Proceedings of the ASME 2009 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference
IDETC/CIE 2009
August 30 - September 2, 2009, San Diego, California, USA

DETC2009-86947

A METHOD FOR ESTIMATING PHYSICAL PROPERTIES OF A COMBINED BICYCLE AND RIDER

Jason K. Moore*
Mont Hubbard
Sports Biomechanics Laboratory
Department of Mechanical and Aeronautical Engineering
University of California
One Shields Avenue
Davis, California 95616
Email: jkmoor@ucdavis.edu
mhubbard@ucdavis.edu

J. D. G. Kooijman
A. L. Schwab
Laboratory for Engineering Mechanics
Faculty of 3mE
Delft University of Technology
Mekelweg 2, 2628CD Delft
The Netherlands
Email: jodikooijman@gmail.com
a.l.schwab@tudelft.nl

ABSTRACT

A method is presented to estimate and measure the geometry, mass, centers of mass and the moments of inertia of a typical bicycle and rider. The results are presented in a format for ease of use with the benchmark bicycle model [1]. Example numerical data is also presented for a typical male rider and city bicycle.

INTRODUCTION

Meijaard et al. [1] recently provided not only a complete review of the bicycle literature but also a concise summary of the equations of motion of the Whipple model [2] as well as benchmark calculations for comparison with other authors' numerical results. Kooijman [3] presented an experimental verification of the weave eigenvalue of Whipple [2] vs. speed. More recently Sharp [4] has reviewed the stability and control of the bicycle by applying optimal control schemes to the model. Building on published bicycle research [1–4], a recent investigation into handling qualities of a bicycle [5] has begun by examining rider control during normal bicycling. As [1–4] make clear, all theoretical or computational models of bicycle dynamics depend crucially on a sound and accurate knowledge of the inertial and geometric parameters of the vehicle and rider.

A non-minimum set of 25 physical parameters is needed to compute solutions to the equations of motion. The present paper outlines a method to estimate these from experiment. They are calculated from the geometry, mass, center of mass locations, and moments of inertia of both the bicycle and rider. We use the methods described in [3] for experimentally measuring the properties of the bicycle. By combining that method with one that estimates the rider's physical properties based on representing the rider as a collection of geometrical shapes we can obtain an estimate of the parameters for the combined bicycle and rider. As an example, the methods are used to calculate the necessary inputs to the benchmark model for a Dutch city bicycle and a male rider that were used in the experiments in [5]. The Netherlands boasts one of the highest percentages of bicycle trips of any country and the bicycle we chose is commonly used for travel.

BICYCLE MEASUREMENTS

The geometry, mass, centers of mass, and moments of inertia of a 2008 Batavus Browser city bicycle were measured using the experimental methods described in [3]. Estimates of these properties can be determined with a detailed CAD model but we chose to measure the quantities for accuracy and time considerations. The bicycle was assumed to be made up of four rigid bod-

*Address all correspondence to this author.

ies: the rear frame (Bf), the front wheel (F), the rear wheel (R) and the handlebar/fork assembly (H).

Geometry

Fifteen geometrical measurements (Fig. 1) of the bicycle were taken using a ruler (± 0.002 m) and an angle gage (± 0.5 deg). Only five of the measurements, w, c, λ[1], r_{R} and r_{F}, are required for the benchmark model (Tab. 12). The rest of the measurements are used to estimate the seated position of the rider described in the *HUMAN PARAMETER ESTIMATION* section. We use the same global coordinate system as the benchmark model. The origin is at the rear wheel contact point with the X-axis pointing forward along the ground, the Z-axis downward and the Y-axis to the right (Fig. 1). All of the dimensions were taken as if they were projections into the XZ-plane except for the hub widths[2]. Note that in the model the top tube is assumed to be horizontal and the measurements were taken from the intersections of tube centerlines. The wheel radii were measured by rolling the bicycle forward with the rider seated on the bicycle for nine revolutions of the wheel. The distance traversed along the ground was measured with a ruler, divided by nine and converted to wheel radii using the relationship between radius and circumference, $r = \frac{c}{2\pi}$. The head tube angle λ_{ht} and the seat tube angle λ_{st} were measured using an electronic angle gage while the bicycle was fixed in the upright position. The trail c was measured by aligning a straightedge along the centerline of the steering axis and measuring the distance along the ground between the front wheel contact point and the end of the straight edge. The values from the measurements of the Batavus Browser are shown in Tab. 1.

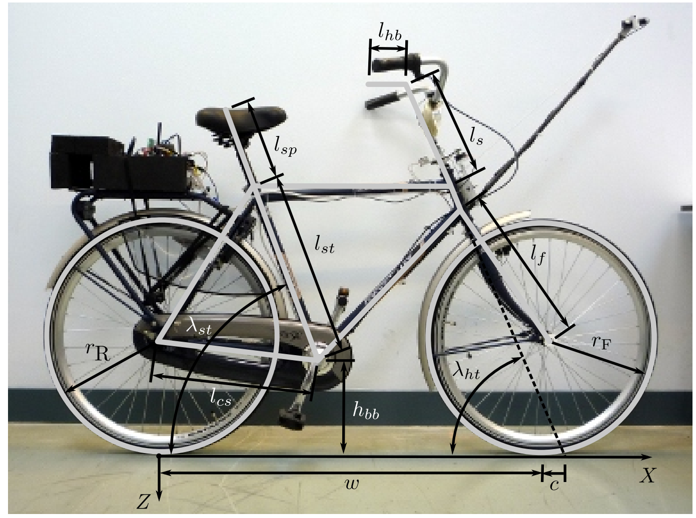

Figure 1. GEOMETRICAL DIMENSIONS OF THE BATAVUS BROWSER BICYCLE SHOWN WITH DATA AQUISITION EQUIPMENT.

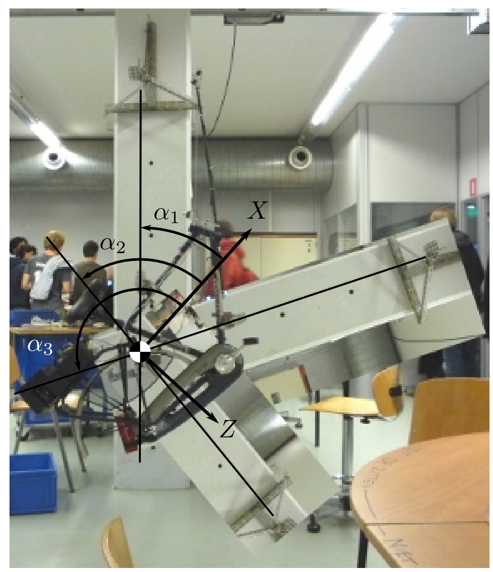

Figure 2. SUPERIMPOSED PHOTOGRAPHS OF THE BICYCLE FRAME HANGING IN THREE ORIENTATIONS FROM THE TORSIONAL PENDULUM SHOWING THE CENTER OF MASS LOCATION AND ORIENTATION ANGLES.

Mass

The bicycle was then disassembled into four parts representing four rigid bodies (rear wheel, front wheel, rear frame, and the handlebar/fork assembly) to facilitate the measurement of the properties of each individual body. The parts' masses (Tab. 2) were measured using a large tabletop scale with an accuracy of ± 0.02 kg.

Center of Mass Locations

The rear frame and handlebar/fork assembly centers of mass were estimated by hanging the parts from a torsional pendulum at three different orientations through the assumed XZ-plane of symmetry (Fig. 2). They were photographed at each orientation and the photos were then pasted into a drafting software package, scaled and rotated such that the part was in the normal upright orientation. The angles, α_i, from the ground plane (XY-plane) to the pendulum axis were estimated with a ± 1 degree accuracy. The centers of mass were located where the pendulum axes intersected each other. The location relative to the benchmark coordinate system was recorded with a ± 0.02 m accuracy (Tab. 3). The centers of mass of the wheels were assumed to be at their geometric centers as dictated by the benchmark model.

[1] $\lambda = 90^\circ - \lambda_{ht}$
[2] Not shown in the figure.

Table 1. BATAVUS BROWSER BICYCLE DIMENSIONS (ACCURACY OF ±0.002 M AND ±0.5 DEG).

Description	Symbol	Value	Units
bottom bracket height	h_{bb}	0.295	m
chain stay length	l_{cs}	0.460	m
fork length	l_f	0.455	m
front hub width	w_{fh}	0.100	m
front wheel radius	r_{F}	0.342	m
handlebar length	l_{hb}	0.190	m
head tube angle	λ_{ht}	68.5	deg
rear hub width	w_{rh}	0.130	m
rear wheel radius	r_{R}	0.342	m
seat post length	l_{sp}	0.240	m
seat tube angle	λ_{st}	68.5	deg
seat tube length	l_{st}	0.530	m
stem length	l_s	0.250	m
trail	c	0.055	m
wheel base	w	1.120	m

Table 2. BATAVUS BROWSER BICYCLE MASSES (ACCURACY OF ±0.02 KG).

Description	Symbol	Value	Units
front wheel mass	m_{F}	2.02	kg
handlebar/fork mass	m_{H}	4.35	kg
rear frame mass	$m_{\mathrm{B}f}$	14.05	kg
rear wheel mass	m_{R}	3.12	kg

Moments of Inertia

Three measurements were made to estimate the globally referenced moments and products of inertia (I_{xx}, I_{xz} and I_{zz}) of the rear frame and handlebar/fork assembly . The same torsional pendulum used in [3] was used to measure the averaged period $\overline{T}_i$ of oscillation of the rear frame and handlebar/fork assembly at three different orientation angles α_i, where $i = 1, 2, 3$, as shown in Fig. 2. The parts were perturbed lightly, less than 1 degree, and allowed to oscillate about the pendulum axis through at least ten periods. The time of oscillation was recorded via a stopwatch (±1 s). This was done three times for each frame and the recorded times were averaged. The coefficient of elasticity k for the torsional pendulum had previously been measured in [3] and found to be $k = 5.01 \pm 0.01\ \frac{\mathrm{Nm}}{\mathrm{rad}}$. Three moments of inertia J_i about the pendulum axes were calculated with

$$J_i = \frac{k\overline{T}_i^2}{4\pi^2} \tag{1}$$

and the numerical values are shown in Tab. 4.

The moments and products of inertia of the rear frame and handlebar/fork assembly with reference to the benchmark coordinate system were calculated by formulating the relationship between inertial frames

$$\mathbf{J}_i = \mathbf{R}_i^T \mathbf{I} \mathbf{R}_i \tag{2}$$

where $\mathbf{J}_i$ is the inertia tensor about the pendulum axes, $\mathbf{I}$, is the inertia tensor in the global reference frame and $\mathbf{R}$ is the rotation

Table 3. POSITION OF THE CENTERS OF MASS OF THE REAR FRAME AND HANDLEBAR/FORK ASSEMBLY (ACCURACY OF ±0.02 M).

Description	Symbol	Value	Units
handlebar/fork	$(x_{\mathrm{H}}, z_{\mathrm{H}})$	(0.88, -0.78)	(m, m)
rear frame	$(x_{\mathrm{B}f}, z_{\mathrm{B}f})$	(0.25, -0.62)	(m, m)

Table 4. REAR FRAME AND HANDLEBAR/FORK MEASURED MOMENTS OF INERTIA.

Rear frame			
i	$\overline{T}_i$ (s)	α_i (deg)	J_i (kg m^2)
1	3.60 ± 0.06	41 ± 1	1.65 ± 0.05
2	3.40 ± 0.06	81 ± 1	1.47 ± 0.05
3	2.50 ± 0.06	150 ± 1	0.79 ± 0.04
Handlebar/fork assembly			
i	$\overline{T}_i$ (s)	α_i (deg)	J_i (kg m^2)
1	1.50 ± 0.06	37 ± 1	0.29 ± 0.02
2	0.70 ± 0.03	105 ± 1	0.06 ± 0.01
3	1.20 ± 0.06	139 ± 1	0.18 ± 0.02

Table 5. REAR FRAME AND HANDLEBAR/FORK INERTIA TENSORS.

Symbol	Value	Units
$\mathbf{I}_{\mathrm{B}f}$	$\begin{bmatrix} 1.12 & -0.44 \\ -0.44 & 1.34 \end{bmatrix} \pm \begin{bmatrix} 0.06 & 0.04 \\ 0.04 & 0.06 \end{bmatrix}$	kg m^2
$\mathbf{I}_{\mathrm{H}}$	$\begin{bmatrix} 0.35 & -0.04 \\ -0.04 & 0.06 \end{bmatrix} \pm \begin{bmatrix} 0.03 & 0.02 \\ 0.02 & 0.01 \end{bmatrix}$	kg m^2

matrix relating the two frames. The global inertia tensor is defined as

$$\mathbf{I} = \begin{bmatrix} I_{xx} & -I_{xz} \\ -I_{xz} & I_{zz} \end{bmatrix}. \tag{3}$$

The inertia tensor can be reduced to a 2×2 matrix because the I_{yy} component is not needed in the linear formulation of the benchmark bicycle[3] and the bicycle is assumed to be symmetric about the XZ-plane. The simple rotation matrix about the Y-axis can similarly be reduced to a 2×2 matrix where $s_{\alpha i}$ and $c_{\alpha i}$ are defined as $\sin \alpha_i$ and $\cos \alpha_i$, respectively.

$$\mathbf{R} = \begin{bmatrix} c_{\alpha i} & s_{\alpha i} \\ -s_{\alpha i} & c_{\alpha i} \end{bmatrix} \tag{4}$$

The first entry of $\mathbf{J}_i$ in Eq. 2 is the moment of inertia about the pendulum axis and is written explicitly as

$$J_i = c_{\alpha i}^2 I_{xx} + 2 s_{\alpha i} c_{\alpha i} I_{xz} + s_{\alpha i}^2 I_{zz}. \tag{5}$$

Calculating all three J_i allows one to form

$$\begin{bmatrix} J_1 \\ J_2 \\ J_3 \end{bmatrix} = \begin{bmatrix} c_{\alpha 1}^2 & 2 s_{\alpha 1} c_{\alpha 1} & s_{\alpha 1}^2 \\ c_{\alpha 2}^2 & 2 s_{\alpha 2} c_{\alpha 2} & s_{\alpha 2}^2 \\ c_{\alpha 3}^2 & 2 s_{\alpha 3} c_{\alpha 3} & s_{\alpha 3}^2 \end{bmatrix} \begin{bmatrix} I_{xx} \\ I_{xz} \\ I_{zz} \end{bmatrix} \tag{6}$$

and the unknown global inertia tensor can be solved for. The numerical results are given in Tab. 5.

Finding the inertia tensors of the wheels is less complex because the wheels are symmetric about three orthogonal planes so there are no products of inertia. The $I_{xx} = I_{zz}$ moments of inertia

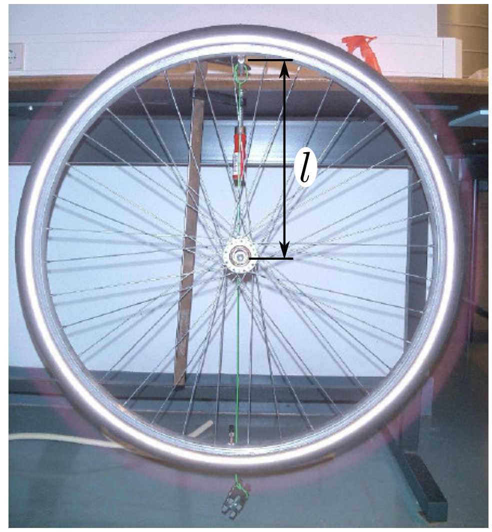

Figure 3. FRONT BICYCLE WHEEL MOUNTED IN A COMPOUND PENDULUM FROM [3].

Table 6. WHEEL MEASURED INERTIA COMPONENTS.

Front wheel	
$\overline{T}$(s)	I (kg m^2)
0.78 ± 0.06	$I_{\mathrm{F}xx} = 0.08 \pm 0.01$
1.37 ± 0.06	$I_{\mathrm{F}yy} = 0.16 \pm 0.03$
Rear wheel	
$\overline{T}$(s)	I (kg m^2)
0.79 ± 0.06	$I_{\mathrm{F}xx} = 0.08 \pm 0.01$
1.51 ± 0.06	$I_{\mathrm{F}yy} = 0.16 \pm 0.04$

were calculated by measuring the averaged period of oscillation about an axis in the XZ-plane using the torsional pendulum setup and Eq. 1. The I_{yy} moment of inertia was calculated with a compound pendulum as described in [3] and shown in Fig. 3 using

$$I_{yy} = \left(\frac{\overline{T}}{2\pi} \right)^2 mgl - ml^2 \tag{7}$$

where $l = 0.303 \pm 0.002$ m is the pendulum length, m is the mass of the wheel, $\overline{T}$ is the averaged period and g is the local acceleration due to gravity. Table 6 gives the calculated values.

[3]The pitch of the rear frame and handlebar/fork assembly are quadratic functions of the lean and steer [6], so the pitch becomes zero in the linear model.

HUMAN PARAMETER ESTIMATION

The measurement of the physical properties of a human is more difficult than for a bicycle because the human body parts are not as easily described as rigid bodes with defined joints and inflexible geometry. Döhring [7] measured the moments of inertia and centers of mass of a combined rider and motor-scooter with a large measurement table, but this is not always practical. The validity of the present method could be determined if such data existed for a bicycle and rider.

Many methods exist for estimating the geometry, centers of mass and moments of inertia of a human including cadaver measurements [8–10], photogrammetry, ray scanning techniques [11, 12], water displacement [13], and mathematical geometrical estimation of the body segments [14]. We estimated the physical properties of the rider in a seated position using a simple mathematical geometrical estimation similar in idea to [14] in combination with mass data from [8].

Several measurements of the human rider were needed to calculate the physical properties. The mass of the rider was measured along with fourteen anthropomorphic measurements of the body (Tab. 7 and Tab. 8). These measurements in combination with the geometrical bicycle measurements taken in the previous section (Tab. 1) are used to define a model of the rider made up of simple geometrical shapes (Fig. 4). The legs and arms are represented by cylinders, the torso by a cuboid and the head by a sphere. The feet are positioned at the center of the pedaling axis to maintain symmetry about the XZ-plane.

All but one of the anthropomorphic measurements were taken when the rider was standing casually on flat ground. The lower leg length l_{ll} is the distance from the floor to the knee joint. The upper leg length l_{ul} is the distance from the knee joint to the hip joint. The length from hip to hip l_{hh} and shoulder to shoulder l_{ss} are the distances between the two hip joints and two shoulder joints, respectively. The torso length l_{to} is the distance between hip joints and shoulder joints. The upper arm length l_{ua} is the distance between the shoulder and elbow joints. The lower arm length l_{al} is the distance from the elbow joint to the center of the hand when the arm is outstretched. The circumferences are taken at the cross section of maximum circumference (e.g. around the bicep, around the brow, over the nipples for the chest). The forward lean angle λ_{fl} is the approximate angle made between the floor (XY-plane) and the line connecting the center of the rider's head and the top of the seat while the rider is seated normally on the bicycle. This was estimated by taking a side profile photograph of the rider on the bicycle and scribing a line from the head to the top of the seat. The measurements were made as accurately as possible with basic tools but no special attention is given further to the accuracy of the calculations due to the fact that modeling the human as basic geometric shapes already introduces a large error. The values are reported to the same decimal places as the previous section for consistency.

The masses of each segment (Tab. 8) were defined as a proportion of the total mass of the rider $m_{\mathrm{B}r}$ using data from cadaver studies by [8].

Table 7. RIDER ANTRHOPOMORPHIC MEASUREMENTS.

Description	Symbol	Value	Units
chest circumference	c_{ch}	0.94	m
forward lean angle	λ_{fl}	82.9	deg
head circumference	c_h	0.58	m
hip joint to hip joint	l_{hh}	0.26	m
lower arm circumference	c_{la}	0.23	m
lower arm length	l_{la}	0.33	m
lower leg circumference	c_c	0.38	m
lower leg length	l_{ll}	0.46	m
shoulder to shoulder	l_{ss}	0.44	m
torso length	l_{to}	0.48	m
upper arm circumference	c_{ua}	0.30	m
upper arm length	l_{ua}	0.28	m
upper leg circumference	c_{ul}	0.50	m
upper leg length	l_{ul}	0.46	m

Table 8. BODY MASS AND SEGMENT MASSES.

Segment	Symbol	Equation	Value	Unit
mass of rider body	$m_{\mathrm{B}r}$	N/A	72.0	kg
head	m_h	$0.068m_{\mathrm{B}r}$	4.90	kg
lower arm	m_{la}	$0.022m_{\mathrm{B}r}$	1.58	kg
lower leg	m_{ll}	$0.061m_{\mathrm{B}r}$	4.39	kg
torso	m_{to}	$0.510m_{\mathrm{B}r}$	36.72	kg
upper arm	m_{ua}	$0.028m_{\mathrm{B}r}$	2.02	kg
upper leg	m_{ul}	$0.100m_{\mathrm{B}r}$	7.20	kg

The geometrical and anthropomorphic measurements were converted into a set of 31 grid points in three dimensional space that mapped the skeleton of the rider and bicycle (Fig. 4). The position vectors to these grid points are listed in Tab. 10. Several intermediate variables used in the grid point equations are listed in Tab. 11 where f_o is the fork offset and the rest arise from the multiple solutions to the location of the elbow and knee joints and have to be solved for using numeric methods. The correct

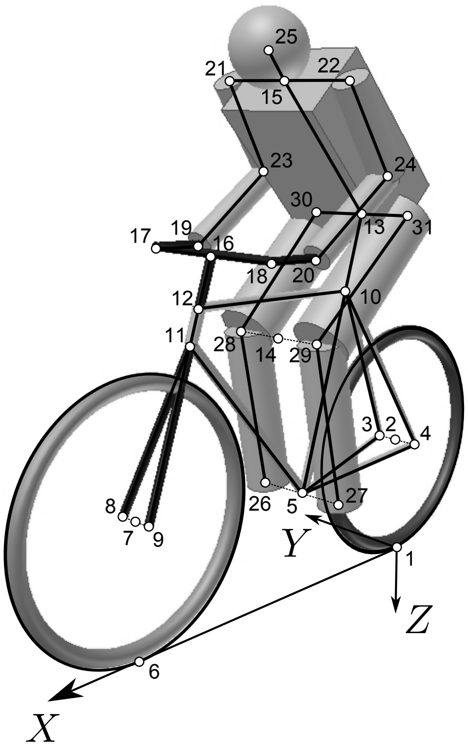

Figure 4. LOCATIONS OF GRID POINTS AND SIMPLE GEOMETRIC SHAPES. SEE ALSO TAB. 10.

solutions are the ones that force the arms and legs to bend in a natural fashion. The grid points mark the center of the sphere and the end points of the cylinders and cuboid. The segments are aligned along lines connecting the appropriate grid points. The segments are assumed to have uniform density so the centers of mass are taken to be at the geometrical centers. The midpoint formula is used to calculate the local centers of mass for each segment in the global reference frame. The total body center of mass can be found from the standard formula

$$\mathbf{r}_{\mathrm{B}r} = \frac{\sum m_i \mathbf{r}_i}{m_{\mathrm{B}r}} = [0.291 \quad 0 \quad -1.109]\,\mathrm{m} \tag{8}$$

where $\mathbf{r}_i$ is the position vector to the centroid of each segment and m_i is the mass of each segment. The local moments of inertia of each segment are determined using the ideal definitions of inertia for each segment type (Tab. 9). The width of the cuboid representing the torso l_y is defined by the shoulder width and upper arm circumference.

$$l_y = l_{ss} - \frac{c_{ua}}{\pi} \tag{9}$$

The cuboid thickness was estimated using the chest circumference measurement and assuming that the cross section of the chest is similar to a stadium shape.

Table 9. SEGMENT INTERIA TENSORS. HERE THE x, y AND z AXES ARE LOCAL.

Segment	Moment of Inertia
cuboid	$\frac{1}{12}m\begin{bmatrix} l_y^2+l_z^2 & 0 & 0 \\ 0 & l_x^2+l_z^2 & 0 \\ 0 & 0 & l_x^2+l_y^2 \end{bmatrix}$
cylinder	$I_x, I_y = \frac{1}{12}m\left(\frac{3c^2}{4\pi^2}+l^2\right), I_z = \frac{mc^2}{8\pi^2}$
sphere	$I_x, I_y, I_z = \frac{mc^2}{10\pi^2}$

$$l_x = \frac{c_{ch} - 2l_y}{\pi - 2} \tag{10}$$

The local $\hat{\mathbf{z}}_i$ unit vector for the segments was defined along the line connecting the associated grid points from the lower numbered grid point to the higher numbered grid point. The local unit vector in the y direction was set equal to the global $\hat{\mathbf{Y}}$ unit vector with the $\hat{\mathbf{x}}_i$ unit vector following from the right hand rule. The rotation matrix needed to rotate each of the moments of inertia to the global reference frame can be calculated by dotting the global unit vectors $\hat{\mathbf{X}}$, $\hat{\mathbf{Y}}$, $\hat{\mathbf{Z}}$ with the local unit vectors $\hat{\mathbf{x}}_i$, $\hat{\mathbf{y}}_i$, $\hat{\mathbf{z}}_i$ for each segment.

$$\mathbf{R}_i = \begin{bmatrix} \hat{\mathbf{X}}\cdot\hat{\mathbf{x}}_i & \hat{\mathbf{X}}\cdot\hat{\mathbf{y}}_i & \hat{\mathbf{X}}\cdot\hat{\mathbf{z}}_i \\ \hat{\mathbf{Y}}\cdot\hat{\mathbf{x}}_i & \hat{\mathbf{Y}}\cdot\hat{\mathbf{y}}_i & \hat{\mathbf{Y}}\cdot\hat{\mathbf{z}}_i \\ \hat{\mathbf{Z}}\cdot\hat{\mathbf{x}}_i & \hat{\mathbf{Z}}\cdot\hat{\mathbf{y}}_i & \hat{\mathbf{Z}}\cdot\hat{\mathbf{z}}_i \end{bmatrix} \tag{11}$$

The local inertia matrices are then rotated to the global reference frame with

$$\mathbf{I}_i = \mathbf{R}_i \mathbf{J}_i \mathbf{R}_i^T. \tag{12}$$

The local moments of inertia can then be translated to the center of mass of the entire body using the parallel axis theorem

$$\mathbf{I}_i^* = \mathbf{I}_i + m_i \begin{bmatrix} d_y^2+d_z^2 & -d_x d_y & -d_x d_z \\ -d_x d_y & d_z^2+d_x^2 & -d_y d_z \\ -d_x d_z & -d_y d_z & d_x^2+d_y^2 \end{bmatrix} \tag{13}$$

where d_x, d_y and d_z are the distances along the the X, Y and Z axes, respectively, from the local center of mass to the global center of mass. Finally, the local translated and rotated moments of inertia are summed to give the total moment of inertia of the rider by

$$\mathbf{I}_{\mathrm{B}r} = \sum \mathbf{I}_i^* = \begin{bmatrix} 8.00 & 0 & -1.93 \\ 0 & 8.07 & 0 \\ -1.93 & 0 & 2.36 \end{bmatrix} \text{kg m}^2. \tag{14}$$

COMBINED REAR FRAME AND RIDER

The mass, center of mass and moment of inertia is calculated similarly to what was previously described. The total mass is

$$m_{\mathrm{B}} = m_{\mathrm{B}f} + m_{\mathrm{B}r}. \tag{15}$$

The center of mass position is

$$\mathbf{r}_B = \frac{m_{\mathrm{B}f}\mathbf{r}_{\mathrm{B}f} + m_{\mathrm{B}r}\mathbf{r}_{\mathrm{B}r}}{m_{\mathrm{B}}}. \tag{16}$$

The two moments of inertia are translated to the center of mass location using the parallel axis theorem (Eq. 13) and the components summed to find the final moments of inertia.

RESULTS

The final results are presented in the form used by the benchmark model (Tab. 12). These can be used to populate the canonical form

$$\mathbf{M}\ddot{\mathbf{q}} + v\mathbf{C}_1\dot{\mathbf{q}} + \left[g\mathbf{K}_0 + v^2\mathbf{K}_2\right]\mathbf{q} = 0 \tag{17}$$

of the linear benchmark equations of motion presented in [1]. The coefficient matrices for the example rider and bicycle follow in Eqs. 18-21 along with the standard eigenvalue plot for the Whipple model (Fig. 5).

$$\mathbf{M} = \begin{bmatrix} 106.87 & 1.41 \\ 1.41 & 0.22 \end{bmatrix} \tag{18}$$

$$\mathbf{C}_1 = \begin{bmatrix} 0 & 27.06 \\ -0.57 & 0.97 \end{bmatrix} \tag{19}$$

$$\mathbf{K}_0 = \begin{bmatrix} -93.73 & -1.58 \\ -1.58 & -0.58 \end{bmatrix} \tag{20}$$

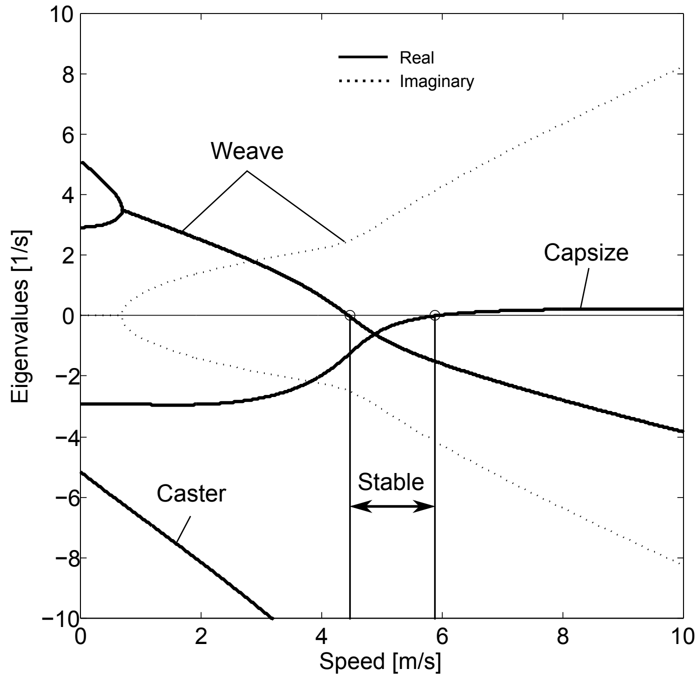

Figure 5. EIGENVALUES OF THE EXAMPLE BICYCLE AND RIDER AS A FUNCTION OF SPEED.

$$\mathbf{K}_2 = \begin{bmatrix} 0 & 78.72 \\ 0 & 1.48 \end{bmatrix} \tag{21}$$

CONCLUSIONS

A simple new and different method of estimating the physical properties of a combined bicycle and rider for use with the linearized benchmark bicycle was presented. The methods described allow one to obtain reasonable estimations of the parameters used to predict the dynamic modes of the benchmark model with minimal experimental equipment and effort. This is unlike the more general methods described in the references because it is specific for a bicycle and rider.The accuracy of the bicycle moment of inertia measurements can be improved by measuring time more accurately with a rate gyro and simple DAQ system and measuring the pendulum angles more accurately with a precision level. The estimations of the human's properties can be improved but not without more time consuming measurement and modeling techniques as described in some of the references.

NOMENCLATURE

α pendulum orientation angle
λ geometric angle

c circumference except for the definition of trail that matches the benchmark model from [1] and the abbreviation for cos

c_α $\cos\alpha$

d distance

f_o fork offset

g local acceleration due to gravity

h height

k pendulum torsional stiffness

l length

m mass

r radius

s, t, u, v intermediate variables, v is also used for forward speed

s_α $\sin\alpha$

w width except for the definition of wheelbase that matches the benchmark model

x center of mass x coordinate for the benchmark bicycle

z center of mass z coordinate for the benchmark bicycle

I global inertia component

J inertia component

T period

$\mathbf{q}$ state vector

$\mathbf{r}$ position vector defined relative to the benchmark reference frame $[r_X \quad r_Y \quad r_Z]$ or to a local reference frame $[r_x \quad r_y \quad r_z]$

xyz local axes

$\mathbf{R}$ rotation matrix

$\mathbf{I}$ globally referenced inertia matrix

$\mathbf{J}$ inertia matrix

$\mathbf{M}, \mathbf{C}_1, \mathbf{K}_0, \mathbf{K}_2$ benchmark canonical matrices

XYZ global axes

REFERENCES

[1] Meijaard, J. P., Papadopoulos, J. M., Ruina, A., and Schwab, A. L., 2007. "Linearized dynamics equations for the balance and steer of a bicycle: A benchmark and review". *Royal Society of London Proceedings Series A,* **463**, August, pp. 1955–1982.

[2] Whipple, F. J. W., 1899. "The stability of the motion of a bicycle". *Quarterly Journal of Pure and Applied Mathematics,* **30**, pp. 312–348.

[3] Kooijman, J. D. G., 2006. "Experimental validation of a model for the motion of an uncontrolled bicycle". MSc thesis, Delft University of Technology.

[4] Sharp, R. S., 2008. "On the stability and control of the bicycle". *Applied Mechanics Reviews,* **61**(6), November, p. 24.

[5] Kooijman, J. D. G., and Schwab, A. L., 2008. "Some observations on human control of a bicycle". In 11th mini Conference on Vehicle System Dynamics, Identification and Anomalies (VSDIA2008), Budapest, Hungary, I. Zobory, ed., Budapest University of Technology and Economincs, p. 8.

[6] Petersen, D. L., and Hubbard, M., 2007. "Analysis of the holonomic constraint in the Whipple bicycle model". In The Engineering of Sport: 7, M. Estivalet and P. Brisson, eds., Springe.

[7] Döhring, E., 1953. "Über die stabilität und die lenkkräfte von einspurfahrzeugen". PhD thesis, Technical University Braunschweig, Germany.

[8] Dempster, W. T., 1955. Space requirements of the seated operator, geometrical, kinematic and mechanical aspects of the body with special reference to the limbs. Technical Report WADC 55-159, Wright-Patterson AFB, Ohio.

[9] Clauser, C. E., McConville, J. T., and Young, J. W., 1969. Weight, volume and center of mass of segments of the human body. Tech. Rep. AMRL TR 69-70, Wright-Patterson Air Force Base, Ohio. NTIS No. AD-710 622.

[10] Chandler, R. F., Clauser, C. E., McConville, J. T., Reynolds, H. M., and Young, J. W., 1975. Investigation of inertial properties of the human body. Tech. Rep. AMRL TR 74-137, Wright-Patterson Air Force Base, Ohio. NTIS No. AD-A016 485.

[11] Zatsiorsky, V., and Seluyanov, V., 1983. "The mass and inertia characteristics of the main segments of the human body". In Biomechanics VIII-B, H. Matsui and K. Kobayashi, eds., Human Kinetic, pp. 1152–l 159.

[12] Zatsiorsky, V., Seluyanov, V., and Chugunova, L., 1990. "In vivo body segment inertial parameters determination using a gamma-scanner method". In Biomechanics of Human Movement: Applications in Rehabilitation, Sports and Ergonomics, N. Berme and A. Cappozzo, eds., Bertec, pp. 186–202.

[13] Park, S. J., Kim, C., and Park, S. C., 1999. "Anthropometric and biomechanical characteristics on body segments of Koreans". *Applied Human Sciences,* **18**(3), May, pp. 91–9.

[14] Yeadon, M. R., 1990. "The simulation of aerial movement-II. A mathematical inertia model of the human body". *Journal of Biomechanics,* **23**, pp. 67–74.

Table 10. SKELETON GRID POINTS WITH RESPECT TO THE GLOBAL FRAME. SEE FIG. 4

Description	Equation	Value (m)		
rear contact point	$\mathbf{r}_1 = [0 \quad 0 \quad 0]$	[0	0	0]
rear wheel center	$\mathbf{r}_2 = [0 \quad 0 \quad -r_\mathrm{R}]$	[0	0	−0.342]
right rear hub center	$\mathbf{r}_3 = \mathbf{r}_2 + \left[0 \quad \frac{w_{rh}}{2} \quad 0\right]$	[0	0.065	−0.342]
left rear hub center	$\mathbf{r}_4 = \mathbf{r}_2 + \left[0 \quad -\frac{w_{rh}}{2} \quad 0\right]$	[0	−0.065	−0.342]
bottom bracket center	$\mathbf{r}_5 = \left[\sqrt{l_{cs}^2 - (r_\mathrm{R} - h_{bb})^2} \quad 0 \quad -h_{bb}\right]$	[0.458	0	−0.295]
front wheel contact point	$\mathbf{r}_6 = [w \quad 0 \quad 0]$	[1.120	0	0]
front wheel center	$\mathbf{r}_7 = \mathbf{r}_6 + [0 \quad 0 \quad -r_\mathrm{F}]$	[1.120	0	−0.342]
right front hub center	$\mathbf{r}_8 = \mathbf{r}_7 + \left[0 \quad \frac{w_{fh}}{2} \quad 0\right]$	[1.120	0.050	−0.342]
left front hub center	$\mathbf{r}_9 = \mathbf{r}_7 + \left[0 \quad -\frac{w_{fh}}{2} \quad 0\right]$	[1.120	−0.050	−0.342]
left front hub center	$\mathbf{r}_{10} = \mathbf{r}_5 + [-l_{st}\cos\lambda_{st} \quad 0 \quad -l_{st}\sin\lambda_{st}]$	[0.263	0	−0.788]
top of seat tube	$\mathbf{r}_{11} = \mathbf{r}_7 + \left[-f_o\sin\lambda_{ht} - \cos\lambda_{ht}\sqrt{l_f^2 - f_o^2} \quad 0 \quad f_o\cos\lambda_{ht} - \sin\lambda_{ht}\sqrt{l_f^2 - f_o^2}\right]$	[0.887	0	−0.733]
top of head tube	$\mathbf{r}_{12} = \left[r_{X11} - \frac{r_{Z11} - r_{Z10}}{\tan\lambda_{ht}} \quad 0 \quad r_{Z10}\right]$	[0.865	0	−0.788]
top of seat	$\mathbf{r}_{13} = \mathbf{r}_{10} + [-l_{sp}\cos\lambda_{st} \quad 0 \quad -l_{sp}\sin\lambda_{st}]$	[0.175	0	−1.011]
center of knees	$\mathbf{r}_{14} = \mathbf{r}_5 + [s \quad 0 \quad -t]$	[0.551	0	−0.746]
shoulder midpoint	$\mathbf{r}_{15} = \mathbf{r}_{13} + [l_{to}\cos\lambda_{fl} \quad 0 \quad -l_{to}\sin\lambda_{fl}]$	[0.235	0	−1.488]
top of stem	$\mathbf{r}_{16} = \mathbf{r}_{12} + [-l_s\cos\lambda_{ht} \quad 0 \quad -l_s\sin\lambda_{ht}]$	[0.773	0	−1.021]
right handlebar	$\mathbf{r}_{17} = \mathbf{r}_{16} + \left[0 \quad \frac{l_{ss}}{2} \quad 0\right]$	[0.773	0.220	−1.021]
left handlebar	$\mathbf{r}_{18} = \mathbf{r}_{16} + \left[0 \quad -\frac{l_{ss}}{2} \quad 0\right]$	[0.773	−0.220	−1.021]
right hand	$\mathbf{r}_{19} = \mathbf{r}_{17} + [-l_{hb} \quad 0 \quad 0]$	[0.583	0.220	−1.021]
left hand	$\mathbf{r}_{20} = \mathbf{r}_{18} + [-l_{hb} \quad 0 \quad 0]$	[0.583	−0.220	−1.021]
right shoulder	$\mathbf{r}_{21} = \mathbf{r}_{15} + \left[0 \quad \frac{l_{ss}}{2} \quad 0\right]$	[0.235	0.220	−1.488]
left shoulder	$\mathbf{r}_{22} = \mathbf{r}_{15} + \left[0 \quad -\frac{l_{ss}}{2} \quad 0\right]$	[0.235	−0.220	−1.488]
right elbow	$\mathbf{r}_{23} = \mathbf{r}_{19} + \left[-u \quad \frac{l_{ss}}{2} \quad -v\right]$	[0.321	0.220	−1.222]
left elbow	$\mathbf{r}_{24} = \mathbf{r}_{23} + [0 \quad -l_{ss} \quad 0]$	[0.321	−0.220	−1.222]
center of head	$\mathbf{r}_{25} = \mathbf{r}_{15} + \left[\frac{c_h}{2\pi}\cos\lambda_{fl} \quad 0 \quad -\frac{c_h}{2\pi}\sin\lambda_{fl}\right]$	[0.246	0	−1.579]
right foot	$\mathbf{r}_{26} = \mathbf{r}_5 + \left[0 \quad \frac{l_{hh}}{2} \quad 0\right]$	[0.458	0.130	−0.295]
left foot	$\mathbf{r}_{27} = \mathbf{r}_5 + \left[0 \quad -\frac{l_{hh}}{2} \quad 0\right]$	[0.458	−0.130	−0.295]
right knee	$\mathbf{r}_{28} = \mathbf{r}_{14} + \left[0 \quad \frac{l_{hh}}{2} \quad 0\right]$	[0.551	0.130	−0.746]
left knee	$\mathbf{r}_{29} = \mathbf{r}_{14} + \left[0 \quad -\frac{l_{hh}}{2} \quad 0\right]$	[0.551	−0.130	−0.746]
right hip	$\mathbf{r}_{30} = \mathbf{r}_{13} + \left[0 \quad \frac{l_{hh}}{2} \quad 0\right]$	[0.175	0.130	−1.011]
left hip	$\mathbf{r}_{31} = \mathbf{r}_{13} + \left[0 \quad -\frac{l_{hh}}{2} \quad 0\right]$	[0.175	−0.130	−1.011]

Table 11. GRID POINT INTERMEDIATE VARIABLES.

Symbol	Equation
f_o	$r_{\mathrm{F}} \cos\lambda_{ht} - c \sin\lambda_{ht}$
s	$0 = l_{ul}^2 - l_{ll}^2 - (r_{Z13} - r_{Z5})^2 - (r_{X5} - r_{X13})^2 - 2(r_{Z13} - r_{Z5})\sqrt{(l_{ll}^2 - s^2)} - 2s(r_{X5} - r_{X13})$
t	$\sqrt{l_{ll}^2 - s^2}$
u	$0 = l_{la}^2 - l_{ua}^2 + (r_{Z21} - r_{Z19})^2 + (r_{X19} - r_{X21})^2 + 2(r_{Z21} - r_{Z19})\sqrt{(l_{la}^2 - u^2)} - 2u(r_{X19} - r_{X21})$
v	$\sqrt{l_{la}^2 - u^2}$

Table 12. COMBINED BICYCLE AND RIDER PARAMETER VALUES.

Parameter	Symbol	Value
wheel base	w	1.120 m
trail	c	0.055 m
steer axis tilt ($\pi/2 - \lambda_{ht}$)	λ	0.38 rad
gravity	g	9.81 N kg^{-1}
forward speed	v	various m s^{-1}
R*ear wheel R*		
radius	r_{R}	0.342 m
mass	m_{R}	3.12 kg
mass moments of inertia	$(I_{\mathrm{R}xx}, I_{\mathrm{R}yy})$	(0.08, 0.16) kg m^2
rear **B***ody and frame B*		
position center of mass	$(x_{\mathrm{B}}, z_{\mathrm{B}})$	(0.28, −1.03) m
mass	m_{B}	86 kg
mass moments of inertia	$\begin{bmatrix} I_{\mathrm{B}xx} & 0 & I_{\mathrm{B}xz} \\ 0 & I_{\mathrm{B}yy} & 0 \\ I_{\mathrm{B}xz} & 0 & I_{\mathrm{B}zz} \end{bmatrix}$	$\begin{bmatrix} 11.89 & 0 & -2.13 \\ 0 & I_{\mathrm{B}yy} & 0 \\ -2.13 & 0 & 3.73 \end{bmatrix}$ kg m^2
front **H***andlebar and fork assembly H*		
position center of mass	$(x_{\mathrm{H}}, z_{\mathrm{H}})$	(0.88, −0.78) m
mass	m_{H}	4.35 kg
mass moments of inertia	$\begin{bmatrix} I_{\mathrm{H}xx} & 0 & I_{\mathrm{H}xz} \\ 0 & I_{\mathrm{H}yy} & 0 \\ I_{\mathrm{H}xz} & 0 & I_{\mathrm{H}zz} \end{bmatrix}$	$\begin{bmatrix} 0.35 & 0 & -0.04 \\ 0 & I_{\mathrm{H}yy} & 0 \\ -0.04 & 0 & 0.07 \end{bmatrix}$ kg m^2
F*ront wheel F*		
radius	r_{F}	0.342 m
mass	m_{F}	2.02 kg
mass moments of inertia	$(I_{\mathrm{F}xx}, I_{\mathrm{F}yy})$	(0.08, 0.16) kg m^2