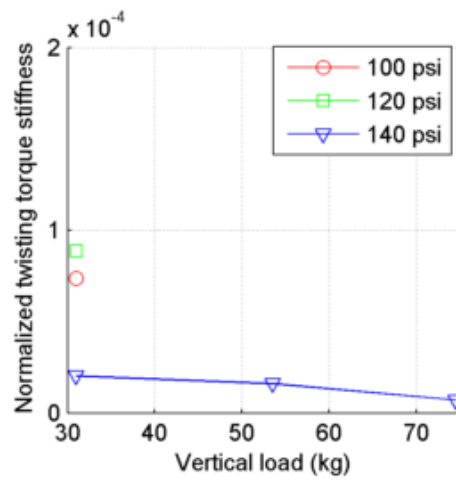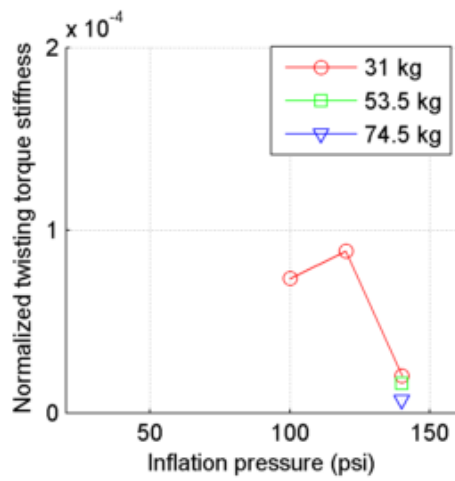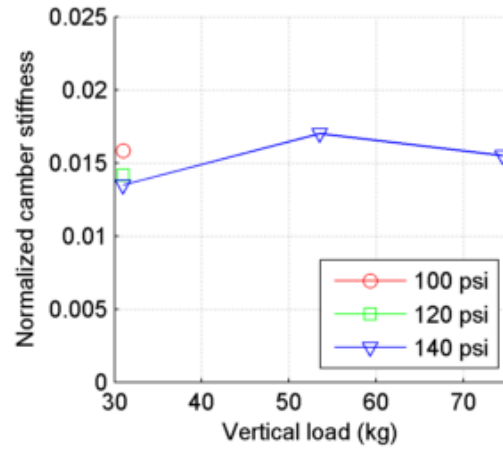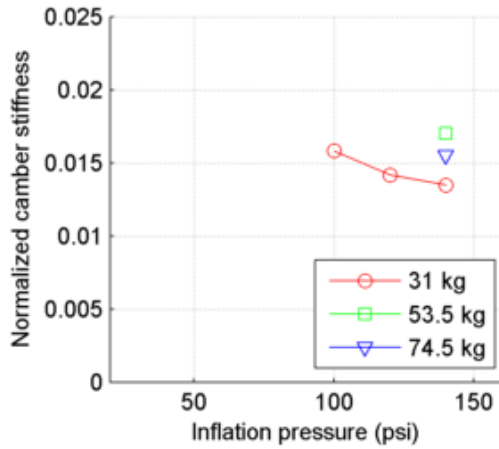
summary
part 2 of 2

Michelin Dynamic xx-622
tire radius = 12.5 mm, rim width = 13.4 mm
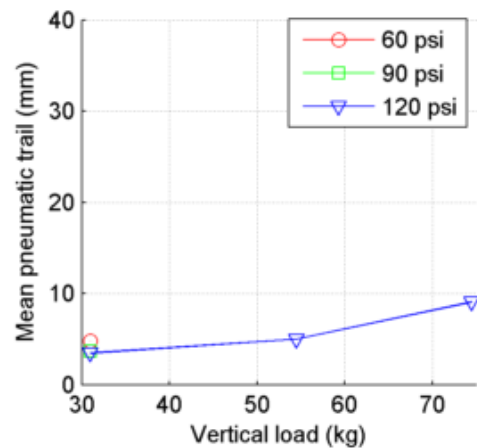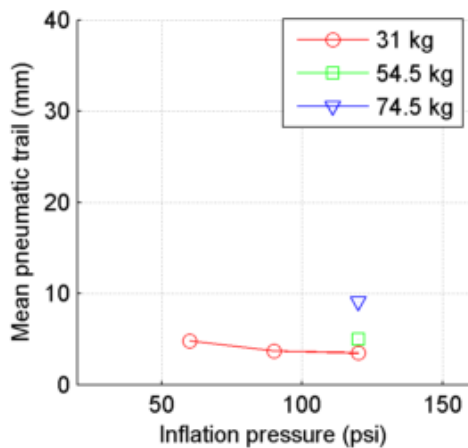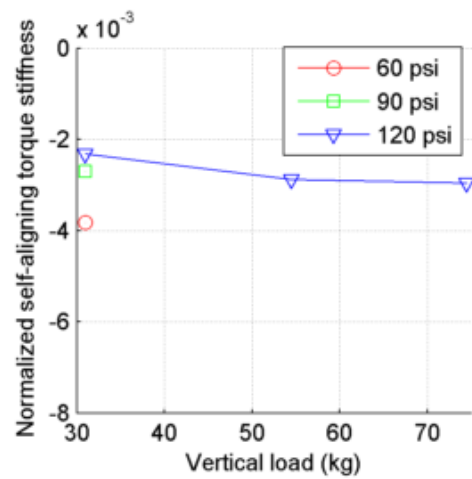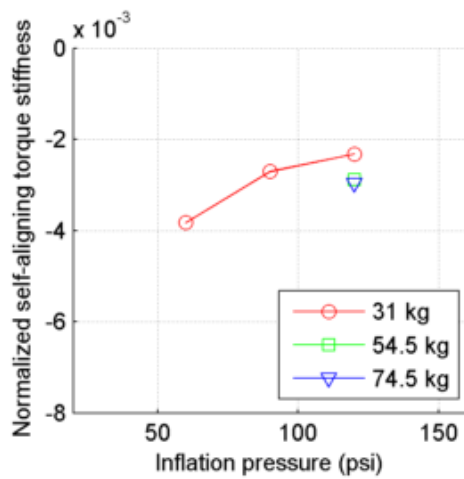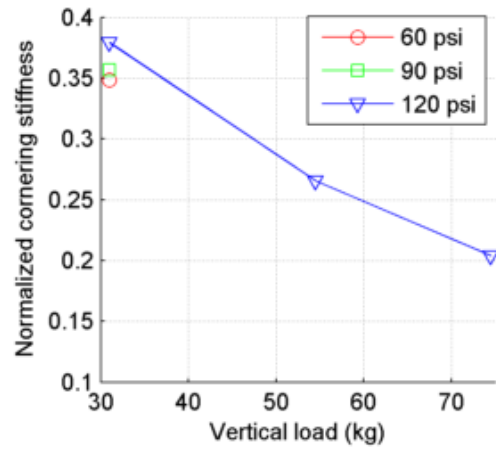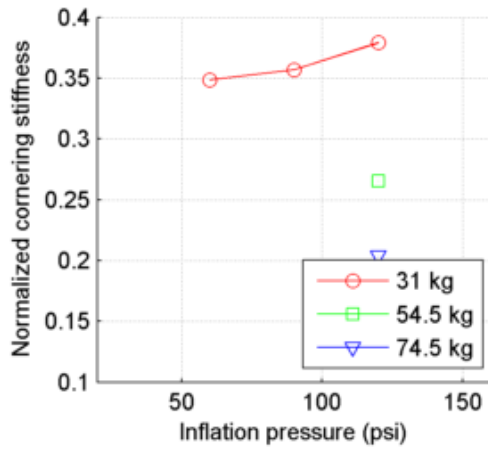
summary
part 1 of 2

Michelin Dynamic xx-622
tire radius = 12.5 mm, rim width = 13.4 mm
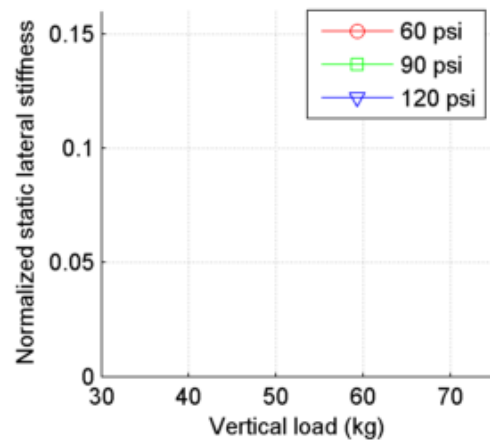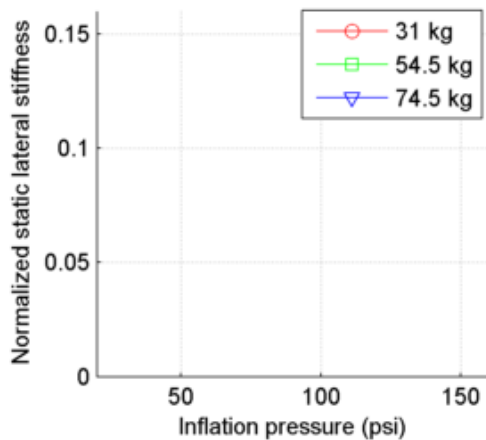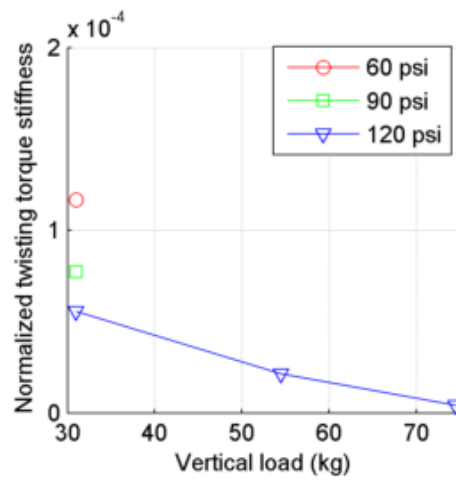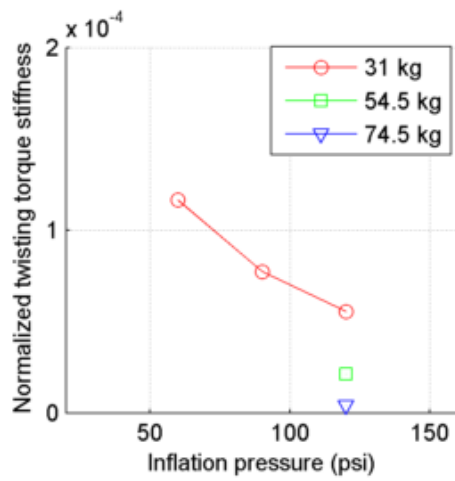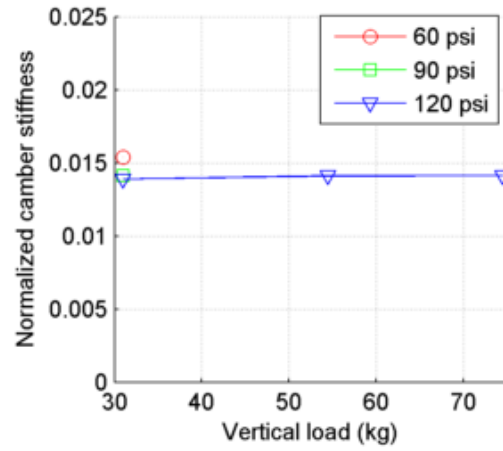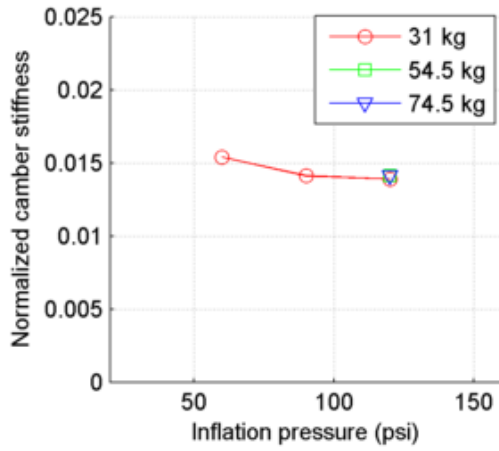
summary
part 2 of 2

Schwalbe Big Apple 55-622
tire radius = 23.5 mm, rim width = 18.7 mm
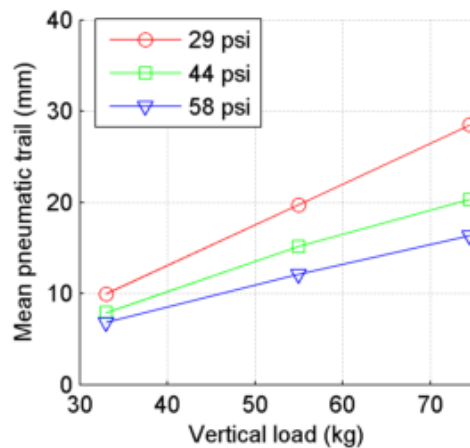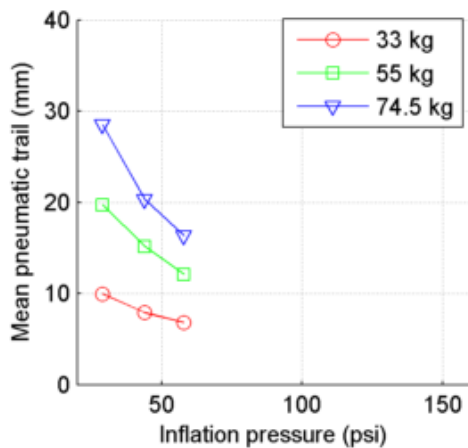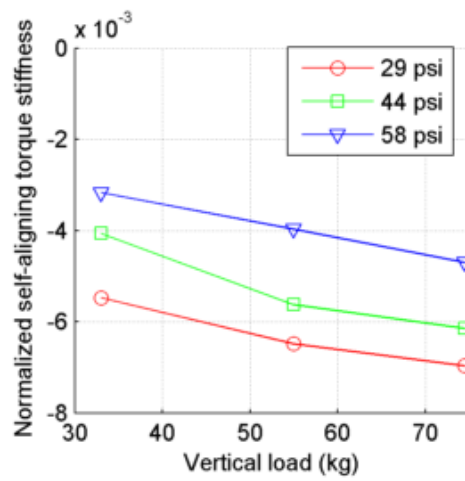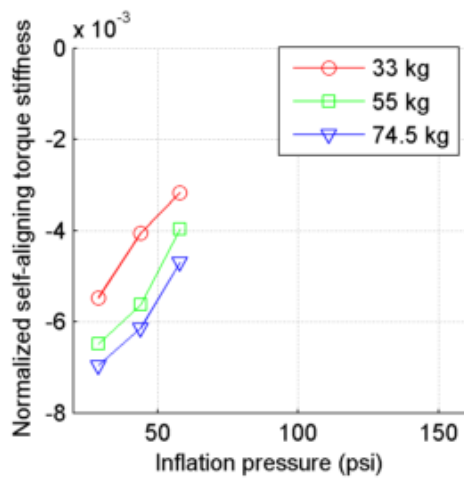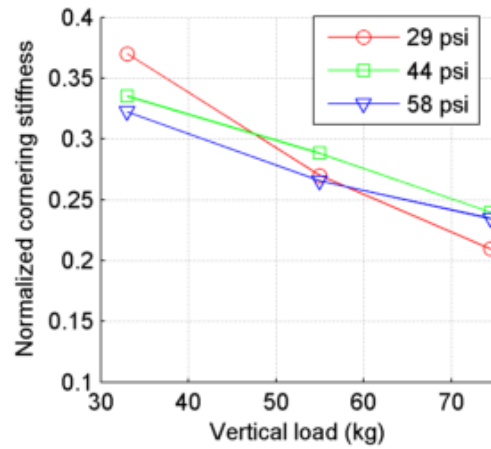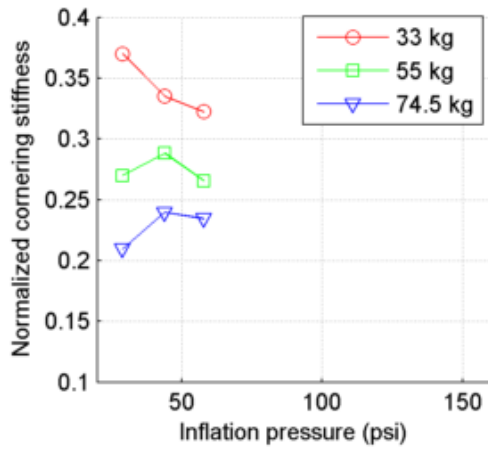
summary
part 1 of 2

Schwalbe Big Apple 55-622
tire radius = 23.5 mm, rim width = 18.7 mm
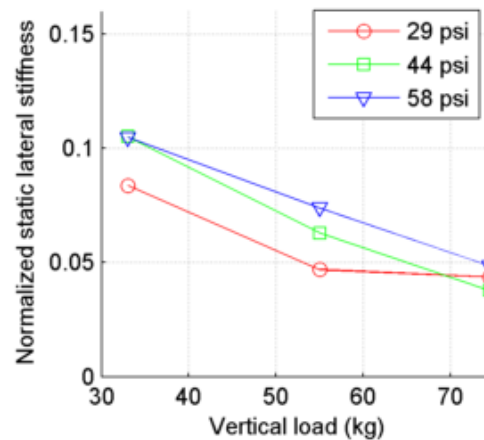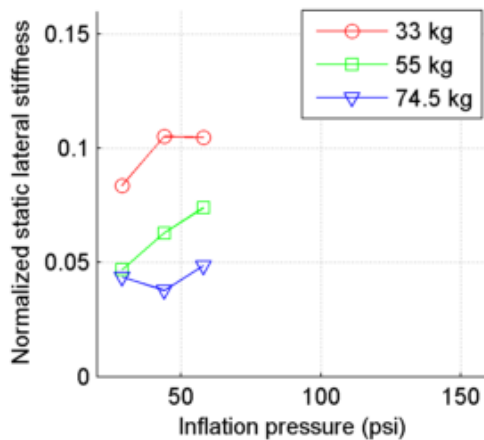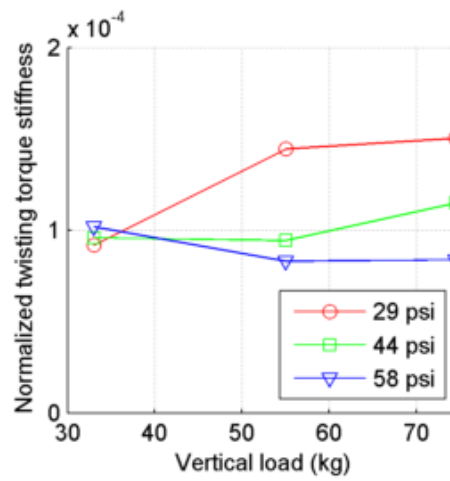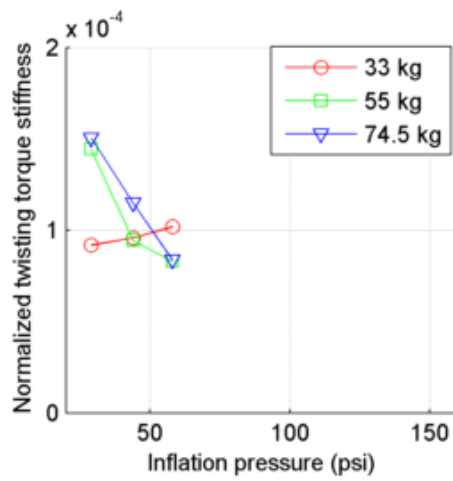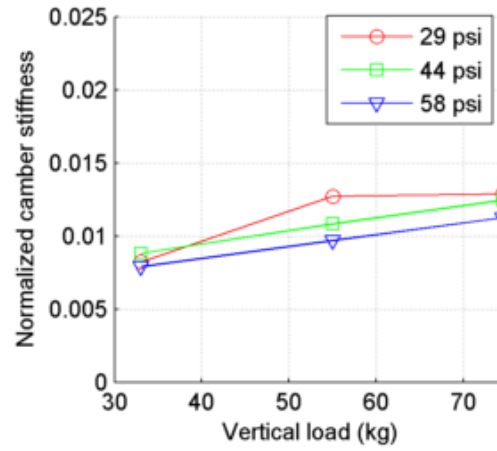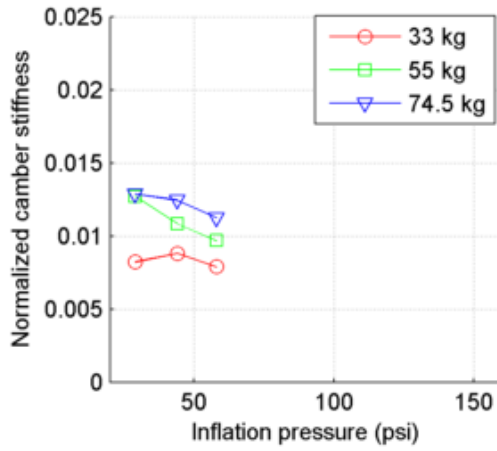
summary
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Schwalbe Kojak 35-622
tire radius = 15.7 mm, rim width = 18.7 mm
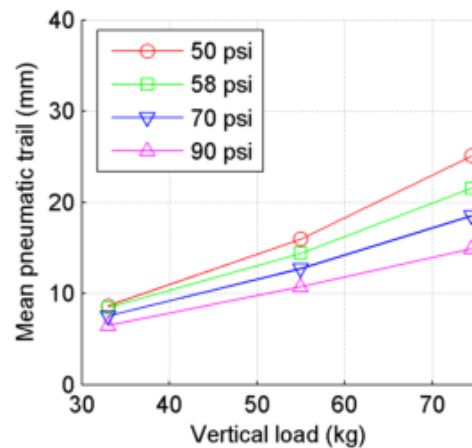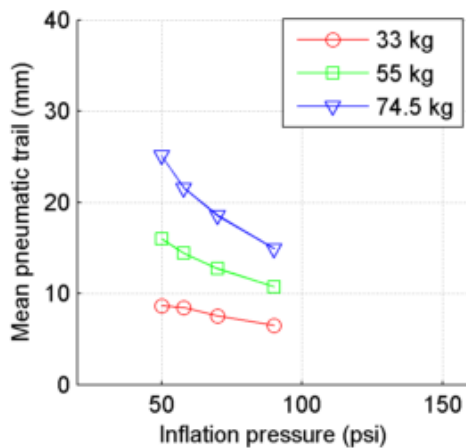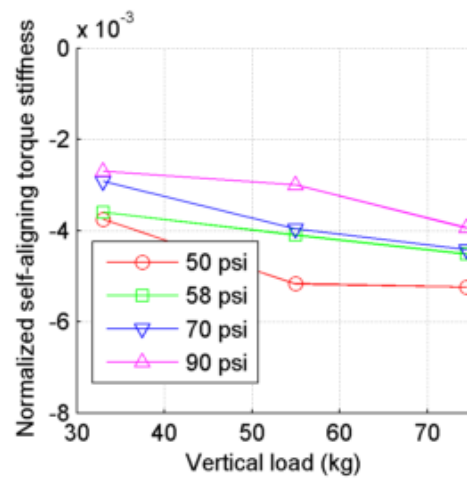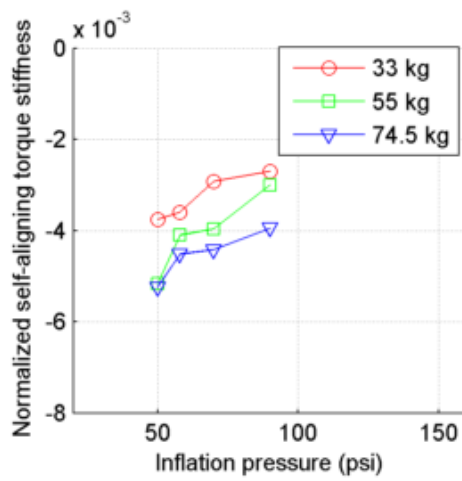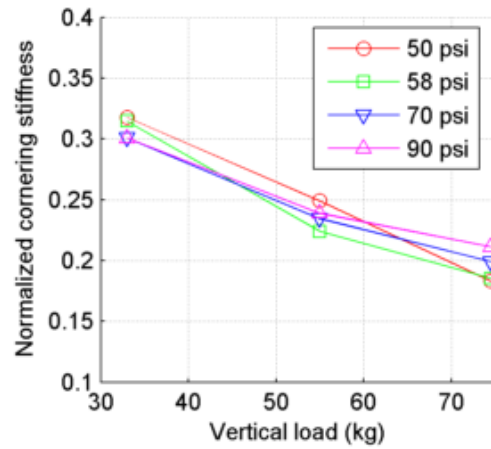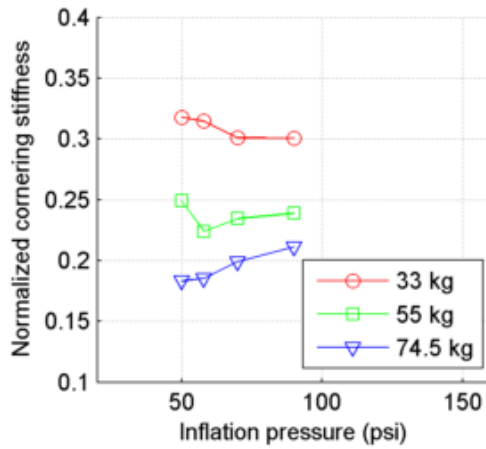
summary
part 1 of 2

Schwalbe Kojak 35-622
tire radius = 15.7 mm, rim width = 18.7 mm
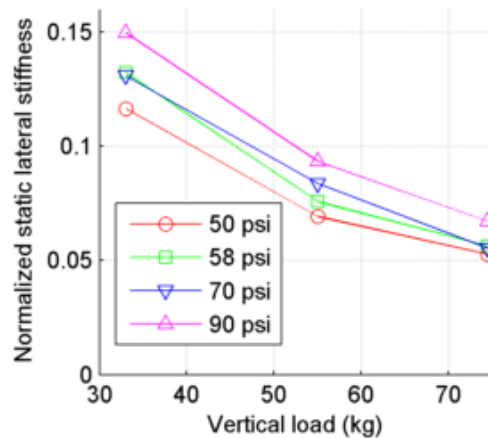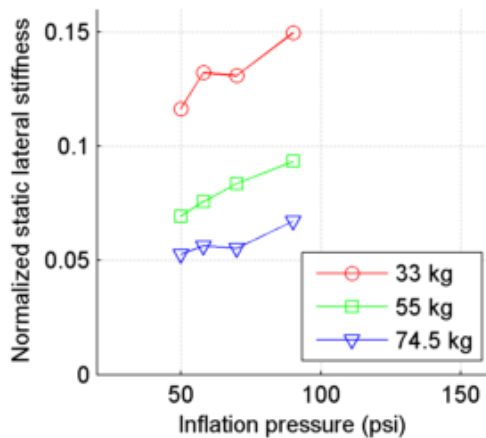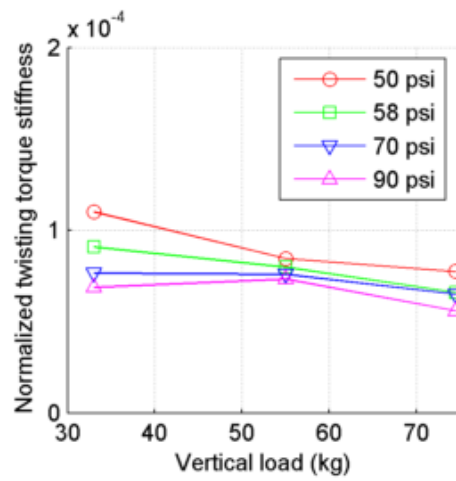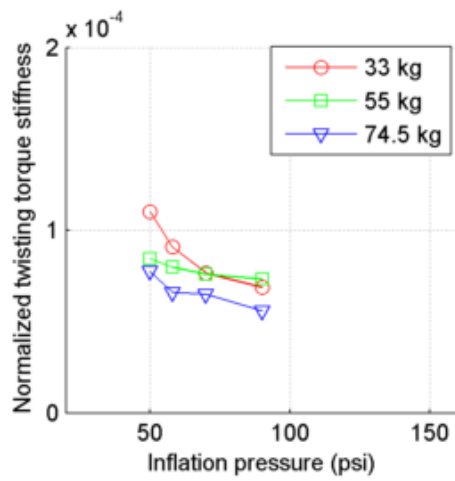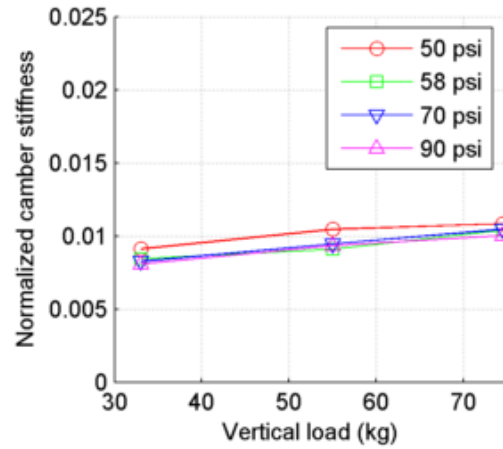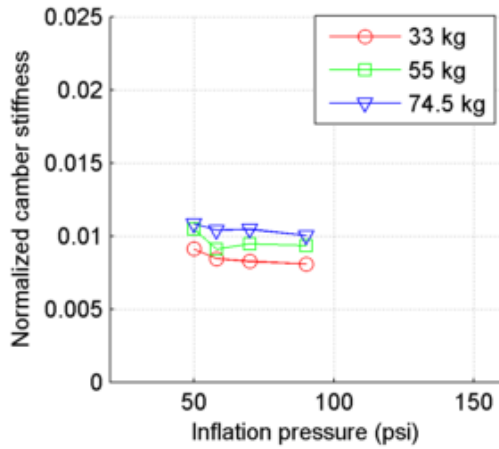
summary
part 2 of 2

Schwalbe Marathon Plus 37-622
tire radius = 14.9 mm, rim width = 18.7 mm
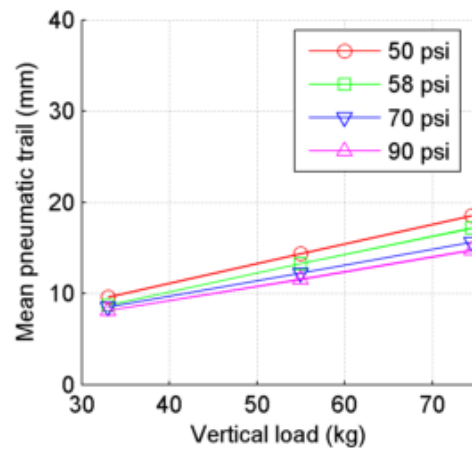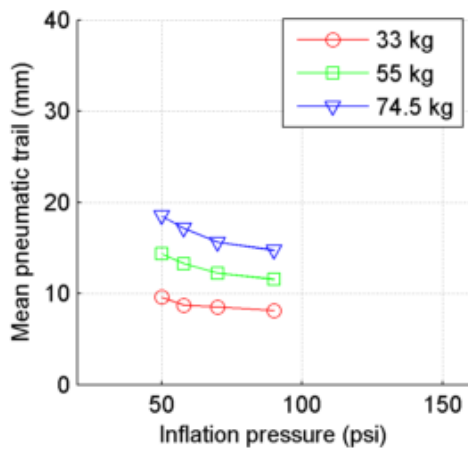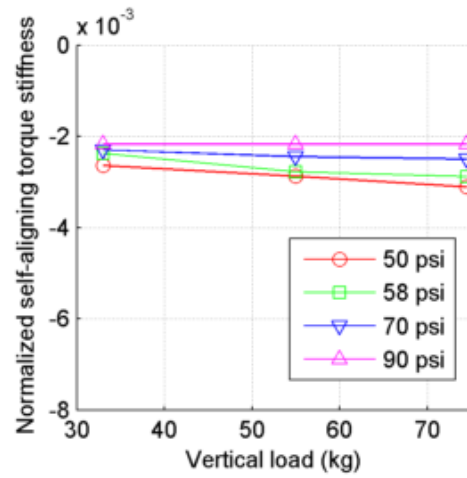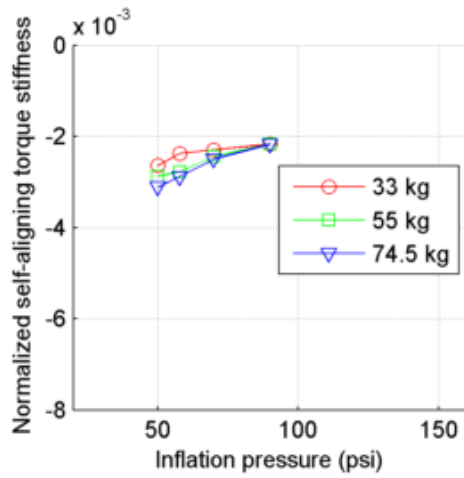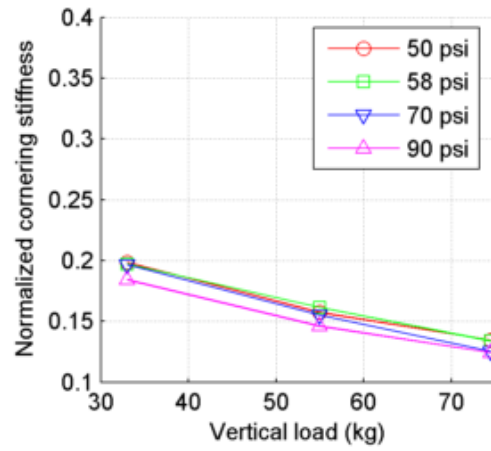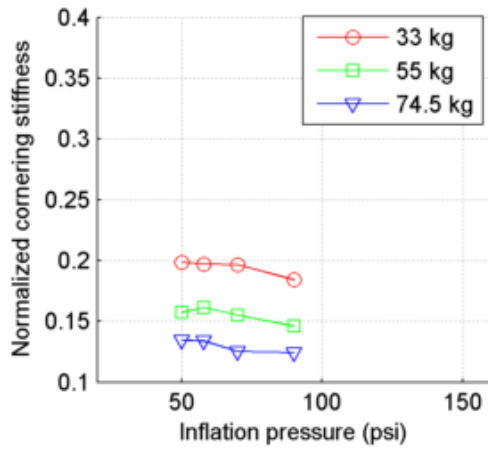
summary
part 1 of 2

Schwalbe Marathon Plus 37-622
tire radius = 14.9 mm, rim width = 18.7 mm
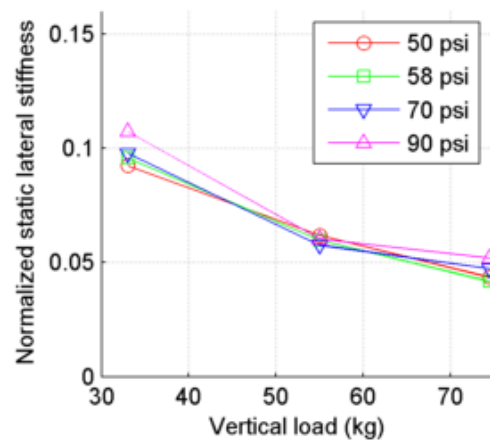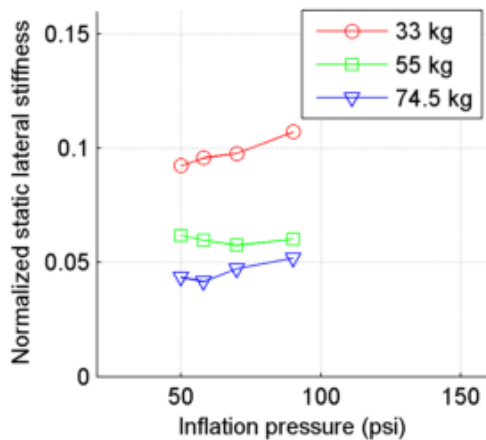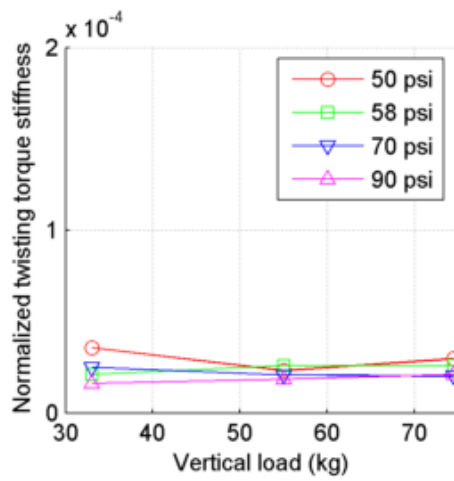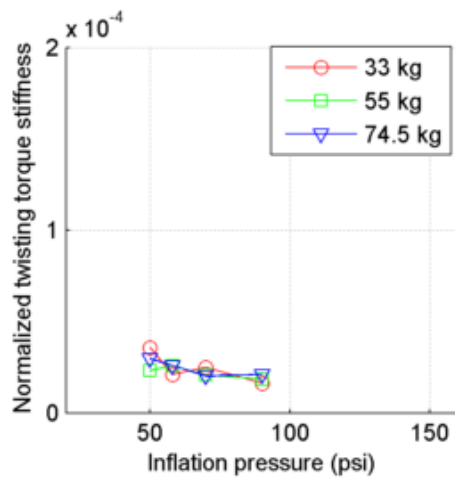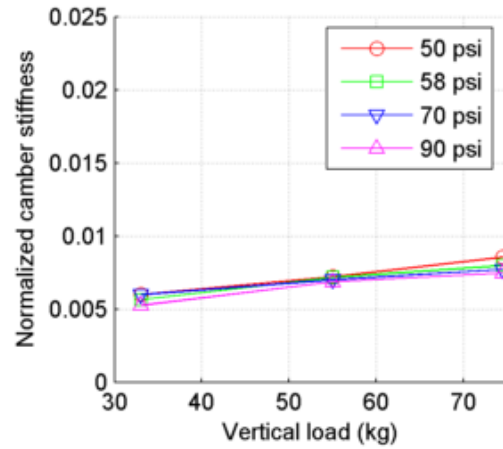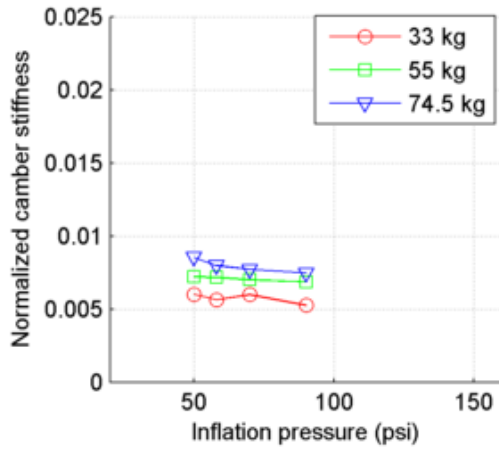
summary
part 2 of 2

Vittoria Randonneur Hyper 37-622
tire radius = 17.5 mm, rim width = 18.7 mm

summary
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Vittoria Randonneur Hyper 37-622
tire radius = 17.5 mm, rim width = 18.7 mm

summary
part 2 of 2

Vredestein Fortezza DuoComp 23-622
tire radius = 11.5 mm, rim width = 13.4 mm
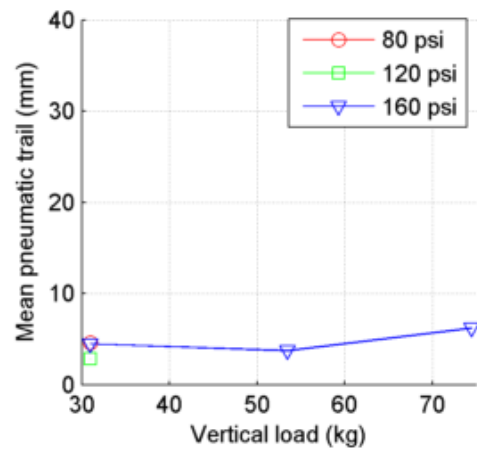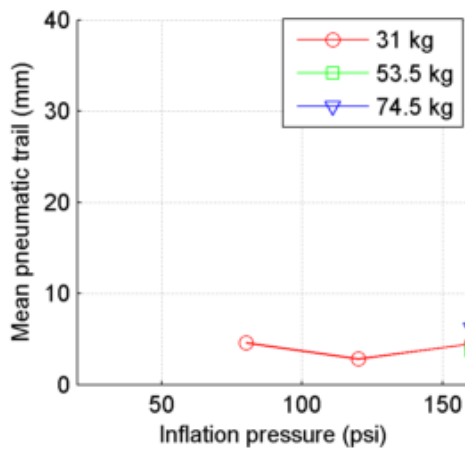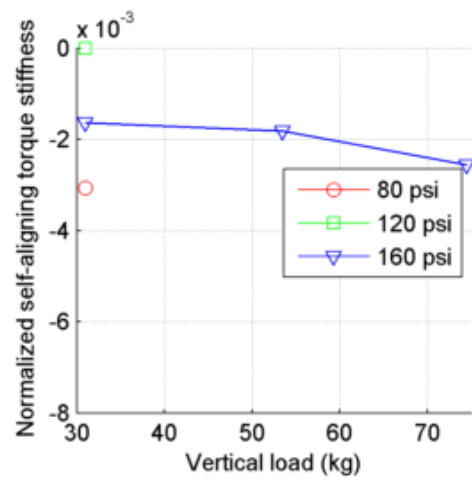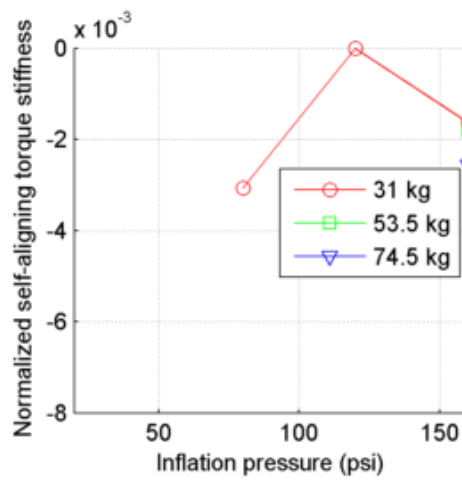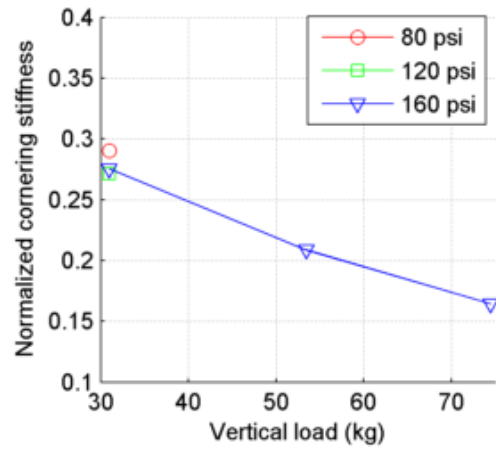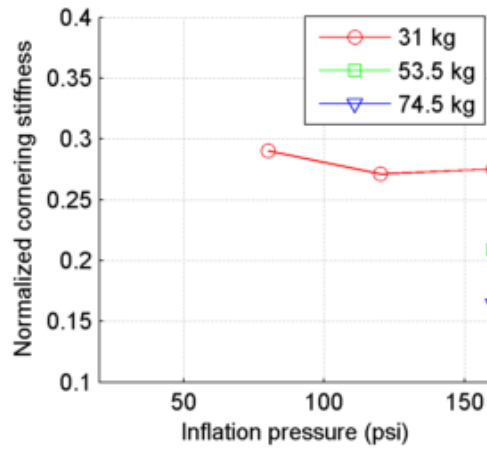
summary
part 1 of 2

Vredestein Fortezza DuoComp 23-622
tire radius = 11.5 mm, rim width = 13.4 mm
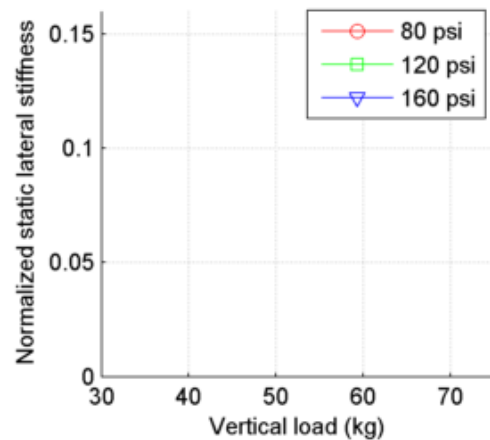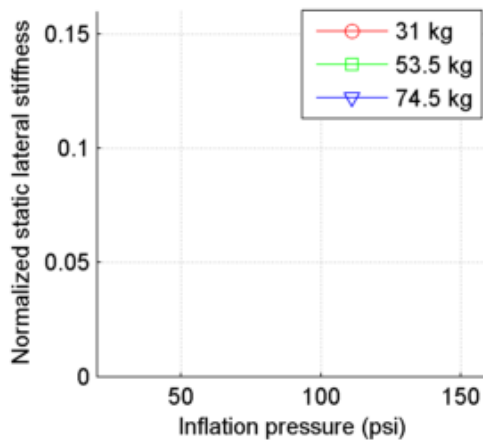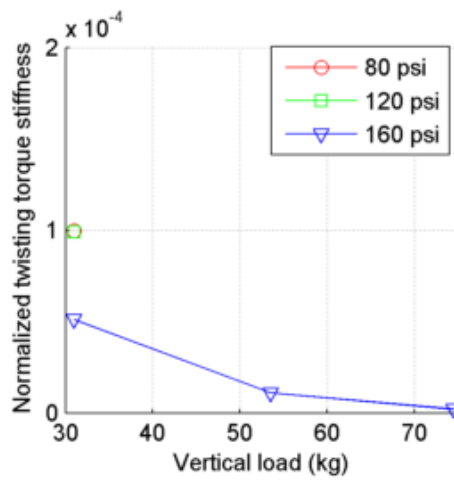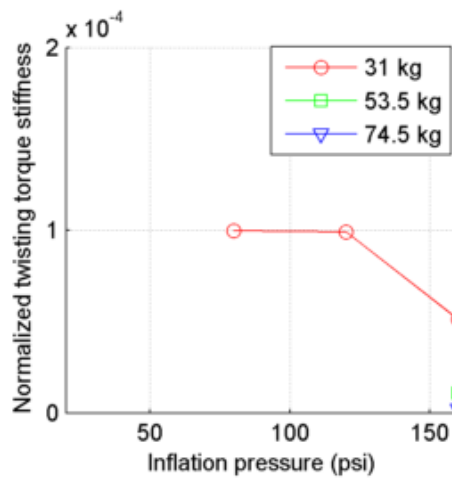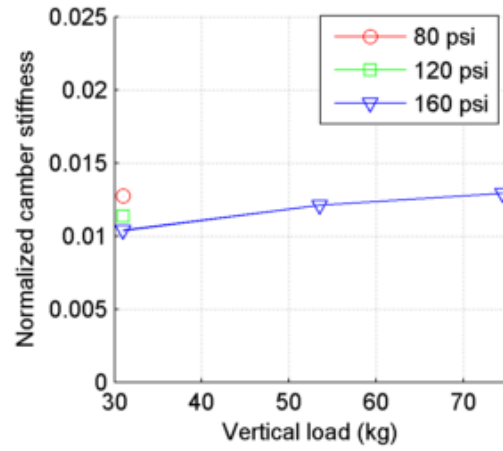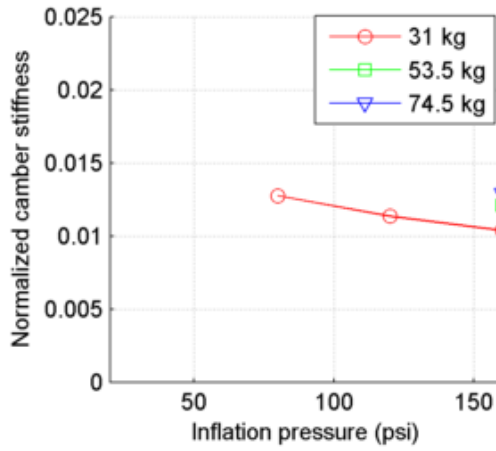
summary
part 2 of 2

Vredestein Fortezza TriComp 23-622
tire radius = 11.5 mm, rim width = 13.4 mm
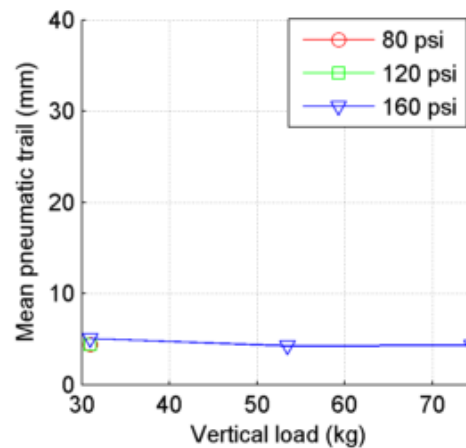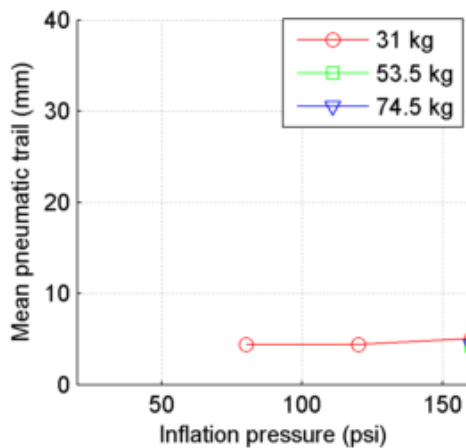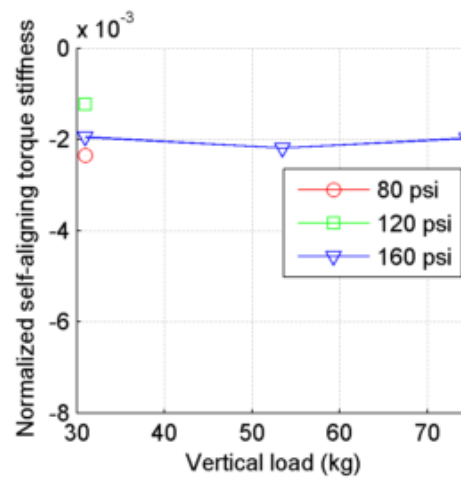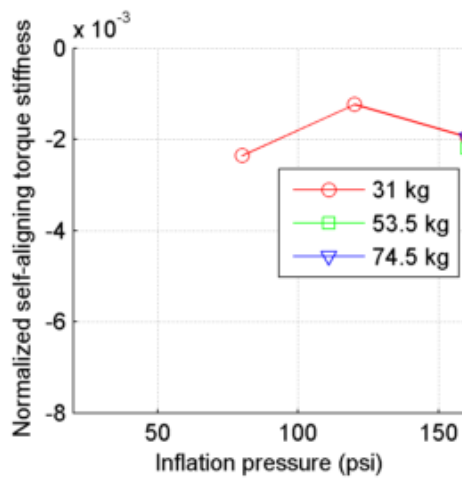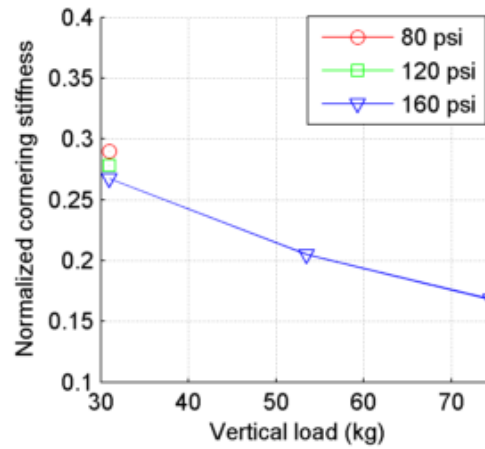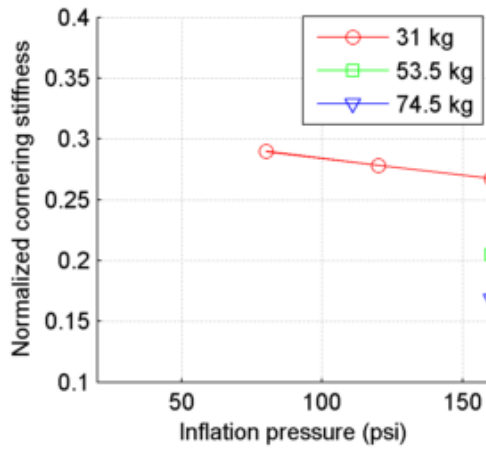
summary
part 1 of 2

Vredestein Fortezza TriComp 23-622
tire radius = 11.5 mm, rim width = 13.4 mm

summary
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Vredestein Perfect Tour 37-622
tire radius = 16.8 mm, rim width = 18.7 mm
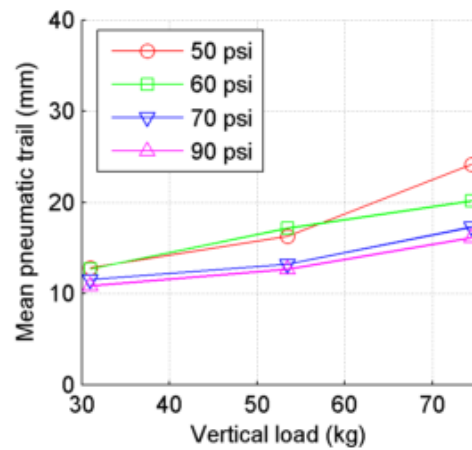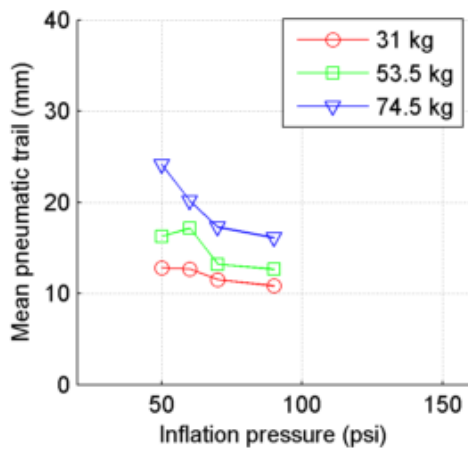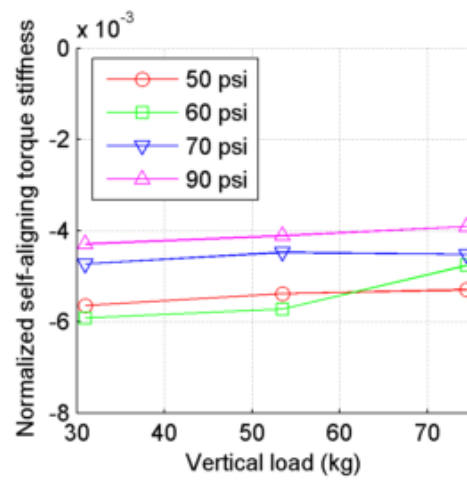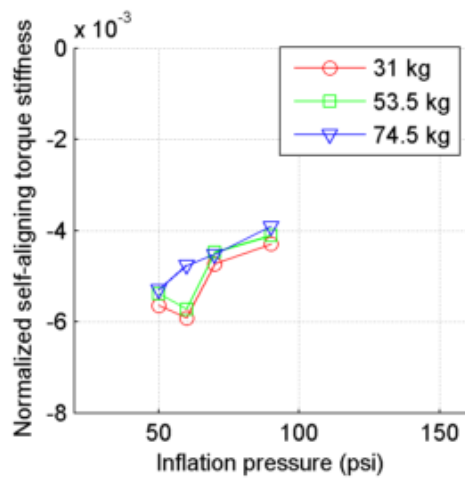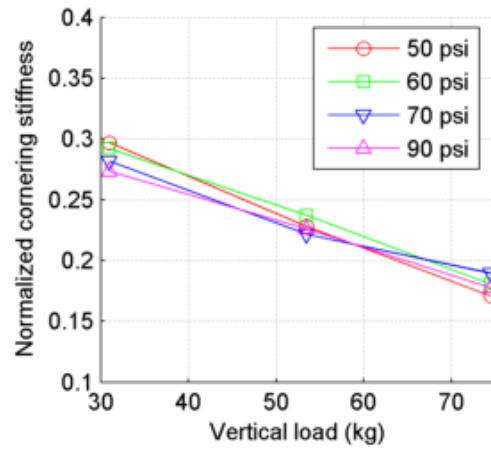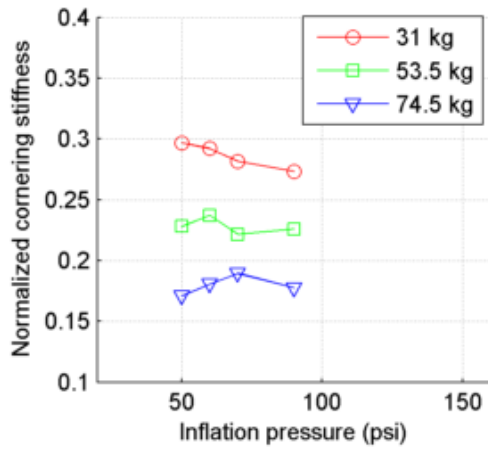
summary
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Vredestein Perfect Tour 37-622
tire radius = 16.8 mm, rim width = 18.7 mm

summary
part 2 of 2

Vredestein Fortezza TriComp 23-622   Vredestein Perfect Tour 37-622
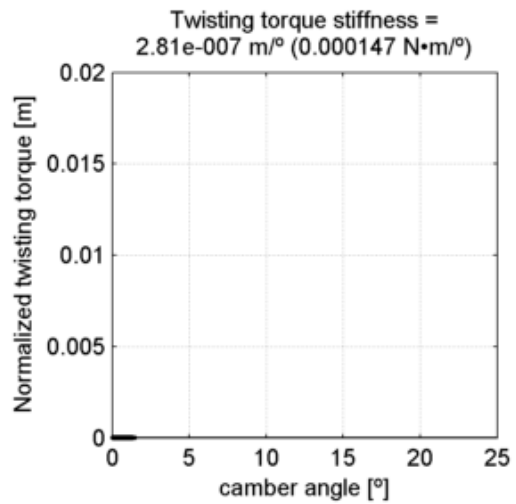
```matlab
% *************************************************************************
% Per Rotta's paper, positive horizontal deflections move the rim towards
% the left (so tire moves relatively towards the right), and positive
% camber angles tilt the rim towards the right.
% *************************************************************************
function Rotta_model
    try
    tstart = tic;            % keep track of time
    clc; format compact      % establish desired initial environment
    disp('Rotta model.')
    rc = Rotta_constants;     % define all necessary constants

        % ****************************************************************
        % Specify what to do
        % ****************************************************************
        rip.plot_target_vert_force_search_progess = 0;
        rip.plot_single_tire_config = 1; % plot one set of parameters.
        rip.plot_wheel_horz_d = 0;
        rip.run_silent = 0;       % display status after various steps
        rip.post_patch = 1;       % do calculate tire shape beyond contact patch
        rip.adjust_wheel_horz_d = 0;
        read_tire_specs = 0;      % read from a file of tire specs, or not
        tire_specs_filename = 'Rotta_tire_specs_April_7_format.csv';

        % ****************************************************************
        % Specify the number and span of cross sections to define half tire
        % ****************************************************************
        rip.num_slices = 201;               % number of radial slices to evaluate
        % finding exact end of contact patch and having horz_d -> 0 perfectly
        % at the end of non-zero camber contact patch is sensitive to this #
        rip.last_slice_angle = 15*pi/180; % how far around tire to go

        if read_tire_specs == 0

            % ****************************************************************
            % Specify slip and camber angles
            % ****************************************************************
            rip.slip_angle = 0.0*pi/180;       % angle contact patch center line angle
            rip.camber_angle = 45*pi/180;   % positive = rim tilts to right
            % positive = contact patch pushed to right, rim to left. Contact patch
            % front (top of plot) moves right, rear (bottom of plot) moves left

            % ****************************************************************
            % Specify the rates at which wheel horizontal displacement decays
            % once sliding or tire is not longer in contact with ground
            % ****************************************************************
            rip.relaxation_rate_out(2) = 0.993; % before touches ground
            rip.relaxation_rate_in(2) = 0.993; % just after touching ground
            rip.relaxation_rate_in(1) = 0.993; % just before leaving ground
            rip.relaxation_rate_out(1) = 0.993; % after leaving ground
            rip.relaxation_rate = 0.993;

            rip.friction_coefficient = inf; % inf for no slip
%           rip.friction_coefficient = 1;   % 1 for non-skid tape

            rip.drum_radius = 0;             % flat track at UWM (note use 0 for flat!
%           rip.drum_radius = 2500/2;        % 2.5 meter diameter drum at TU Delft

            rip.disk_radius = 0;         % straight track at UWM (note use of 0 for straight!
%           rip.disk_radius = 3*1000/2;   % 3.0 meter diameter disk at Padua
```
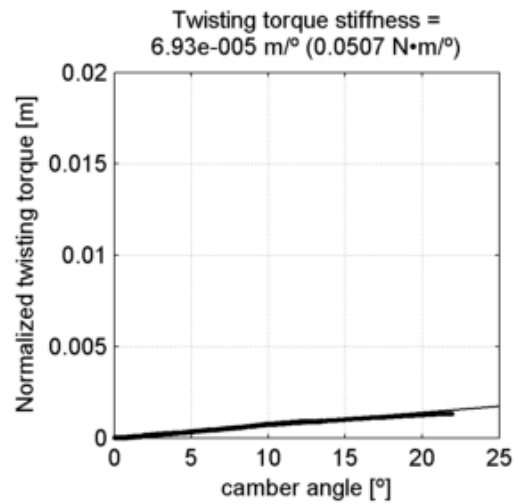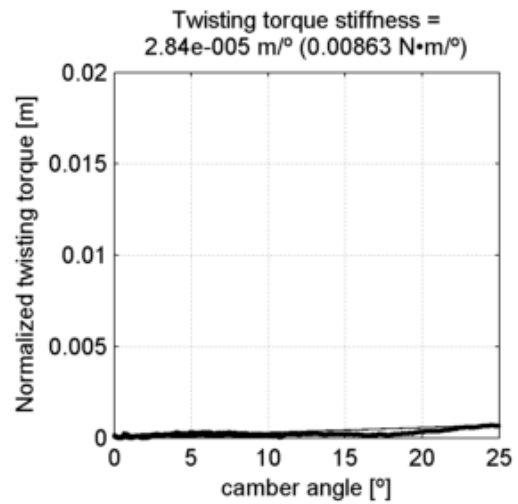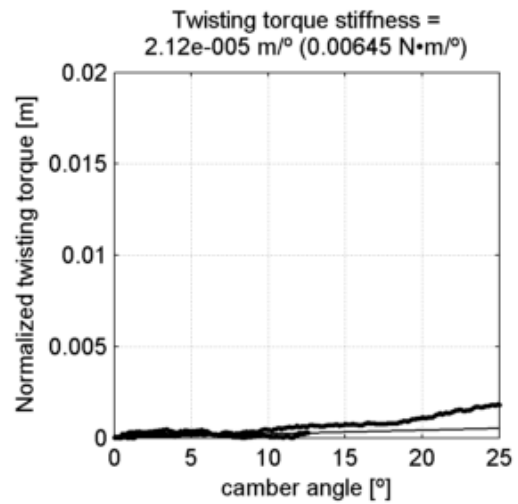
```matlab
        % *******************************************************************
        % Specify the maximum vertical and horizontal deflections
        % *******************************************************************
        rip.wheel_vert_d = 5; % positive = rim sags towards ground
        % wheel_vert_d ignored if target_vert_force ~= 0

        rip.wheel_horz_d = 0; %
        % Positive values push contact patch to the right, or rim to left, and
        % positive values reduce camber thrust deficiency.
        % *******************************************************************
        % define wheel and tire parameters
        % *******************************************************************
        rip.rim_diameter = 622;         % 700C has "bead seat diameter" = 622
        rip.rim_width = 18.7;           % interior width from tire bead to bead
        rip.tire_size = 37;
        rip.tire_width = 0;%128.2;          % estimate this below
        rip.inf_press = 4 * rc.MPa_per_bar; % inflation pressue
        rip.target_vert_force = 400; % * rc.N_per_kg;
%         rip.target_vert_force = 0;

        % *******************************************************************
        % try to make a smart first guess
        % *******************************************************************
        rip.initial_sinkage_guess = 0.1 * rip.target_vert_force / rc.g;

        rows = 1;                       % just do one, as specified above
    else % read_tire_specs == 1

        fprintf('Reading tire specs from %s\n', tire_specs_filename);
        % *******************************************************************
        % Read in a comma-delimited file of tire specifications, skipping
        %   first row of text column headers
        % *******************************************************************
        tire_specs = csvread(tire_specs_filename, 1, 0);
        [rows, cols] = size(tire_specs);
    end

    % *******************************************************************
    % Default to going as fast as possible when doing more than one
    % *******************************************************************
    if rows > 1
        rip.plot_target_vert_force_search_progess = 0;
        rip.plot_single_tire_config = 0;
        rip.plot_wheel_horz_d = 0;
        rip.run_silent = 1;
    end

    % *******************************************************************
    % Evaluate all the tire specifications requested
    % *******************************************************************
    tire_specs_evaluated = 0;
    for row = 1:rows

        if read_tire_specs == 1
            % *******************************************************
            % decide whether to skip or not, and let user know what's up
            % *******************************************************
            do_this_row = tire_specs(row,1);
            if read_tire_specs == 1 && do_this_row == 1
                fprintf(' Starting row %d of %d\n', row, rows)
            elseif read_tire_specs == 1 && do_this_row == 0
                fprintf(' Skipping row %d of %d\n', row, rows)
                continue
            end
            % *******************************************************
            % stuff tire specs from file into parameters expected below
            % *******************************************************
            rip.drum_radius = tire_specs(row,2);
            rip.disk_radius = tire_specs(row,3);
```

```matlab
        rip.rim_diameter = tire_specs(row,4);
        rip.rim_width = tire_specs(row,5);
        rip.tire_number = tire_specs(row,6);
        rip.tire_size = tire_specs(row,7);
        rip.tire_radius = tire_specs(row,8);
        rip.tire_width = tire_specs(row,9);
        rip.tire_thickness = tire_specs(row,10);
        rip.inf_press = tire_specs(row,11) * rc.MPa_per_psi;
        rip.target_vert_force = tire_specs(row,12) * rc.N_per_kg;
        rip.wheel_vert_d = tire_specs(row,13);
        rip.wheel_horz_d = tire_specs(row,14);
        rip.slip_angle = tire_specs(row,15)*pi/180;
        rip.camber_angle = tire_specs(row,16)*pi/180;
        rip.friction_coefficient = tire_specs(row, 17);
        rip.relaxation_rate_out(1) = tire_specs(row,18);
        rip.relaxation_rate_in(1) = tire_specs(row,19);
        rip.relaxation_rate_in(2) = tire_specs(row,20);
        rip.relaxation_rate_out(2) = tire_specs(row,21);
        rip.relaxation_rate = sum(tire_specs(row,18:21))/4; % average
        measured_lateral_stiffness = tire_specs(row,22);
        measured_cornering_stiffness = tire_specs(row,23);
        measured_camber_stiffness = tire_specs(row,24);
        if ~isnan(rip.target_vert_force) && rip.target_vert_force ~= 0
            rip.initial_sinkage_guess = 0.1 * rip.target_vert_force / rc.g;
        end
    end

    % ******************************************************************
    % Calculate additional wheel and tire dimensions
    % Note that:
    %   1. "rip.rim_width" is distance from one inner rim edge to other
    %   2. "rip.rim_angle" is angle swept by tire from rim to rim
    %   3. "rip.tire_size" is number printed on the size
    %   4. "rip.tire_radius" is actual radius of tire cicular arc
    %   5. "rip.tire_width" is cirumference of tire cross section
    % ******************************************************************
    if rip.tire_width == 0 % not yet specified? Estimate from tire size

        rip.tire_radius = rip.tire_size / 2;
        rip.rim_angle = 2 * asin(rip.rim_width / (2 * rip.tire_radius));
        rip.rim_height = rip.tire_radius * (1 + cos(rip.rim_angle / 2));
        rip.tire_width = rip.tire_radius * (2*pi - rip.rim_angle);

    else % rip.tire_width ~= 0, use it to calculate tire radius and "size"
        % no closed-form solution, have to iterate. At least let ML do it.
        rip.rim_angle = fzero(@(t) (rip.rim_width/rip.tire_width - ...
                        (2*sin(0.5*t))/(2*pi - t)), pi/4);
        rip.tire_radius = rip.rim_width / (2*sin(0.5*rip.rim_angle));
        rip.tire_size = rip.tire_radius * 2;
        rip.rim_angle = 2 * asin(rip.rim_width / (2 * rip.tire_radius));
        rip.rim_height = rip.tire_radius * (1 + sin((pi - rip.rim_angle) / 2));
    end

    % ******************************************************************
    % calculate all the rotta stuff
    % ******************************************************************
    [rop] = Rotta_core(rip);
    tire_specs_evaluated = tire_specs_evaluated + 1;

    % ******************************************************************
    % If test compatible with stiffnesses,
    %   calculate those values
    % ******************************************************************
    norm_lateral_stiffness = NaN; norm_cornering_stiffness = NaN; ...
        norm_camber_stiffness = NaN;
    if rip.wheel_horz_d ~= 0 && rip.camber_angle == 0 && rip.slip_angle == 0
        norm_lateral_stiffness = (rop.horz_force / rop.vert_force) / ...
                                 rip.wheel_horz_d;
    elseif rip.slip_angle ~= 0 && rip.camber_angle == 0
        norm_cornering_stiffness = (rop.horz_force / rop.vert_force) / ...
```

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                                    (rip.slip_angle*180/pi);
        elseif rip.slip_angle == 0 && rip.camber_angle ~= 0
            norm_camber_stiffness = (rop.horz_force / rop.vert_force) / ...
                                    (rip.camber_angle*180/pi);
        end

        if read_tire_specs == 1
            tire_specs(row, (cols + 1)) = rop.vert_force;
            tire_specs(row, (cols + 2)) = rop.wheel_vert_d;
            tire_specs(row, (cols + 3)) = 100*rop.wheel_vert_d/rip.rim_height;
            tire_specs(row, (cols + 4)) = rop.wheel_horz_d;
            tire_specs(row, (cols + 5)) = rop.cp_length;
            tire_specs(row, (cols + 6)) = rop.cp_width;
            tire_specs(row, (cols + 7)) = rop.cp_area;
            tire_specs(row, (cols + 8)) = rop.ellipse_cp_length;
            tire_specs(row, (cols + 9)) = rop.ellipse_cp_width;
            tire_specs(row, (cols + 10)) = rop.ellipse_cp_area;
            tire_specs(row, (cols + 11)) = rop.horz_force;
            tire_specs(row, (cols + 12)) = rop.net_force_orientation;
            tire_specs(row, (cols + 13)) = rop.relaxation_length;
            tire_specs(row, (cols + 14)) = rop.pneumatic_trail;
            tire_specs(row, (cols + 15)) = rop.self_aligning_torque;
            tire_specs(row, (cols + 16)) = norm_lateral_stiffness;
            tire_specs(row, (cols + 17)) = norm_cornering_stiffness;
            tire_specs(row, (cols + 18)) = norm_camber_stiffness;
            tire_specs(row, (cols + 19)) = (norm_lateral_stiffness - ...
                            measured_lateral_stiffness)/measured_lateral_stiffness;
            tire_specs(row, (cols + 20)) = (norm_cornering_stiffness - ...
                            measured_cornering_stiffness)/measured_cornering_stiffness;
            tire_specs(row, (cols + 21)) = (norm_camber_stiffness - ...
                            measured_camber_stiffness)/measured_camber_stiffness;
        end

end % for row = 1:rows
% ********************************************************************
% Write tire specs previously read along with new values calculated
%   back out to a comma-delimited file
% Pad first row digits to make room for row of text column headings
%   that will be overwritten next
% ********************************************************************
if read_tire_specs == 1
    % read first row of text column headings
    h = fopen(tire_specs_filename, 'rt');
    tire_specs_header = fgetl(h);
    fclose(h);
    % append additional column headings for newly calculated values
    tire_specs_header = [tire_specs_header ...
        ',Rotta vertical force (N),Rotta vertical displacement (mm)' ...
  ',Rotta sinkage (%),Rotta wheel horz disp (mm),Rotta contact patch length (mm)' ...
        ',Rotta contact patch width (mm),Rotta contact patch area (mm^2)' ...
',Rotta contact patch ellipse length (mm),Rotta contact patch ellipse width (mm)' ...
        ',Rotta contact patch ellipse area (mm^2),Rotta horz force (N)' ...
        ',Rotta net force orientation (degrees),Rotta relaxation length (mm)' ...
        ',Rotta pneumatic trail (mm),Rotta self aligning torque (N-mm)' ...
        ',Rotta norm lat stiffness (1/mm),Rotta norm corn stiffness (1/deg)' ...
        ',Rotta norm camb stiffness (1/deg)'];
    % write out first row of text column headings
    output_filename = ['out_' tire_specs_filename];
    h = fopen(output_filename, 'wt');
    while h == -1
        beep;
        ButtonName = questdlg(sprintf('Could not open %s\n Retry?', ...
                              output_filename), 'Error', ...
                              'Yes', 'No', 'Yes');
        switch ButtonName
            case 'Yes'
                h = fopen(output_filename, 'wt');
            case 'No'
                break
        end
```

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        end
        if h == -1
            fprintf('Could not open %s so no results written\n', ...
                                  output_filename)
        else
            fprintf(h, '%s\n', tire_specs_header);
            fclose(h);
            % write out matrix of values and padded digits
            dlmwrite(output_filename, tire_specs, '-append')
            fprintf('Results written to %s\n', output_filename)
        end
    end % if read_tire_specs == 1
    t = toc(tstart);
    fprintf(['All done. Evaluated %d set of tire specs in %0.4g seconds' ...
             ', at a rate of %0.3g seconds per tire spec.\n'], ...
        tire_specs_evaluated, t, t/tire_specs_evaluated)
catch me
    fprintf('\n\nUh oh. Something went wrong.\n\n')
    fprintf('\n%s\n', getReport(me))
    % *******************************************************************
    % Write tire specs previously read along with new values calculated
    %  back out to a comma-delimited file
    % Pad first row digits to make room for row of text column headings
    %   that will be overwritten next
    % *******************************************************************
    if read_tire_specs == 1
        fprintf('\n\nWill now try to write results so far.\n\n')
        % read first row of text column headings
        h = fopen(tire_specs_filename, 'rt');
        tire_specs_header = fgetl(h);
        fclose(h);
        % append additional column headings for newly calculated values
        tire_specs_header = [tire_specs_header ...
            ',Rotta vertical force (N),Rotta vertical displacement (mm)' ...
     ',Rotta sinkage (%),Rotta wheel horz disp (mm),Rotta contact patch length (mm)' ...
            ',Rotta contact patch width (mm),Rotta contact patch area (mm^2)' ...
    ',Rotta contact patch ellipse length (mm),Rotta contact patch ellipse width (mm)' ...
            ',Rotta contact patch ellipse area (mm^2),Rotta horz force (N)' ...
            ',Rotta net force orientation (degrees),Rotta relaxation length (mm)' ...
            ',Rotta pneumatic trail (mm),Rotta self aligning torque (N-mm)' ...
            ',Rotta norm lat stiffness (1/mm),Rotta norm corn stiffness (1/deg)' ...
            ',Rotta norm camb stiffness (1/deg)'];
        % write out first row of text column headings
        output_filename = tire_specs_filename;
        h = fopen(output_filename, 'wt');
        while h == -1
            beep;
            ButtonName = questdlg(sprintf('Could not open %s\n Retry?', ...
                                  output_filename), 'Error', ...
                                  'Yes', 'No', 'Yes');
            switch ButtonName
                case 'Yes'
                    h = fopen(output_filename, 'wt');
                case 'No'
                    break
            end
        end
        if h == -1
            fprintf('Could not open %s so no results written\n', ...
                                  output_filename)
        else
            fprintf(h, '%s\n', tire_specs_header);
            fclose(h);
            % write out matrix of values and padded digits
            dlmwrite(output_filename, tire_specs, '-append')
            fprintf('Results written to %s\n', output_filename)
        end
    end % if read_tire_specs == 1
    t = toc(tstart);
    fprintf(['All done. Evaluated %d set of tire specs in %0.4g seconds' ...
```

```matlab
                   ', at a rate of %0.3g seconds per tire spec.\n'], ...
            tire_specs_evaluated, t, t/tire_specs_evaluated)
    end
return
% ***************************************************************************
% ***************************************************************************
% Extract everything from Rotta model
%
% rip = Rotta Input Parameters structure
% rop = Rotta Output Parameters structure
%
% ***************************************************************************
% ***************************************************************************
function [rop] = Rotta_core(varargin)
    rc = Rotta_constants;
    % **********************************************************************
    % Check for correct input arguments, and complain nicely if missing
    % **********************************************************************
    if size(varargin,2) ~= 1
        clc; fprintf(['Input argument missing.\n' ...
                      '  Use "Rotta_model.m" or "Rotta_with_beam_model.m" instead.\n'])
        return
    else
        rip =  varargin{1};
        if ~isstruct(rip)
        clc; fprintf(['Input argument incorrect.\n' ...
                      '  Use "Rotta_model.m" or "Rotta_with_beam_model.m" instead.\n'])
            return
        end
    end

    if rip.relaxation_rate == 0 && rip.friction_coefficient ~= inf
        clc; fprintf(['Finite coefficient of friction with zero '...
                      'relaxation rate is problematic.\n'])
        rop.cp_horz_force = 0;
        return
    end
    % ************************************************************
    % If no displacements specified, make it explicite
    % ************************************************************
    if ~isfield(rip, 'displacements')
        rip.displacements = [];
    end

    % ************************************************************
    % Check for and fill in missing parameters
    % ************************************************************
    if ~isfield(rip, 'relaxation_rate_out')
        rip.relaxation_rate_out = [1 1] * rip.relaxation_rate;
    end
    if ~isfield(rip, 'relaxation_rate_in')
        rip.relaxation_rate_in = [1 1] * rip.relaxation_rate;
    end
    if ~isfield(rip, 'adjust_wheel_horz_d')
        rip.adjust_wheel_horz_d = 0;
    end

    % **********************************************************************
    % Define radial cross section orientations for half tire
    % **********************************************************************
    rip.slice_angles = linspace(0, rip.last_slice_angle, rip.num_slices);

    % **********************************************************************
    % Indicate no known contact patch horzontal force, yet.
    % **********************************************************************
    rip.cp_horz_force = 0;

    % **********************************************************************
    % Non-zero slip angle with zero horizontal displacement?
    % Guess a good enough horizontal displacement to get things going.
```

```matlab
% ***********************************************************************
if rip.wheel_horz_d == 0 && rip.slip_angle ~= 0
    rip.wheel_horz_d = rip.slip_angle*180/pi;
end

% ***********************************************************************
% Search for sinkage value that produces desired vert load
% ***********************************************************************
if ~isnan(rip.target_vert_force) && rip.target_vert_force ~= 0
    % keyboard
    [rop] = find_sinkage(rip);
    % copy found wheel vertical displacement from output struct to input
    rip.wheel_vert_d = rop.wheel_vert_d;
end
sinkage = rip.wheel_vert_d/rip.rim_height;
% ***********************************************************************
% Prepare to iterate if both friction and relaxtion rate specified
% ***********************************************************************
if rip.relaxation_rate ~= 0 && rip.friction_coefficient ~= inf
    plot_single_tire_config = rip.plot_single_tire_config; % local copy
    rip.plot_single_tire_config = 0; % suppress plotting for first time
end
iterations = 0; cp_horz_forces = []; horz_forces = [];
while 1
    if iterations == 2 && rip.plot_single_tire_config == 1
        close(gcf);
    end

    % *******************************************************************
    % Calculate everything for one complete set of tire parameters
    % *******************************************************************

    [rop] = calculate_tire_contact_patch(rip); % do the work
    fprintf(['\n  Rotta predicts horizontal force = %0.3g N with ' ...
     ' wheel_horz_d = %0.3g mm\n\n'], rop.horz_force, rip.wheel_horz_d)
    iterations = iterations + 1;
    cp_horz_forces(iterations) = rop.cp_horz_force;
    horz_forces(iterations) = rop.horz_force;
    if rip.relaxation_rate ~= 0 && rip.friction_coefficient ~= inf
        if abs(rip.cp_horz_force - rop.cp_horz_force) < 0.1 || ...
            abs(rip.cp_horz_force - rop.cp_horz_force) / ...
            abs(rop.cp_horz_force) < 0.1
            break
        else
            if iterations >= 1 % only after first iteration
                rip.plot_single_tire_config = plot_single_tire_config;
            end
        end
        % ***************************************************************
        % Check for oscillations and try to damp them out.
        % ***************************************************************
        if iterations >= 3 && ...
                abs((cp_horz_forces(iterations - 2) - ...
                    cp_horz_forces(iterations - 1)) + ...
                   (cp_horz_forces(iterations - 1) - ...
                    cp_horz_forces(iterations - 0))) < 0.1
            rip.cp_horz_force = (cp_horz_forces(iterations - 1) + ...
                                 cp_horz_forces(iterations - 0)) / 2;
            rip.horz_force = (horz_forces(iterations - 1) + ...
                              horz_forces(iterations - 0)) *2 / 3;
            fprintf(['\n    Oscillations detected, trying to damp ' ...
                     'them. Reducing horizontal force to %0.3g\n'], ...
                     rip.horz_force);

        else % no oscillations detected, continue...
            rip.cp_horz_force = rop.cp_horz_force;
            rip.horz_force = rop.horz_force;
            % ***********************************************************
            % Attempt to pick "correct" lateral displacement
            % ***********************************************************
```

```matlab
                    if rip.adjust_wheel_horz_d == 1
                        if rip.slip_angle ~= 0 && rip.camber_angle == 0
                        sinkage = (rip.wheel_vert_d/rip.rim_height);
%                            s_over_p = (sinkage / rip.inf_press);
%                            rip.wheel_horz_d = (4.4598 * s_over_p + 0.2598)*1.15;
                        kludge = (sinkage / rip.inf_press) * rip.tire_width^2.57;
                        rip.wheel_horz_d = (7.1515e-05 * kludge + 2.4252e-01)*1.17;
                        end
                        if rip.slip_angle == 0 && rip.camber_angle ~= 0
                            fudge_factor = 50.0 * 0.14 / sinkage * ...
                                               (rip.target_vert_force / 31*rc.N_per_kg);
                            rip.wheel_horz_d = fudge_factor * ...
                                               (max(rop.cp_center_line(1,:)) - ...
                                                min(rop.cp_center_line(1,end)));
                        end
                    end
                end
        else % rip.relaxation_rate == 0 || rip.friction_coefficient == inf
            break
        end

        if iterations == 10
            disp('          Reached 10 iterations, giving up.')
            break
        end

    end % while 1

    % *************************************************************
    % Set this up for whomever needs it
    % *************************************************************
    if ~isfield(rop, 'wheel_vert_d')
        rop.wheel_vert_d = rip.wheel_vert_d;
        rop.wheel_horz_d = rip.wheel_horz_d;
    end

return

% ***************************************************************************
% ***************************************************************************
% Find sinkage necessary for specified inflation pressure and vertical load
% ***************************************************************************
% ***************************************************************************
function [rop] = find_sinkage(rip)
    rc = Rotta_constants;
    if rip.initial_sinkage_guess == 0
        disp('Bad guess of initial sinkage')
        keyboard
    end
    tstart = tic; % keep track of time to see how much we are wasting
    % ******************************************************************
    % plot progress for debugging purposes
    % ******************************************************************
    if rip.plot_target_vert_force_search_progess == 1
        figure; hold on; grid on; set(gcf, 'name', 'Rotta');
        xlabel('vertical displacement (mm)'); ylabel('vertical force (N)');
    end
    fprintf(['     Searching for vertical displacement that produces the ' ...
            '%0.4g N target vertical force.\n'], rip.target_vert_force)

    % ******************************************************************
    % set up for searching
    % ******************************************************************
    rip.plot_single_tire_config = 0; % no plotting
    rip.post_patch = 0; % don't calculate anything after contact patch
    vert_forces = zeros(1,10); % initialize for speed
    wheel_vert_ds = zeros(1,10); % initialize for speed
    tries = 1;
    wheel_vert_ds(tries) = rip.initial_sinkage_guess; % use given initial guess
    while 1
```

```matlab
        % **************************************************************
        % Calculate everything for one complete set of tire parameters
        % **************************************************************
        rip.wheel_vert_d = wheel_vert_ds(tries);
        [rop] = calculate_tire_contact_patch(rip);
        vert_forces(tries) = rop.vert_force;
        if rip.plot_target_vert_force_search_progess == 1
            plot(wheel_vert_ds(tries), vert_forces(tries), 'k.'); drawnow
        end
        if abs(100*(vert_forces(tries) - rip.target_vert_force) / ...
                rip.target_vert_force) <= rc.percent_force_tolerance
            break
        end
        wheel_vert_ds(tries+1) = wheel_vert_ds(tries) * ...
            rip.target_vert_force / vert_forces(tries);
        if tries >= 10
            disp('           Quit after 10 tries.')
            break
        end
        tries = tries + 1;
    end
    t = toc(tstart);
    str = sprintf(['%d iterations and %0.4g seconds required to find %0.4g N,' ...
                   '\n  which is within %0.4g%% of target %0.4g N force'], ...
            tries, t, rop.vert_force, ...
            100*(vert_forces(tries) - rip.target_vert_force)/rip.target_vert_force, ...
            rip.target_vert_force);
    if rip.plot_target_vert_force_search_progess == 1
        plot(wheel_vert_ds(tries), vert_forces(tries), 'bo');
        if tries >= 2
            plot(wheel_vert_ds(1:2), vert_forces(1:2), 'b');
        end
        hold off
        title(str);
    end
    if rip.run_silent == 0
        disp(str)
    end
    rop.wheel_vert_d = wheel_vert_ds(tries);
return
% ***************************************************************************
% ***************************************************************************
% Find contact patch for one tire
% ***************************************************************************
% ***************************************************************************
function [rop] = calculate_tire_contact_patch(rip)
    rc = Rotta_constants; % define all necessary constants

    % ***********************************************************************
    % Calculate additional wheel and tire dimensions
    % ***********************************************************************
    rip.rim_height = rip.tire_radius * (1 + cos(rip.rim_angle / 2));
    axle_height = rip.rim_diameter / 2 + rip.rim_height; % from axle to ground

    % ***********************************************************************
    % Define undeformed tire cross-section
    % ***********************************************************************
    t = linspace(0, 2*pi - rip.rim_angle, rc.tire_fineness) + pi/2 + rip.rim_angle/2;
    x = rip.tire_radius*cos(t); z = rip.tire_radius*sin(t);

    % ***********************************************************************
    % Start plots
    % ***********************************************************************
    tire_colors = {'r', 'm', 'r'}; % tire_lines = {'-', ':', '--'};

    str = sprintf(['  Tire radius = %0.4g mm, tire width = %0.4g, rim width = %0.4g' ...
                   'rim diameter = %0.4g mm,\n' ...
                   '    rim height = %0.4g mm, ' ...
                   'inflation pressure = %0.4g N/mm^2 (%0.4g bar, %0.4g psi)\n' ...
                   '        vertical displacement = %0.4g mm (%0.4g%% sinkage), ' ...
```

```matlab
                'horizontal displacement = %0.4g mm,\n' ...
                '        camber angle = %0.4gº, ' ...
                'slip angle = %0.4gº, ' ...
                'relaxation rate = %0.4g, ' ...
                'friction coefficient = %0.4g'], ...
            rip.tire_radius, rip.tire_width, rip.rim_width, rip.rim_diameter, ...
            rip.rim_height, rip.inf_press, (rip.inf_press / rc.MPa_per_psi) *...
            rc.bar_per_psi, rip.inf_press / rc.MPa_per_psi, ...
            rip.wheel_vert_d, 100*rip.wheel_vert_d/rip.rim_height, ...
            rip.wheel_horz_d, rip.camber_angle*180/pi, rip.slip_angle*180/pi, ...
            rip.relaxation_rate, rip.friction_coefficient);
if rip.run_silent == 0
    disp(str)
end
if rip.plot_single_tire_config == 1
    figure; set(gcf, 'name', 'Rotta');
    subplot(2,3,1); cla; axis equal; grid on; hold on;
    % plot3([x(1), x(end)], [z(1) z(end)]*0, [z(1) z(end)], 'c', x, z*0, z, 'm');
    xlim([-1 1]* rip.tire_radius * 1.5); zlim([-1 1]* rip.tire_radius * 1.5);
    xlabel('x (mm)'); ylabel('y (mm)'); zlabel('z (mm)')
    title(str); view(-1,1)

    subplot(2,3,2); cla; hold on; axis equal; grid on;
    xlabel('x (mm)'); ylabel('y (mm)');

    subplot(2,3,4); cla; hold on; grid on;
 xlabel('Horizontal displacement (blue in mm), dS (black) and dN (red) (both in N)');
    ylabel('y, distance along long axis of tire and contact patch (mm)');

    subplot(2,3,5); cla; hold on; grid on;
    xlabel('ground reaction force angle, from the vertical (º)');
    ylabel('y, distance along long axis of contact patch (mm)');

    subplot(2,3,[3 6]); cla; hold on; axis equal; grid on;
    xlabel('x (mm)'); ylabel('y (mm)'); zlabel('z (mm)')
end

% ********************************************************************
% Predefine 2D rotation matrices
% ********************************************************************
R2z = [ cos(rip.camber_angle) sin(rip.camber_angle);
       -sin(rip.camber_angle) cos(rip.camber_angle)];
uR2z = [ cos(-rip.camber_angle) sin(-rip.camber_angle);
       -sin(-rip.camber_angle) cos(-rip.camber_angle)];

% ********************************************************************
% Prepare to loop through both halves (ends) of the contact patch
% ********************************************************************
max_x = NaN; min_x = NaN; max_y = NaN;
min_y = NaN; max_z = NaN; min_z = NaN;

% ********************************************************************
% Preallocate for speed, but necessary size unknown
% ********************************************************************
slice_vert_ds = zeros(1, 1); slice_horz_ds = zeros(1, 1);
% can't pre-allocate these because exact size unknown and extra spaces
% screw up subsequent calculations and plotting
cpexs = []; cpeys = [];
rop.horz_force = 0; rop.cp_area = 0;
rop.relaxation_length = NaN;

% ********************************************************************
% Figure out how many halves are necessary
% ********************************************************************
if rip.post_patch == 1
    halves = 2;
else
    if rip.slip_angle ~= 0
        halves = 2;
    else
```

```matlab
            halves = 1;
        end
    end

    % ************************************************************************
    % Format input displacements into expected shape
    % ************************************************************************
    if ~isempty(rip.displacements)
        displacements = ...
            [fliplr(rip.displacements(1:length(rip.displacements)/2)); ...
             rip.displacements(length(rip.displacements)/2+1:end)];
    end

    % ************************************************************************
    % Loop through the necessary halves
    % ************************************************************************
    for half = 1:halves
        rip.relaxation_rate = rip.relaxation_rate_out(half);
        % ********************************************************************
        % Orient slip angle properly for each half of contact patch
        % ********************************************************************
        half_rip.slip_angle = rip.slip_angle * (-1)^half;

        % ********************************************************************
        % Prepare to loop through all cross sections (rip.num_slices) in this half
        % ********************************************************************
        status = 1;                      % start with everything okay
        relaxing = 0;       % not in the range of tire relaxing from cp yet
        told_about_sliding = 0;
        cp_guess = rip.wheel_horz_d * 0.75; % make first guess. This works for
        % vert_d = 5, horz_d < 17, and rip.camber_angle < 50
        % pre-allocate these for speed
        dy(half,:) = zeros(rip.num_slices, 1); da(half,:) = dy(half,:);
        dS(half,:) = dy(half,:); dN(half,:) = dy(half,:);
        cphxs = zeros(1,4); cphys = cphxs;

        % ********************************************************************
        % Calculate each cross section (slice)
        % ********************************************************************
%         for slice = 1:rip.num_slices
        slice = 0; % need to use while so can manipulate slice number
        while slice < rip.num_slices
            slice = slice + 1;

            % ****************************************************************
            % Calculate vertical deflection of this cross section (slice).
            % This is close, but not exact.
            % ****************************************************************
            slice_vert_ds(half, slice) = cos(rip.camber_angle) * ...
                    (axle_height - (axle_height - ...
                        rip.wheel_vert_d) / (cos(rip.slice_angles(slice))));
            % ****************************************************************
            % Calculate effect of drum curvature, if radius ~= 0
            % ****************************************************************
            if rip.drum_radius ~=0 % allows for negative radius
                % this is an approximation that should be good enough for
                % large wheel and drum radii compared to wheel vertical
                % deflection, which is the case with narrow bicycle tires
                drum_angle = asin(axle_height * sin(rip.slice_angles(slice)) / ...
                            rip.drum_radius);

                slice_vert_ds(half, slice) = slice_vert_ds(half, slice) - ...
                    rip.drum_radius * (1 - cos(drum_angle)) / ...
                    cos(rip.slice_angles(slice));
            end

            if isnan(slice_vert_ds(half, slice))
                disp('slice_vert_ds(half, slice) = NaN')
                keyboard
            end
```

```matlab
% **************************************************************
% Still all okay? Then keep on finding contact patch edges
% **************************************************************
if status == 1 % everything okay
    if rip.camber_angle ~= 0
        % ************************************************
        % Calculate horizontal deflection of this slice
        % ************************************************
        slice_horz_ds(half, slice) = rip.wheel_horz_d + ...
            tan(rip.camber_angle) * slice_vert_ds(half, slice);
        if slice == 1 && half == 1
            delta = rip.tire_radius * sin(rip.camber_angle);
            % ********************************************
            % Can't make it perfect, iterate instead
            % ********************************************
            correction = 0; % set this just in case
            while 1 % set up to loop through deltas
                % disp('Testing delta')
                [status, rim, tire, cp, ~, ~] = ...
                    calculate_slice_contact_patch(rip, ...
                        slice_vert_ds(half, slice), ...
                        slice_horz_ds(half, slice), cp_guess, delta);
                if status == 1 % everything okay
                    if abs(cp(1,2) - slice_horz_ds(half, slice)) > ...
                                                    rc.min_lat_disp
                        correction = (cp(1,2) - slice_horz_ds(half, slice));
                        delta = delta + correction;
                    else
                        break
                    end
                end
            end % while 1
        end
    else % rip.camber_angle == 0
        slice_horz_ds(half, slice) = rip.wheel_horz_d;
        delta = 0; % both of these are specific to
        correction = 0; % rip.camber_angle ~= 0
    end
    % **************************************************************
    % Calculate effect of disk curvature, if radius ~<= 0
    % **************************************************************
    if rip.disk_radius ~= 0 % allows for negative radius
        % this is an approximation that should be good enough for
        % large wheel and disk radii compared to wheel vertical
        % deflection, which is the case with narrow bicycle tires
        disk_angle = asin(axle_height * sin(rip.slice_angles(slice)) / ...
                        rip.disk_radius);
        slice_horz_ds(half, slice) = slice_horz_ds(half, slice) + ...
                sign(rip.disk_radius) * 1.4 - ...
                rip.disk_radius * (1 - cos(disk_angle)) / ...
                cos(rip.slice_angles(slice));
    end

    % **********************************************************
    % Apply slip angle
    % **********************************************************
    slice_horz_ds(half, slice) = slice_horz_ds(half, slice) -  ...
                    (axle_height - slice_vert_ds(half, slice)) * ...
                    sin(rip.slice_angles(slice)) * sin(half_rip.slip_angle);

    friction_repeat = 1; % force the first iteration
    friction_repeat_count = 0;
    while friction_repeat == 1 % set up to loop in case of sliding
        friction_repeat = 0; % default to not repeating
        friction_repeat_count = friction_repeat_count + 1;
        % **********************************************************
        % Find contact patch for one cross-sectional slice
        % **********************************************************
        phase = 1;
```

```matlab
            [status, rim, tire, cp, radii, ~] = ...
                calculate_slice_contact_patch(rip, ...
                    slice_vert_ds(half, slice), ...
                    slice_horz_ds(half, slice), cp_guess, delta);
            if status == 1 % everything okay
                % **************************************************
                % See if contact patch edges have crossed
                % **************************************************
                if cp(1,1) >= cp(1,4)
                    if rip.run_silent == 0
                        disp('            Contact patch edges have crossed.')
                    end
                    status = 0; % not okay
                    cp_end_slice(half) = slice;

                else
                    % *********************************************
                    % If everything still okay, update guess for
                    %   next time: between current edges, calculate
                    %   lateral and normal forces, and plot them
                    % *********************************************
                    cp_guess = (cp(1,1) + cp(1,4))/2;

                    % *********************************************
                    % Calc area and forces of this cntct patch sect
                    % *********************************************
                    if slice > 1
                        dy(half, slice) = (axle_height - ...
                                            slice_vert_ds(half, slice)) * ...
                                    sin(rip.slice_angles(slice)) - ...
                                    (axle_height - slice_vert_ds(half, slice)) * ...
                                    sin(rip.slice_angles(slice-1));
                        dS(half, slice) = rip.inf_press * (radii(1) - radii(2)) * ...
                                    dy(half, slice);
                        da(half, slice) = abs(cp(1, 1) - cp(1,4)) * dy(half, slice);
                        dN(half, slice) = rip.inf_press * da(half, slice);

                        % *************************************
                        % fill in gap that was left by skipping
                        % over slice 1 and not having a slice 0
                        % *************************************
                        if slice == 2
                            dy(half, 1) = dy(half, 2);
                            dS(half, 1) = dS(half, 2);
                            da(half, 1) = da(half, 2);
                            dN(half, 1) = dN(half, 2);
                            if rip.plot_single_tire_config == 1
                                y = dy(half, 1)*(-1)^(half);
                                subplot(2,3,4);
                                plot(dS(half, 1) / dy(half, 1), ...
                                    y, 'k.', dN(half, 1), y, 'r.');
                                subplot(2,3,5); plot( ...
                                    90 - atan2(dN(half, 1), ...
                                    dS(half, 1))*180/pi, y, 'k.');
                                if half == 1
                                    subplot(2,3,4);
                                    plot(dS(half, 1) / dy(half, 1), ...
                                        0, 'k.', dN(half, 1), 0, 'r.');
                                    subplot(2,3,5); plot( ...
                                        90 - atan2(dN(half, 1), ...
                                        dS(half, 1))*180/pi, 0, 'k.');
                                end
                            end
                        end

                        % *************************************
                        % if additional force from beyond cp exists
                        % *************************************
                        if rip.cp_horz_force == 0
                            dS_f = dS(half, slice);
```

```matlab
                        else
                            dS_f = dy(half, slice) * ...
                                (rip.horz_force / rip.cp_horz_force);
                        end

                        % ******************************************
                        % check if available friction exceeded
                        % ******************************************
                        if friction_repeat_count == 1 && ...
                                abs(dS_f) > abs(dN(half, slice) * ...
                                rip.friction_coefficient)
                            % "periods per year" = "slices per mm"
                            n = length(rip.slice_angles) / ...
                                (rip.slice_angles(end)*axle_height);
                            % calculate slide at the relaxation rate
                            slide = slice_horz_ds(half, slice-1) * ...
                                (1 + ((rip.relaxation_rate_in(half)-1) / n)) ^ ...
                                    (n * rip.slice_angles(2) * axle_height);
                            % if not too, small go ahead and slide
                            if abs(slide) >= 0.1
                                slice_horz_ds(half, slice) = slide;
                                if told_about_sliding == 0
                                    if rip.run_silent == 0
                                        disp('         Sliding...')
                                    end
                                    told_about_sliding = 1;
                                end
                            end
                            friction_repeat = 1;
                        else % if dS_f > dN(half, slice) * ...
                            if rip.plot_single_tire_config == 1
                                subplot(2,3,4);
                                    plot(dS(half, slice) / dy(half, slice), ...
                                    sum(dy(half,:))*(-1)^(half), 'k.', ...
                                 dN(half, slice), sum(dy(half,:))*(-1)^(half), 'r.');
                                subplot(2,3,5); plot(...
                                    90 - atan2(dN(half, slice), ...
                                    dS(half, slice))*180/pi, ...
                                    sum(dy(half,:))*(-1)^(half), 'k.');
                            end % if rip.plot_single_tire_config == 1
                        end % if dS_f > dN(half, slice) * ...
                    end % if slice > 1
                end % if cp(1,1) >= cp(1,4)
            end % if status == 1
        end % while 1
    end % if status == 1
    if status ~= 1
        if rip.post_patch ~= 1
            if rip.run_silent == 0
                disp('      Skipping tire calculations beyond contact patch.')
            end
            break
        else
        % **********************************************************
        % Tire is no longer in contact with ground
        % **********************************************************
        cp_len = max(max(cphys)) - min(min(cphys));

        % **********************************************************
        % Decay displacement back to zero at specified rate
        % **********************************************************
        if ~isempty(rip.displacements) || rip.relaxation_rate ~= 0 ...
                && abs(slice_horz_ds(half, slice-1)) > rc.min_lat_disp
            % **********************************************************
            % If available, apply beam displacements
            % **********************************************************
            if ~isempty(rip.displacements)
                    slice_horz_ds(half, slice) = slice_horz_ds(half, slice-1) * ...
                        displacements(half, slice) / displacements(half, slice-1);
                slice_vert_ds(half, slice) = slice_vert_ds(half, slice-1) * ...
```

```matlab
                        displacements(half, slice) / displacements(half, slice-1);
                else % isempty(rip.displacements)
                    if relaxing == 0 % if the first time?
                        relaxing = 1; % it won't be the next time

                        % keep track of this for calculating relaxation len
                        unrelaxed_slice(half) = slice-1;
                        if rip.run_silent == 0
                            disp('          Relaxing horizontal displacement.')
                        end
                    end
                    % *********************************************
                    % Relax vert and horz displacement for this
                    % slice, and must do both. This works much
                    % better than just relaxing horz disp, and
                    % this method is independent of slice width.
                    % *********************************************
                    % y(i) = y0*exp((r-1)*x(i));
                    e = exp((rip.relaxation_rate-1) * ...
                            (rip.slice_angles(slice) - ...
                             rip.slice_angles(unrelaxed_slice(half))) * ...
                             axle_height);

                    slice_horz_ds(half, slice) = ...
                        slice_horz_ds(half, unrelaxed_slice(half)) * e;
                    if ~isreal(slice_horz_ds(half, slice))
                        slice_horz_ds(half, slice) = 0;
                    end
                    slice_vert_ds(half, slice) = ...
                        slice_vert_ds(half, unrelaxed_slice(half)) * e;
                    if ~isreal(slice_vert_ds(half, slice))
                        slice_vert_ds(half, slice) = 0;
                    end
                    % only do this once
                    if half == 1 && isnan(rop.relaxation_length)
                        if slice_horz_ds(half, slice) <= ...
                                slice_horz_ds(half, unrelaxed_slice(half))/exp(1)
                            rop.relaxation_length = ...
                                axle_height * ...
                                    (sin(rip.slice_angles(slice)) - ...
                                     sin(rip.slice_angles(unrelaxed_slice(half))));
                        end
                    end % if half == 1 && isnan(rop.relaxation_length)
                end % else % isempty(rip.displacements)

                % ****************************************************
                % Calculate shape for this relaxed horizontal disp
                % ****************************************************
                phase = 2;
                [~, rim, tire, cp, radii, ~] = ...
                    calculate_slice_contact_patch(rip, slice_vert_ds(half, slice), ...
                        slice_horz_ds(half, slice), cp_guess, delta);

                dy(half, slice) = axle_height * (sin(rip.slice_angles(slice)) - ...
                                        sin(rip.slice_angles(slice-1)));
                dS(half, slice) = rip.inf_press * (radii(1) - radii(2)) * ...
                            dy(half, slice);
                da(half, slice) = 0;
                dN(half, slice) = rip.inf_press * da(half, slice);
                if rip.plot_single_tire_config == 1
                    subplot(2,3,4); plot(dS(half, slice) / dy(half, slice), ...
                                sum(dy(half,:))*(-1)^(half), 'k.', ...
                                dN(half, slice), sum(dy(half,:))*(-1)^(half), 'r.');
                    subplot(2,3,5); plot(...
                        90 - atan2(dN(half, slice), dS(half, slice))*180/pi, ...
                        sum(dy(half,:))*(-1)^(half), 'k.');
                end
            else % if rip.relaxation_rate ~= 0 && abs(slice_horz_ds(half, slice-1)) > 0.1
                phase = 3;
```

```matlab
            rim = [x(1) x(end); z(1) z(end)];
            tire = [x; z];
            [~, min_tire_z_idx ] = min(tire(2,:));
            cp = [tire(1, min_tire_z_idx); tire(2, min_tire_z_idx)];
            rim = R2z*rim;
            tire = R2z*tire;
            %cp = R2z*cp;
            slice_horz_ds(half, slice) = NaN; % 'NaN' to aid plotting
     end % if rip.relaxation_rate ~= 0 && abs(slice_horz_ds(half, slice-1)) > 0.1
        end % if rip.post_patch ~= 1
end % if status == 1

% ****************************************************************
% Plot everything
% ****************************************************************
if rip.plot_single_tire_config == 1 && half == 1
    subplot(2,3,1);
    y = -axle_height * sin(rip.slice_angles(2)) * slice;
    plot3(rim(1,:), rim(2,:)*0+y, rim(2,:), 'b', ...
          tire(1,:), tire(2,:)*0+y, tire(2,:), ...
          char(tire_colors(phase)), ...
          cp(1,:), cp(2,:)*0+y, cp(2,:), '.k');
    % ****************************************************************
    % If cp has 4 columns, the middle two are center lines,
    % so don't connect them with lines when they cross outside.
    % ****************************************************************
    [~,c] = size(cp);
    if c == 4
        if cp(1,2) > cp(1,1) && cp(1,2) < cp(1,4)
            plot3(cp(1,:), cp(2,:)*0+y, cp(2,:), 'g');
        else
            plot3(cp(1,[1 c]), cp(2,[1 c])*0+y, cp(2,[1 c]), 'g');
        end
    else
        plot3(cp(1,:), cp(2,:)*0+y, cp(2,:), 'g');
    end
end % if rip.plot_single_tire_config == 1

% ****************************************************************
% Un-rotate camber in 2D so that it can be re-rotated in 3D
% ****************************************************************
rim = uR2z*rim;
tire = uR2z*tire;
cp = uR2z*cp;

% ****************************************************************
% translate and expand to 3D
% ****************************************************************
rim3 = [rim(1,:); [0 0]; rim(2,:) - (axle_height - rip.tire_radius)];
tire3 = [tire(1,:); zeros(1,length(tire)); ...
         tire(2,:) - (axle_height - rip.tire_radius)];
cp3 = [cp(1,:); zeros(1,length(cp(1,:))); ...
       cp(2,:) - (axle_height - rip.tire_radius)];
% rotate about x-axis: axle
R3x = [1                  0                      0;
       0  cos(rip.slice_angles(slice)*(-1)^(half+1)) ...
          sin(rip.slice_angles(slice)*(-1)^(half+1));
       0 -sin(rip.slice_angles(slice)*(-1)^(half+1)) ...
          cos(rip.slice_angles(slice)*(-1)^(half+1))];
% rotate about y-axis: camber (could come out of this loop)
R3y = [ cos(rip.camber_angle) 0 sin(rip.camber_angle);
       0                  1                      0;
       -sin(rip.camber_angle) 0 cos(rip.camber_angle)];
R3 = R3y*R3x;
rim3 = R3*rim3;
tire3 = R3*tire3;
cp3 = R3*cp3;

% ****************************************************************
% Keep track of contact patch edges
```

```matlab
        % ****************************************************************
        if status == 1 % everything okay
            cphxs(slice,:) = cp3(1,:);
            cphys(slice,:) = cp3(2,:);
        end

        % ****************************************************************
        % Plot rim, tire, and contact patch edges in 3D
        % ****************************************************************
        if rip.plot_single_tire_config == 1
            subplot(2,3,[3 6]);
            plot3(rim3(1,:), rim3(2,:), rim3(3,:), 'b');
            plot3(tire3(1,:), tire3(2,:), tire3(3,:), ...
                                    char(tire_colors(phase)));
            plot3(cp3(1,:), cp3(2,:), cp3(3,:), '.k');

            % ************************************************************
            % Extract contact patch edge coordinates from 3D model of tire
            %   If cp3 has 4 columns, middle two are center lines,
            %      so don't connect them with lines when they cross outside.
            % ************************************************************
            [~,c] = size(cp3);
            if c == 4
                if cp3(1,2) > cp3(1,1) && cp3(1,2) < cp3(1,4)
                    plot3(cp3(1,:), cp3(2,:), cp3(3,:), 'g');
                else
                    plot3(cp3(1,[1 c]), cp3(2,[1 c]), cp3(3,[1 c]), 'g');
                end
            else
                plot3(cp3(1,:), cp3(2,:), cp3(3,:), 'g');
            end

            % ************************************************************
            % Keep track of these for setting limits at the end
            % ************************************************************
            max_x = max([max_x tire3(1,:)]);
            min_x = min([min_x tire3(1,:)]);
            max_y = max([max_y tire3(2,:)]);
            min_y = min([min_y tire3(2,:)]);
            max_z = max([max_z tire3(3,:)]);
            min_z = min([min_z tire3(3,:)]);
        end
end % for slice = 1:rip.num_slices

[r,~] = size(cphxs); % get number of rows
if r > 1 % if results exist, not just first row of zeros
    % ****************************************************************
    % Assemble complete contact patch boarder to calculate area,
    % from two, assymmetrical halves (created by slip angle)
    % ****************************************************************
    cph_cl = [cphxs(:,2)'; cphys(:,2)']; % extract center line
    if half == 1
        cphxs = [cphxs(:,1); flipud(cphxs(:,2)); ...
                  cphxs(:,3); flipud(cphxs(:,4))];
        cphys = [cphys(:,1); flipud(cphys(:,2)); ...
                  cphys(:,3); flipud(cphys(:,4))];
    else
        cphxs = [cphxs(:,1); flipud(cphxs(:,2)); ...
                  cphxs(:,3); flipud(cphxs(:,4))];
        cphys = [cphys(:,1); flipud(cphys(:,2)); ...
                  cphys(:,3); flipud(cphys(:,4))];
    end
    rop.cp_area = rop.cp_area + polyarea(cphxs, cphys);
    cpexs = [cpexs; cphxs];
    cpeys = [cpeys; cphys];
    if half == 1
        cp_cl = fliplr(cph_cl);
    else
        cp_cl = [cp_cl cph_cl];
    end
```

```matlab
            % ****************************************************************
            % Sum up lateral force and contact patch length
            % ****************************************************************
            cp_half_length = (max(max(cphys)) - min(min(cphys)));
            ys(half,:) = cumsum(dy(half,:));

            % calculate pneumatic trail for this half of contact patch
            rop.pneumatic_trails(half) = (sum(ys(half, ys(half,:) <= cp_half_length) ...
                                    .* dS(half, ys(half,:) <= cp_half_length)) / ...
                                     sum(dS(half, ys(half,:) ...
                                    <= cp_half_length)))*(-1)^(half+1);
            rop.horz_forces(half) = sum(dS(half,:));
        else
            disp('Something went wrong and there are no results to plot.')
            break
        end
    end % for half = 1:2
    if halves == 1 && rip.run_silent == 0
        disp('            Skipping second half of symmetrical contact patch.')
    end

    % ****************************************************************
    % Put everything into format expected by caller
    % ****************************************************************
    rop.dS = [fliplr(dS(1,:)) dS(halves,:)];
    rop.dN = [fliplr(dN(1,:)) dN(halves,:)];
    rop.slice_horz_ds = [fliplr(slice_horz_ds(1,:)) slice_horz_ds(halves,:)];
    rop.slice_vert_ds = [fliplr(slice_vert_ds(1,:)) slice_vert_ds(halves,:)];
    rop.horz_force = sum(rop.horz_forces);
    % combine pneumatic trails from both halfs
    rop.pneumatic_trail =
sum(abs(rop.horz_forces).*rop.pneumatic_trails)/sum(abs(rop.horz_forces));
    rop.self_aligning_torque = rop.pneumatic_trail * rop.horz_force;
    rop.cp_area = rop.cp_area * 2 / halves; % double area if only did one half
    rop.cp_length = max(max(cpeys)) - min(min(cpeys)) * 2 / halves;
    rop.cp_width = max(max(cpexs)) - min(min(cpexs));
    rop.cp_center_line = cp_cl;
    rop.axle_height = axle_height;
    if halves == 2 % only if we did both sizes
        rop.cp_horz_force = sum(rop.dS(rip.num_slices-cp_end_slice(1) : ...
                                rip.num_slices+1+cp_end_slice(2))); % in N
    else
        rop.cp_horz_force = 0;
    end
    rop.cp_end_slice = cp_end_slice;
    rop.vert_force = rop.cp_area * rip.inf_press;
    rop.net_force_orientation = atan2(rop.horz_force, rop.vert_force) * 180/pi;
    str = sprintf(['    Vertical force = %0.4g N (%0.4g kg, %0.4g lb)\n' ...
                   '        Horizontal force = %0.4g N (%0.4g kg, %0.4g lb),' ...
                   ' which normalizes to %0.4g\n' ...
                   '        Resultant force orientation = %0.4g° from' ...
                   ' vertical, (%0.4g%% of camber angle)\n' ...
                   '        Relaxation length = %0.4g mm'], ...
                   rop.vert_force, rop.vert_force / rc.g, rop.vert_force / rc.N_per_lb,
...
                   rop.horz_force, rop.horz_force / rc.g, rop.horz_force / rc.N_per_lb,
...
                   rop.horz_force/rop.vert_force, ...
                   rop.net_force_orientation, ...
                   100 * rop.net_force_orientation / (rip.camber_angle*180/pi), ...
                   rop.relaxation_length);
    if rip.wheel_horz_d ~= 0 && rip.slip_angle == 0 && rip.camber_angle == 0
        static_lateral_stiffness = rop.horz_force/rip.wheel_horz_d;
        str = [str ...
               sprintf([', Static lateral stiffness = %0.4g N/mm' ...
                        ' (%0.4g 1/mm)']', ...
                        static_lateral_stiffness, ...
                        static_lateral_stiffness/rop.vert_force)];
    elseif rip.wheel_horz_d ~= 0 && rip.slip_angle ~= 0 && rip.camber_angle == 0
```

```matlab
        cornering_stiffness = rop.horz_force/(rip.slip_angle*180/pi);
        str = [str ...
                sprintf([', Cornering stiffness = %0.4g N/degree' ...
                        ' (%0.4g 1/degree)'], ...
                        cornering_stiffness, ...
                        cornering_stiffness/rop.vert_force)];
    elseif rip.wheel_horz_d ~= 0 && rip.slip_angle == 0 && rip.camber_angle ~= 0
        camber_stiffness = rop.horz_force/(rip.camber_angle*180/pi);
        str = [str ...
                sprintf([', Camber stiffness = %0.4g N/degree' ...
                        ' (%0.4g 1/degree)'], ...
                        camber_stiffness, ...
                        camber_stiffness/rop.vert_force)];
    end
    if rip.plot_single_tire_config == 1
        subplot(2,3,[3 6]); title(str);

        subplot(2,3,4); title(sprintf( ...
        ['Horizontal (dS) and vertical (dN) force per unit lenght (N/mm)\n' ...
            'actual horizontal deflections vary from %0.4g to %0.4g mm'], ...
            max(max(slice_horz_ds)), min(min(slice_horz_ds))));
        cp_force_per_length = rop.horz_force/rop.cp_length;
        plot(cp_cl * 0 + cp_force_per_length, cp_cl, 'g.');
        ylim([-1 1] * axle_height * rip.last_slice_angle);

        subplot(2,3,5); title(sprintf( ...
        ['Ground reaction force orientation (from the vertical) '...
         'per contact patch section\n' ...
         'actual vertical deflections vary from %0.4g to %0.4g mm'], ...
            max(max(slice_vert_ds)), min(min(slice_vert_ds))));

    end
    if rip.run_silent == 0
        disp(str)
        fprintf(['                Contact patch length = %0.4g mm ' ...
                'area = %0.4g mm^2\n'], rop.cp_length, rop.cp_area)
    end

    % *****************************************************************
    % Fit an ellipse to contact patch with same algorithm used
    % for ink prints
    % *****************************************************************
    if rip.camber_angle == 0 && rip.slip_angle == 0 && rip.wheel_horz_d == 0
        ellipse_t = fit_ellipse(cpexs, cpeys);
        t = linspace(0, 2*pi); % redo so it goes all the way around
        ellipse_x = ellipse_t.X0_in + ...
                ellipse_t.long_axis/2 * cos(t) * cos(-ellipse_t.phi) - ...
                ellipse_t.short_axis/2 * sin(t) * sin(-ellipse_t.phi);
        ellipse_y = ellipse_t.Y0_in + ...
                ellipse_t.long_axis/2 * cos(t) * sin(-ellipse_t.phi) + ...
                ellipse_t.short_axis/2 * sin(t) * cos(-ellipse_t.phi);
        ellipse_area = pi*ellipse_t.long_axis*ellipse_t.short_axis/4;
        rop.ellipse_cp_length = ellipse_t.long_axis;
        rop.ellipse_cp_width = ellipse_t.short_axis;
        rop.ellipse_cp_area = ellipse_area;
    else
        rop.ellipse_cp_length = NaN;
        rop.ellipse_cp_width = NaN;
        rop.ellipse_cp_area = NaN;
    end

    if rip.plot_single_tire_config == 1
        subplot(2,3,2); fill(cpexs, cpeys, 'g');
        xlim([min(min(cpeys)) max(max(cpeys))] * 1.05 + cpexs(1,1));
        ylim([min(min(cpeys)) max(max(cpeys))] * 1.05);
        title(sprintf(['Contact patch length = %0.4g mm (%0.4g in),\n' ...
                        'width = %0.4g mm (%0.4g in),\n' ...
                        'area = %0.4g mm^2 (%0.4g in^2)'], ...
                    rop.cp_length, rop.cp_length / rc.mm_per_in, ...
                    rop.cp_width, rop.cp_width / rc.mm_per_in, ...
```

```
                            rop.cp_area, rop.cp_area / rc.mm_per_in^2));

            if rip.camber_angle == 0 && rip.slip_angle == 0 && rip.wheel_horz_d == 0
                hold on; plot(ellipse_y, ellipse_x, 'b', 'linewidth', 2);
                xl = xlim;
                text(xl(1)*0.9, 0, ...
                        sprintf(['Fitted ellipse(in blue)\n' ...
                                  'has length = %0.4g mm\n (%0.4g in) %0.4g%%,\n' ...
                                  'width = %0.4g mm\n (%0.4g in)  %0.4g%%,\n' ...
                                  'area = %0.4g mm^2\n (%0.4g in^2)  %0.4g%%'], ...
                                  ellipse_t.long_axis, ...
                                  ellipse_t.long_axis / rc.mm_per_in, ...
                                  100 * ellipse_t.long_axis / rop.cp_length, ...
                                  ellipse_t.short_axis, ...
                                  ellipse_t.short_axis / rc.mm_per_in, ...
                                  100 * ellipse_t.short_axis / rop.cp_width, ...
                                  ellipse_area, ellipse_area / rc.mm_per_in^2, ...
                                  100 * ellipse_area / rop.cp_area));
                hold off
            end
        end
        % ************************************************************************
        % Wrap up all plots: hold off, set limits and view angle for 3D
        % ************************************************************************
        if rip.plot_single_tire_config == 1
            subplot(2,3,4); xl = xlim; plot(xl, [1 1]*(-rop.pneumatic_trail), 'k');
            text((xl(1) + xl(2))/2.5, -rop.pneumatic_trail+6, ...
                sprintf('Pneumatic trail = %0.3g mm', rop.pneumatic_trail));
            beam_length = 2 * (rip.rim_diameter/2 + rip.tire_radius) * max(rip.slice_angles);
            xint = linspace(0, beam_length/2, rip.num_slices);
            xint = [-fliplr(xint) xint]; % this dupes center point to match rest
            subplot(2,3,4); plot(rop.slice_horz_ds, xint, 'b.');

            % plot measured vs. modeled centerline displacement. matches:
%             rip.wheel_vert_d = 2.866; % positive = rim sags towards ground
%             rip.wheel_horz_d = 1.1;
%             rip.slip_angle = 0.55*pi/180;      % angle contact patch center line angle
%             rip.camber_angle = 0*pi/180;    % positive = rim tilts to right
%             rip.relaxation_rate_out(2) = 0.991; % before touches ground
%             rip.relaxation_rate_in(2) = 0.991; % just after touching ground
%             rip.relaxation_rate_in(1) = 0.98; % just before leaving ground
%             rip.relaxation_rate_out(1) = 0.995; % after leaving ground
%             rip.relaxation_rate = 1;
%             rip.friction_coefficient = 4;    % 1 for non-skid tape
%             h = gcf;
%             load('tire_centerline_lateral_displacement.mat')
%             figure; plot(x-530, y, 'g.'); hold on; grid on
%             plot(-xint, rop.slice_horz_ds, 'b.');
%             plot(-xint(rip.num_slices-unrelaxed_slice(1): ...
%                         rip.num_slices+1+unrelaxed_slice(2)), ...
%                     rop.slice_horz_ds(rip.num_slices-unrelaxed_slice(1): ...
%                         rip.num_slices+1+unrelaxed_slice(2)), 'r.');
%             legend('measured', 'modeled'); set(gcf, 'Position', [38 499 1879 602]);
%             xlabel('distance along tire (mm)'); ylabel('lateral deflection (mm)')
%             figure(h);

            subplot(2,3,1); hold off; subplot(2,3,2); hold off;
            subplot(2,3,4); hold off; yl = ylim;
            subplot(2,3,5); hold off; ylim(yl);
            subplot(2,3,[3 6]); hold off;
            % view(90, 0); % perfect side view
            view(-10, 15); % nice view into open end of section
            xlim([min_x max_x]); ylim([min_y max_y]); zlim([min_z max_z]);
            set(gcf, 'Position', [1 1 1920 1124], 'Color', 'white')
        end
        if rip.plot_single_tire_config == 1 && rip.plot_wheel_horz_d == 1
            figure; set(gcf, 'name', 'Rotta');
            subplot(3,1,1); plot(xint, rop.slice_horz_ds, 'b.-');
            ylabel('Horizontal displacements'); xlabel('Beam location (mm)')
            title(sprintf('Slices = %g, relaxation rate = %g', ...
```

```
                    rip.num_slices, rip.relaxation_rate))
        subplot(3,1,2); plot(xint, rop.dS); ylabel('dS')
        subplot(3,1,3); plot(xint, rop.dN); ylabel('dN')
    end
return

% *************************************************************************
% *************************************************************************
% Find contact patch and tire shape for one cross-sectional slice
% *************************************************************************
% *************************************************************************
function [status, rim, tire, cp, radii, angles] = ...
         calculate_slice_contact_patch(rip, ...
           slice_vert_d, slice_horz_d, cp_guess, delta)

    rc = Rotta_constants; % define all necessary constants

    slice_horz_d = slice_horz_d - delta;

    % *********************************************************************
    % Specify local parameters
    % *********************************************************************
    status = 1;  % assume success

    % *********************************************************************
    % Calculate additional wheel and tire dimensions for use below
    % *********************************************************************
    rip.rim_angle = 2 * asin(rip.rim_width / (2 * rip.tire_radius)); % of
    rip.rim_height = rip.tire_radius * (1 + cos(rip.rim_angle / 2)); % tire
    tire_circumference = rip.tire_radius * (2 * pi - rip.rim_angle); % from
                                                        % rim to rim
    % *********************************************************************
    % define un-translated and un-rotated rim
    % *********************************************************************
    rip.rim_widths = [-1 0 1] * rip.rim_width/2;
    rip.rim_heights = [0 0 0] + (rip.rim_height - rip.tire_radius);

    % *********************************************************************
    % Origin is at center of undeformed tire cross section. Per Rotta,
    % x is horizontal, z is vertical, and y is normal to cross section
    % so camber rotates about z-axis in 2D
    % *********************************************************************
    R2z = [ cos(rip.camber_angle) sin(rip.camber_angle);
            -sin(rip.camber_angle) cos(rip.camber_angle)];
    xy = R2z*[rip.rim_widths; rip.rim_heights];
    rip.rim_widths(1) = xy(1,1);
    rip.rim_widths(2) = xy(1,3);
    rip.rim_widths(3) = xy(1,2);
    rip.rim_heights(1) = xy(2,1);
    rip.rim_heights(2) = xy(2,3);
    rip.rim_heights(3) = xy(2,2);
    % note that mid-point gets moved to end so that side 2 is element 2

    % *********************************************************************
    % pre-allocate, for speed
    % *********************************************************************
    cp_edges = [0 0]; cp_mids = [0 0]; ctr_x = [0 0]; ctr_z = [0 0];
    s = [0 0]; r = [0 0]; a = [0 0];

    for side = 1:2
        % *****************************************************************
        % calculate actual side bulge arc length, radius, and theta
        % try using MATLAB®'s fzero() instead of a while loop
        % Woo Hoo! takes only 25% of time required by while loop
        % first, make sure initial guess is okay. Adjust as necessary
        % *****************************************************************
        if slice_vert_d > 1
            x0 = [cp_guess rip.rim_width*(-1)^side];
            while 1
                for i = 1:2
```

```matlab
                    cp_edge = x0(i);
                    y0(i) = ((((rip.rim_widths(side) - cp_edge)^2 + ...
                        (rip.rim_heights(side) + rip.tire_radius - slice_vert_d)^2) / ...
                        (2 * (rip.rim_heights(side) + rip.tire_radius - slice_vert_d )))
 * ...
                        (pi - asin((rip.rim_widths(side) - cp_edge) * (-1)^(side) / ...
                        (((rip.rim_widths(side) - cp_edge)^2 + ...
                        (rip.rim_heights(side) + rip.tire_radius - slice_vert_d)^2) / ...
                        (2 * (rip.rim_heights(side) + rip.tire_radius - slice_vert_d
 ))))) - ...
                        (tire_circumference/2 - ...
                        cp_edge * (-1)^(side) + ...
                        slice_horz_d * (-1)^(side)));
                end
                if sign(y0(1)) == sign(y0(2))
                    if rip.run_silent == 0
                        disp('Shifting range of fzero()')
                    end
                    x0(1) = x0(1) + (x0(1) - x0(2))/2;
                    x0(2) = x0(2) + (x0(2) - x0(1))/2;
                else
                    break;
                end
            end % while 1
        else
            x0 = cp_guess;
        end
        % *******************************************************************
        % Finally, calculate contact patch edge location
        % Origin of coordinate system used is center of undeformed tire
        % *******************************************************************
        cp_edges(side) = fzero(@(cp_edge) ...
            (((rip.rim_widths(side) - cp_edge)^2 + ...
            (rip.rim_heights(side) + rip.tire_radius - slice_vert_d)^2) / ...
            (2 * (rip.rim_heights(side) + rip.tire_radius - slice_vert_d ))) * ...
            (pi - asin((rip.rim_widths(side) - cp_edge) * (-1)^(side) / ...
            (((rip.rim_widths(side) - cp_edge)^2 + ...
            (rip.rim_heights(side) + rip.tire_radius - slice_vert_d)^2) / ...
            (2 * (rip.rim_heights(side) + rip.tire_radius - slice_vert_d ))))) - ...
            (tire_circumference/2 - ...
            cp_edge * (-1)^(side) + ...
            slice_horz_d * (-1)^(side))), ...
            x0);
        % *******************************************************************
        % subtract slice_horz_d from rubber on side one,
        % so side one is the left side. Camber angle already
        % incorporated in rim location used in next step.
        % *******************************************************************
        s(side) = tire_circumference/2 - ...
                cp_edges(side) * (-1)^(side) + ...
                slice_horz_d * (-1)^(side);


        % *******************************************************************
        % calculate radius of side bulge via
        % r^2 = x^2 + (z - r)^2 => r = (x^2 + z^2)/(2z) where
        % x = rip.rim_width - cp_edge
        % z = rip.rim_height + rip.tire_radius - slice_vert_d
        % *******************************************************************
        r(side) = ((rip.rim_widths(side) - cp_edges(side))^2 + ...
          (rip.rim_heights(side) + rip.tire_radius - slice_vert_d)^2) / ...
          (2 * (rip.rim_heights(side) + rip.tire_radius - slice_vert_d ));

        % *******************************************************************
        % calculate angle swept by radius of side bulge
        % via theta = pi - asin(x/r)
        % *******************************************************************
        a(side) = pi - ...
            asin((rip.rim_widths(side) - cp_edges(side)) * (-1)^(side) / ...
            r(side));
```

```matlab
        % *****************************************************************
        % See if we have exceeded the tire circumference yet
        % *****************************************************************
        if abs(r(side)*a(side) - s(side)) > rc.circumference_tolerance % short?
            disp('        Ran out of tire while calculating bulge arc length.')
            keyboard;
            status = 0; % not okay
        end

        % *****************************************************************
        % Define tire bulge for this side:
        %   First, point at center of curve, straight up from cp edges
        % *****************************************************************
        ctr_x(side) = cp_edges(side);
        ctr_z(side) = r(side) - rip.tire_radius + slice_vert_d;

        % *****************************************************************
        %   Then, define tire arcs about that center point
        % *****************************************************************
        t = linspace(0, a(side), rc.tire_fineness/2 + 1) - pi/2;
        x = ctr_x(side) + r(side) * cos(t)*(-1)^side;
        z = ctr_z(side) + r(side) * sin(t);

        % *****************************************************************
        %   Assemble both sides into single long array for drawing
        % *****************************************************************
        if side == 1
            tire = [fliplr(x); fliplr(z)]; % flip so pnts end up in order
        else
            tire = [tire(1,:) NaN x;
                    tire(2,:) NaN z];
        end

        % *****************************************************************
        % Calcualte where bottom-dead-center ended up
        % add to edge of contact patch location the difference between
        % radius*angle of side bulge and original length of side to bdc
        % *****************************************************************
        cp_mids(side) = x(1) + (r(side)*a(side) - rip.tire_radius * ...
          (pi - 0.5*rip.rim_angle - rip.camber_angle*(-1)^side))*(-1)^side;

    end % for side = 1:2

    % *****************************************************************
    % Format results as expected by calling function
    % *****************************************************************
    rim = [rip.rim_widths(1:2); rip.rim_heights(1:2)];
    cp = [[cp_edges(1) cp_mids(1) cp_mids(2) cp_edges(2)]; ...
        [1 1 1 1]*(-rip.tire_radius + slice_vert_d)];
    radii = r;
    angles = a;

%       % *****************************************************************
%       % Check results
%       % *****************************************************************
%       carcass_width = 0;
%       i = 2;
%       while i < length(tire)
%           if ~isnan(tire(1,i))
%               carcass_width = carcass_width + ...
%                   sqrt((tire(1, i) - tire(1, i-1))^2 + ...
%                       (tire(2, i) - tire(2, i-1))^2);
%                   i = i + 1;
%           else % jump over, but measure, contact patch
%               carcass_width = carcass_width + ...
%                   sqrt((tire(1, i+1) - tire(1, i-1))^2 + ...
%                       (tire(2, i+1) - tire(2, i-1))^2);
%                   i = i + 2;
%           end
%       end
```

```matlab
%       carcass_width_target = r(1)*a(1)+r(2)*a(2) + cp(1,4) - cp(1,1);
%       if abs(carcass_width_target - tire_circumference) > 0.5 || ...
%          abs(carcass_width - tire_circumference) > 0.5
%           str = sprintf(['Uh oh. Tire carcass width came out wrong:\n' ...
%                          '  carcass width = %0.4g mm,\n' ...
%                          '  tire circumference = %0.4g mm\n' ...
%                          '  for a difference = %0.4g mm (%0.4g%%)'], ...
%                          carcass_width, tire_circumference, ...
%                          (carcass_width - tire_circumference), ...
%                          100*(carcass_width - tire_circumference) / ...
%                          tire_circumference);
%           if rip.run_silent == 0
%               disp(str)
%           end
%       end
return
% ************************************************************************
% ************************************************************************
% define some constants for conversion, initialization, etc.
% ************************************************************************
% ************************************************************************
function [h] = Rotta_constants

% ************************************************************************
% Unit conversions
% ************************************************************************
h.MPa_per_psi = 0.00689475728; % Netwons/mm^2 = MPa
h.MPa_per_bar = 0.1;           % Netwons/mm^2 = MPa
h.bar_per_psi = 0.0689475728;  % 1 bar = 0.1 newton/square millimeter
h.N_per_lb = 4.4482216;        % Newtons per pound
h.N_per_kg = 9.81;             % Newtons per kilogram
h.mm_per_in = 25.4;            % millimeters per inch
h.g = 9.81;                    % acceleration of gravity

% ************************************************************************
% Program constants
% ************************************************************************
h.circumference_tolerance = 0.01;% total tire circumference tolerance
h.tire_fineness = 1000;          % number of points used to define tire
h.percent_force_tolerance = 0.1; %
h.min_lat_disp = 0.1;            % when to quit relaxing lateral dispacemnt
return
```

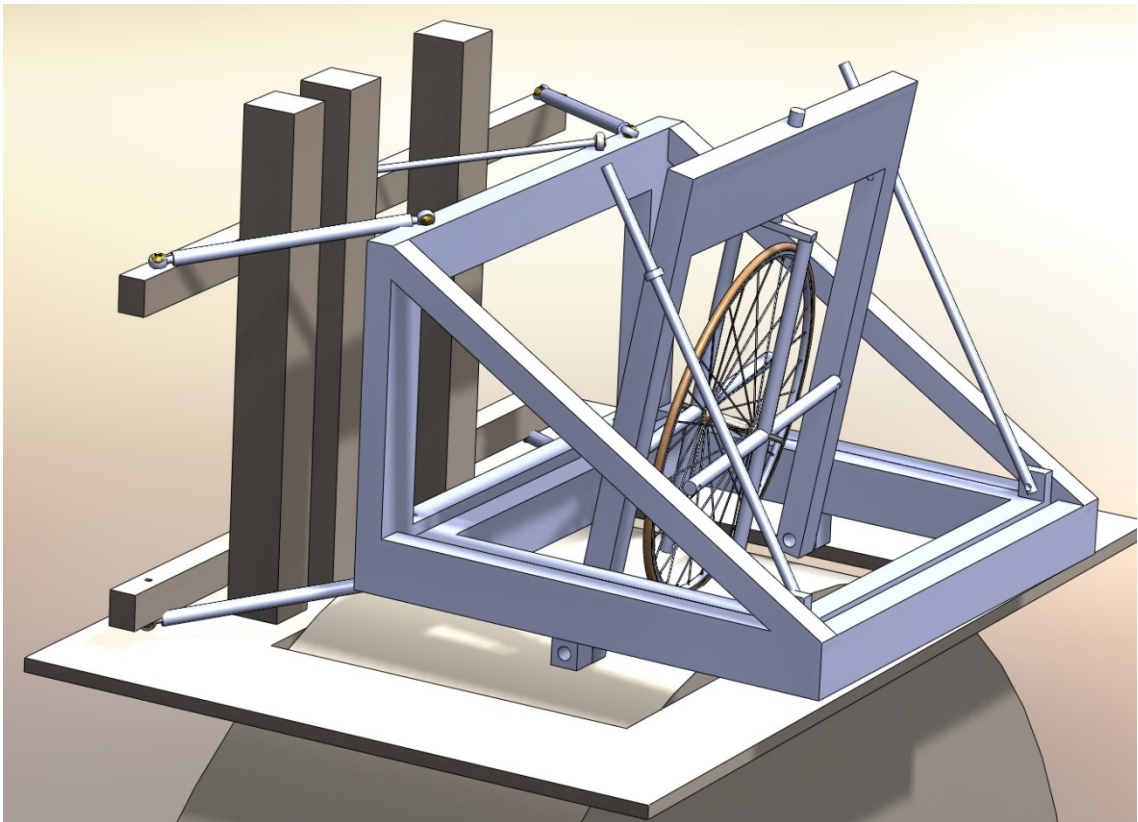# APPENDIX D

ADDITIONAL TEST DEVICE DESIGNS

Figure 8-1: Alternate test device design for TU Delft drum

An early design for a test device to mount above the TU Delft 2.5 meter drum was based on a system of links. It was eventually rejected for being likely too complicated for the budget and manufacturing techniques available.

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Publications

      "Comment on 'On the stability of a bicycle on rollers'", *European Journal of Physics*, 2012

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