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Some recent developments in bicycle dynamics and control

A. L. Schwab[ℵ], J. D. G. Kooijman[ℵ], and J. P. Meijaard[*]

[ℵ]*Laboratory for Engineering Mechanics, Delft University of Technology*
Mekelweg 2, NL 2628 CD Delft, The Netherlands
a.l.schwab@tudelft.nl

[*]*Laboratory of Mechanical Automation and Mechatronics*
Faculty of Engineering Technology, University of Twente
P.O. Box 217, NL 7500 AE Enschede, The Netherlands
j.p.meijaard@utwente.nl

The paper is devoted to the design of a human-like controller for a bicycle model. For control input both steer torque and upper body lean torque are considered. Optimal controller design (LQR) gives unrealistic feedback gains. Intuitive control design gives satisfactory and realistic results. There seems to be little difference in the dynamics and control of a model with fixed rider and a model with leaned upper body motion.

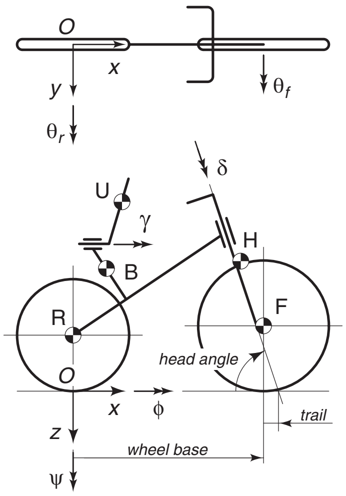
keywords: bicycle dynamics, rider control, optimal control.

1 Introduction

Riding a bicycle is an acquired skill. At low speed the bicycle is highly unstable. But, given some moderate speed the bicycle is easy to stabilize. These observations are confirmed by a stability analysis on a simple dynamical model of an uncontrolled bicycle [1].

This paper is devoted to the design of a human-like control model which is able to stabilize the lateral dynamics of a bicycle. Two known basic features of balancing a bicycle are that a controlling rider can balance a forward-moving bicycle by turning the front wheel in the direction of an undesired lean, and that some uncontrolled bicycles can balance themselves given some initial speed. Two types of controllers will be investigated: an optimal control Linear Quadratic Regulator (LQR) design and an intuitive one based on the steer-into-the-lean concept. Both steer torque and upper body lean torque will be considered as control input.

The literature on rider control for motorcycles is large. An overview can be found in Sharp [2]. Unfortunately, the mass ratio of rider over machine differs significantly between motorcycles and bicycles, which has important implications for the control. The literature on rider control for bicycles is



Rear frame and lower rider body (B)

mass 34 kg

position centre of mass (0.345, 0, -0.765) m

mass moment of inertia $\begin{bmatrix} 3.869 & 0 & 1.300 \\ 0 & 4.667 & 0 \\ 1.300 & 0 & 1.272 \end{bmatrix}$ kg m^2

Rider upper body (U)

mass 51 kg

position centre of mass (0.27, 0, -0.99) m

mass moment of inertia $\begin{bmatrix} 4.299 & 0 & 1.444 \\ 0 & 5.186 & 0 \\ 1.444 & 0 & 1.413 \end{bmatrix}$ kg m^2

position hinge γ (0.3, 0, -0.9) m

Figure 1: Bicycle model with extension of a leaned rider upper body (U) together with degrees of freedom and the upper body extension parameter values from [9]. All other parameter values are from the benchmark bicycle [1]

sparse. Some theoretical and experimental work has been done in the 70s by van Lunteren & Stassen [3], Roland & Lynch [4], and van Zytveld [5]. Getz & Marsden [6] studied control concepts on a highly simplified bicycle model. A recent revival in the study of bicycle dynamics resulted in theoretical studies by Åström *et al.* [7], Sharp [8], and Peterson & Hubbard [9].

2 Bicycle model

The basic mechanical model of the bicycle, see Figure 1, is described in detail in a recent bicycle benchmark paper [1]. In short, it consists of four rigid bodies, viz. the rear frame, the front frame being the front fork and handlebar assembly and the two knife-edge wheels, interconnected by revolute hinges. The rider is assumed to be rigidly connected to the rear frame. As an extension, the rider's upper body lateral lean is added to the system. The contact between the wheels and the flat surface is modelled rigid and non-slipping. The basic model has three velocity degrees of freedom: the roll rate $\dot{\phi}$ of the rear frame, the steering rate $\dot{\delta}$, and the forward speed v which is defined as the angular rate of the rear wheel with respect to the rear frame $\dot{\theta}_r$ times the radius of the wheel. The leaned upper body extension adds one velocity degree of freedom to the basic model, namely the angular rate $\dot{\gamma}$ of the upper body with respect to the rear frame.

The lateral dynamics of the bicycle in the upright configuration can be described by the linearized equations of motion which form a coupled set of second-order differential equations of the form,

$$\mathbf{M}\ddot{\mathbf{q}} + [v\mathbf{C}_1]\dot{\mathbf{q}} + [g\mathbf{K}_0 + v^2\mathbf{K}_2]\mathbf{q} = \mathbf{f}, \tag{1}$$

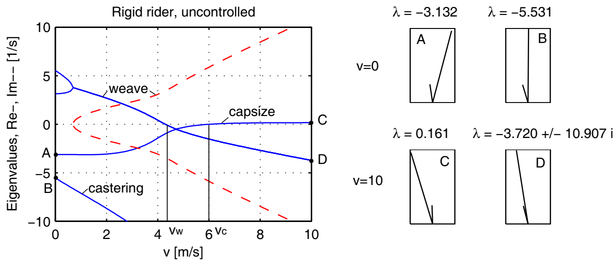


Figure 2: Eigenvalues and eigenmodes for the basic bicycle model with rigid rider, uncontrolled. On the right four eigenmodes are shown, the long line showing the leaned rear frame ϕ and the short line the steer angle δ.

with the forward speed v as a parameter. The degrees of freedom for the basic model are $\mathbf{q} = [\phi, \delta]^T$ and for the extended leaned upper body model $\mathbf{q} = [\phi, \delta, \gamma]^T$. The forcing $\mathbf{f}$ on the right-hand side are the lean torque T_ϕ, the steer torque T_δ, and the upper body lean torque T_γ. The constant entries in matrices $\mathbf{M}$, $\mathbf{C}_1$, $\mathbf{K}_0$ and $\mathbf{K}_2$ are derived by the multibody dynamics software package SPACAR [10] and for the basic bicycle model can be found in [1].

2.1 Uncontrolled motion

The eigenvalues of the uncontrolled motion are shown in Figure 2. For the basic bicycle model there are in principle to four eigenmodes, where oscillatory eigenmodes come in pairs. Two are significant and are traditionally called the *capsize mode* and *weave mode*. The capsize mode corresponds to a real eigenvalue with eigenvector dominated by lean: when unstable, the bicycle just falls over like a capsizing ship. The weave mode is an oscillatory motion in which the bicycle sways about the headed direction. The third remaining eigenmode is the *castering mode* which corresponds to a large negative real eigenvalue with eigenvector dominated by steering. The weave motion is initially unstable but becomes stable at the weave speed $v_w \approx 4.292$ m/s. The capsize motion, initially stable, becomes mildly unstable at $v_c \approx 6.024$ m/s. The bicycle is self-stable for $v_w < v < v_c$ and easy to stabilize above the capsize speed v_c.

3 Controlled motion, rigid rider

Clearly, the bicycle is in need of control below the weave speed and above the capsize speed. The rigid rider model only allows steer torque control since there are no real physical means to exert a lean torque between the rear frame and the ground.

For control purposes, the equations of motion (1) are rewritten into a set of first order differential equations, $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$, with the state vector

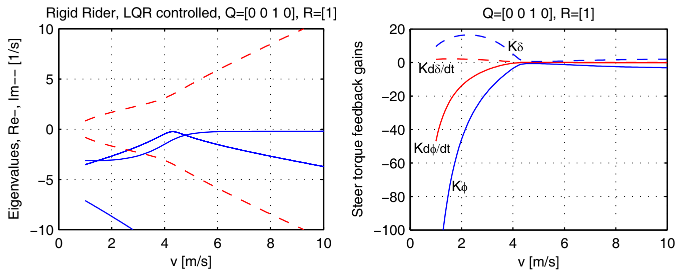


Figure 3: Eigenvalues and feedback gains for the rigid rider with an LQR optimal controller on the steer torque.

$\mathbf{x} = [\dot{\phi}, \dot{\delta}, \phi, \delta]^T$, and the control vector $\mathbf{u} = [T_\delta]$. The control will be assumed to be a linear full-state feedback of the form $\mathbf{u} = -\mathbf{K}\mathbf{x}$, where the linear feedback gains $\mathbf{K}$ will depend on the forward speed.

3.1 Optimal control

Without any prior knowledge of the control strategy, an optimal controller of the Linear Quadratic form (LQR) [11] will be applied. Such a method finds full-state linear feedback gains based on minimizing a performance index, $J = \int_0^\infty (\mathbf{x}^T\mathbf{Q}\mathbf{x} + \mathbf{u}^T\mathbf{R}\mathbf{u})dt$, where the cost in the state is weighted by the matrix $\mathbf{Q}$ and the control cost by the matrix $\mathbf{R}$.

The final goal in balancing a bicycle is to stay upright which can be achieved by minimizing the lean angle, therefore we take $\mathbf{Q} = \text{diag}([0, 0, 1, 0])$. For the control effort we take $\mathbf{R} = [1]$. The resulting eigenvalues and feedback gains are shown in Figure 3. It is clear that above the capsize speed little control is necessary, the feedback gains are low. Contrary, below the weave speed the gains increase without bound, in particular the gain on the lean angle. For a human controller, this is not so realistic. A drawback in the optimal control method is the tendency to stabilize the system by mirroring the real part of the eigenvalues, which can be seen by comparing Figure 3 and Figure 2.

3.2 Intuitive control

The intuitive controller is based on a basic feature of balancing a bicycle: to steer into the undesired lean. The bicycle only needs to be stabilized below the weave speed and above the capsize speed. Above the capsize there is a slow drift from the vertical position and therefore we will use the lean angle as input. Below the weave speed the leanrate suffices as input. The feedback gains will be speed dependent and without any prior knowledge we assume these linear in the forward speed. The intuitive steer-into-the-fall control law then takes on the form, $T_\delta = -K_v(v_{max} - v)\dot{\phi}$ for $v < v_{max}$, and $T_\delta = -K_c(v - v_{max})\phi$ for $v \geq v_{max}$, where v_{max} is a speed in the stable speed range somewhere between the weave and the capsize speed.

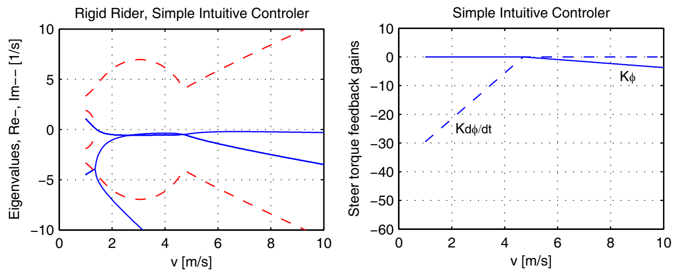


Figure 4: Eigenvalues and feedback gains for the rigid rider with an intuitive steer-into-the-lean controller on the steer torque.

Results for the rigid rider benchmark bicycle are shown in Figure 4, where $v_{max} = 4.7$ m/s, $K_v = -8$ Ns^2/rad, and $K_c = -0.7$ Ns/rad. Compared with the LQR controlled bicycle, Figure 3, the gains are much lower. Note also the qualitatively different eigenvalues. The intuitive controlled bicycle becomes marginally stable over the complete forward speed range.

4 Adding an upper body and control

In the past there has been much debate on the roll of a leaned upper body in the lateral dynamics and control of a bicycle. Since the mass ratio of rider over machine is much larger than that of a motorcycle, there has been speculation on the importance of a leaned upper body opposed to a rigid rider model. Therefore the basic bicycle model is now extended with an upper body which is hinged about a horizontal axis pointing forward, see Figure 1. The leaned upper body adds an extra degree of freedom to the model, the hinge angle γ, together with an extra control torque, T_γ, the action-reaction between the upper body and the rear frame. The state vector now becomes $\mathbf{x} = [\dot{\phi}, \dot{\delta}, \dot{\gamma}, \phi, \delta, \gamma]^T$ and the control vector is now $\mathbf{u} = [T_\delta, T_\gamma]^T$. The properties of the upper body and the rear frame where chosen such that in the case of a fixed hinge they are identical to the rigid rider benchmark problem.

4.1 Uncontrolled motion

The eigenvalues and some eigenmotions of the uncontrolled motion are shown in Figure 5. The passive upper body adds two eigenvalues to the system which turn out to be nearly constant over the complete forward speed range. Moreover, the passive upper body has almost no effect on the remaining eigenvalues, which are nearly the same as those for the rigid rider model, see Figure 2. The weave speed is now $v_w \approx 4.533$ m/s and the capsize speed $v_c \approx 6.037$ m/s, which is less than respectively 6% and 1% change from the rigid rider model. Accordingly, the eigenmodes do not change significantly.

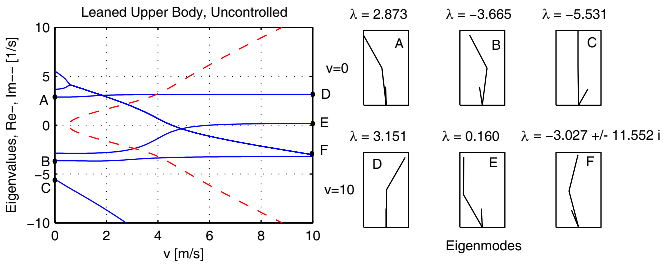


Figure 5: Eigenvalues and eigenmodes for the bicycle model with leaned upper body, uncontrolled. On the right six eigenmodes are shown, the long line shows the leaned rear frame ϕ with on top of that the relatively leaned upper body γ, and the short line shows the steer angle δ.

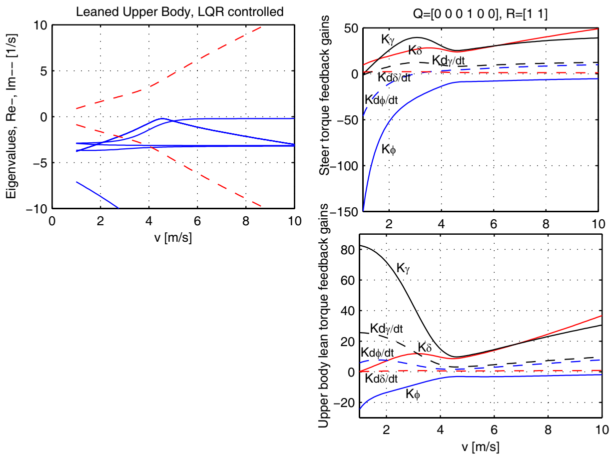


Figure 6: Eigenvalues and feedback gains for leaned rider model with an LQR optimal controller on the steer torque and the upper body lean torque.

4.2 Optimal control

For the optimal controller design we use the same LQR method from Section 3.1. Again the goal is to keep the bicycle upright. Therefore we take the state weight vector as $\mathbf{Q} = \text{diag}([0,0,0,1,0,0])$. The relative control efforts of the steer torque input and the upper body lean torque input can be set by the matrix $\mathbf{R}$, but lacking prior knowledge we use equal control effort weighting $\mathbf{R} = \text{diag}([1,1])$.

The resulting eigenvalues and feedback gains are shown in Figure 6. Above the weave speed we now see considerable control, both in the steer torque and lean torque versus the steer angle and lean angle. Below the weave speed the steer torque gain versus the lean angle becomes very high. Note also the high gain of the lean torque versus the lean angle. The optimal control drawback of mirroring the real part of the eigenvalues is again present.

From a human control perspective these results are quite unsatisfactory.

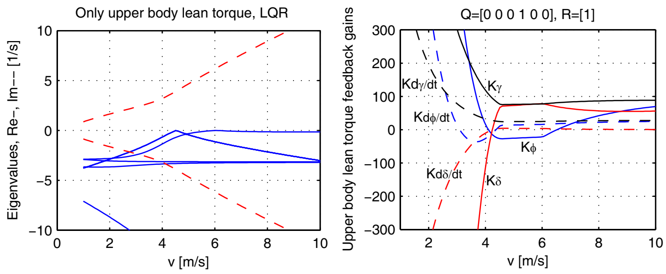


Figure 7: Eigenvalues and feedback gains for leaned rider model with an LQR optimal controller with only upper-body lean-torque control input.

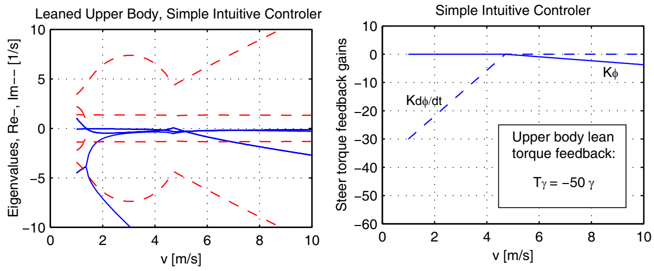


Figure 8: Eigenvalues and feedback gains for a leaned rider model with an intuitive steer-into-the-lean controller on the steer torque and a simple spring-loaded feedback on the upper body.

They also raise the question on the effectiveness of upper body lean control. Therefore we also study a model with only upper-body lean-torque control input, for which the resulting eigenvalues and feedback gains are shown in Figure 7. Clearly, only upper body lean torque control is even worse. The gains are very high for all speeds and below the weave speed they tend to go to infinity.

4.3 Intuitive control

The intuitive controller is based on two concepts: a proportional feedback to stabilize the upper body and steer-into-the-lean. The proportional feedback of the upper body is a simple spring where the spring stiffness is chosen such that we get a realistic frequency for the upper body sway. The steer-into-the-lean controller is identical to the one used in the rigid rider model from Section 3.2. Results for this control model are shown in Figure 8, where the upper body spring stiffness or lean torque feedback gain is $K_\gamma = 50$ Nm/rad, and the steer torque controller uses the identical rigid rider parameters: $v_{max} = 4.7$ m/s, $K_v = -8$ Ns^2/rad, and $K_c = -0.7$ Ns/rad. This intuitive controller gives satisfactory results which are nearly identical to those of the rigid rider model. Note also the stable and slightly damped upper body motion, although there is no real damping in the upper body hinge.

Conclusions

There is little difference between the dynamics and control of a bicycle model with a fixed rider and one with a rider with a lateral leaning upper body. Both models can easily be stabilized by the basic feature of steer-into-the-lean. Such a control model gives realistic feedback gains for realistic stable eigenvalues and eigenmodes. Contrary, a controller designed by an optimal control LQR method gives unrealistic feedback gains and unrealistic stable eigenfrequencies at low speed, although they stabilize the system. Future work is directed to experimental validation of the intuitive controller.

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