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A SIMPLE MULTIBODY DYNAMIC MODEL OF CROSS-COUNTRY SKI-SKATING

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ABSTRACT

The purpose of this paper is to present the development of a simple multibody dynamic model matching the observed movements of the center of mass of a skier performing the skating technique in cross-country skiing. The formulation of the equation of motion was made using the Euler - Lagrange equations applied to a multibody tree-type system in three dimensions. The description of the lower limb of the skier and the ski was completed by employing three bodies, one representing the ski, and two representing the natural movements of the leg of the skier. This simple model is able to show an approximation of the movement of the center of mass of the skier and its velocity behavior allowing to study the effect of the key parameters used to build the model.

INTRODUCTION

Since the year 1890, skiing has developed as a sport activity and assumed its current modern shape. Different skiing techniques have evolved from the traditional Nordic style practiced between 1890 and 1940. These techniques are currently known as alpine, ski jumping, free ride, free style, and cross country Skiing [1].

Among those performed in cross country skiing, the ski-skating technique may be considered a relatively new technique taking into account that the skiing activity can be trace back in time at least 6000 years [2].

The ski-skating technique has been studied extensively from the perspective of physiology, medicine, and training [3]. Nevertheless, as shown in the systematic literature review presented by [4], studies related to the multibody modeling of the skier movement and skiing mechanics of this particular technique have not been developed, and they are still in the phase of being modeled as a point mass body effected by the external forces characteristic of the activity.

Although it is true that these point mass models can provide a relevant amount of information, it is also a fact that known variables that can be practically measured and controlled such as ski leaning angle and, leg extension and angular movement are not possible to study in order to observe their influence on the models.

The number of opportunities that the multibody dynamics field can offer and develop for athletes and teams participating in this discipline is vast.

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Much can be done to achieve optimal performance in the training requirements of elite or high-level competitive skiers with a multibody dynamics model. It is possible to 1) determine in advance all the resultant kinematic parameters associated with the technique, such as velocities and accelerations of the skier or of different parts of the skier's body; 2) describe the complete geometry of the movements of skis, legs, and arms of the athlete; and 3) use these data to adjust the execution of the activity to obtain the maximum output with the least possible effort.

The objective of this paper is to develop a simple multibody dynamic model that reproduces the movement of the center of mass (CM) of a skier performing the ski-skating technique without poles. The model takes as input data not only the measured forces exerted during the skiing phases, but also the ski-skating angle and physiological variables. These include mass, leg length, and its extension.

At the level of simplicity of the present model, it is possible to reproduce the characteristic movement shape of the CM of the skier to achieve close resultant travel velocities and evaluate the influence of the ski-skating angle and the mass of the skier.

The authors of this research work consider that the use of multibody dynamics to simulate the skating technique will open new doors to a better understanding of the occurring phenomena in the execution of the technique. If developed consistently, the model can support athletes in obtaining maximum performance from individual capabilities.

METHODS

Studying the ski-skating technique of cross-country skiing through the lens of simple models is a useful approach to describing and evaluating general questions, ideas, and the specific phenomena under investigation. Also, they can be used as means of evaluating the different paths to follow when the decision of increasing the complexity of the model has become a reality [5].

One of the key aspects of this simplification is to include in the analysis the features that are relevant at the moment for the study. At this stage of the implementation, the following key points have been addressed in order to build and limit the skier model:

-Selection of the multibody dynamic theory to develop the equations of motion of the skier model
-Assumptions to simplify the skier's movements
-Skier's resistance forces: Friction and air drag
-Form of the input of the prescribed motion to represent the movement of the lower limb of the skier

Equation of Motion of the Skier Model

In developing a model of the ski-skating technique, it is necessary to know and depict the coordination pattern of the characteristic movement that an athlete performs during the execution of the physical activity. This description can be made by taking into account that this technique is similar to the one used in ice skating, in which the skater generates the forces by pushing in a sidewards direction [6]. Additionally, in order to begin the formulation of the model, it is necessary to postulate some assumptions which simplify the number of variables and phenomena to be taken into account.

To generate the equations of motion of the skier model, the Euler-Lagrange equations with multipliers [7] are used and, to minimize the effect of the constraint violations that occur during the differentiation of the constraint equations, the Baumgarte stabilization process is applied.

The simplification of the model starts with the selection of the number of bodies used to represent the leg of the skier. The model of the skier can be seen in figure 1.

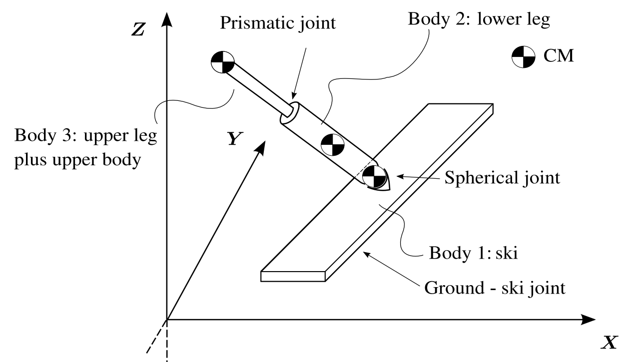

FIGURE 1. DESCRIPTION OF THE MODEL OF THE SKIER [4].

First, as observed in the study made to model the speed skater, it can be considered that the relative motions of the upper body with respect to lower extremities, such as the arm movements and orientation of the trunk, are irrelevant in this first modeling stage. It has been said that the upper body helps to balance the body of the athlete; however, its influence on the kinematical parameters of the movement is not yet clear. What is important to keep in mind is the effect of the position of the upper body when the air drag increases or when it has to be considered as an opposing force to the movement of the skier. The inclusion of the upper body in the model is described next.

Second, the model has to include the natural movement that the leg performs during the execution of the technique without modeling it exactly. To accomplish this, the leg of the skier is modeled as a system formed by three bodies: one that represents the ski, a another that represents the lower part of the leg, and a third that represents the upper leg and upper body.

Third, the joint between the bodies are modeled as follows. The knee joint is modeled as a prismatic joint in order to reduce the input parameters needed to describe it without losing the generality of the movement. The joint between the lower leg and the ski is modeled as a spherical joint resembling the movement of the human ankle. The joint between the ski and the ground will have five restrictions, allowing only the displacement of the ski on a straight path. This is observed on the data taken from the motion capture system.

This last assumed condition of the ski traveling on a straight path is represented by a holonomic constraint. As opposed to the ice skater model where non-holonomic constraints are needed to describe the zero lateral slip, in the case of the skier, it can be observed that the ski has very little steering and will always move in a nearly straight path.

Vector of Generalized Coordinates Because of the fact that the movement of the bodies is represented in a three-dimensional coordinate system, the following generalized coordinates are selected to define completely the location and orientation of the bodies

$$\mathbf{q} = \left[\mathbf{q}_1 \; \mathbf{q}_2 \; \mathbf{q}_3 \right]^{\mathrm{T}} \tag{1}$$

where $\mathbf{q}_i = \left[R_1^i \; R_2^i \; R_3^i \; \varphi^i \; \theta^i \; \psi^i \right]^{\mathrm{T}}$ in which $i = 1,2,3$ represents the bodies of the model, $R_{1\ldots 3}^i$ are the translational coordinate of the origin of the body reference system and φ^i, θ^i and, ψ^i are the Euler angles used to represent the orientation of the body reference system.

The applied sequence of Euler angles is the $Z_1X_2Z_3$. This Euler angle sequence allows to introduce some similar leg angular movements during the performance of the active phase of the skier in accordance with the data obtained from the Vicon motion capture system. Figure 2 presents how the body reference systems are oriented.

Then, the second derivative of the vector of generalized coordinates can be written in the same form as in equation (1). Equation (2) presents the final form of the vector of generalized accelerations.

$$\ddot{\mathbf{q}} = \left[\ddot{\mathbf{q}}_1 \; \ddot{\mathbf{q}}_2 \; \ddot{\mathbf{q}}_3 \right]^{\mathrm{T}} \tag{2}$$

Constraint Equations of the System As it can be seen also in figure 1, it is also necessary to dictate the interaction between the bodies and the environment by the use of constraints.

The first joint to be analyzed will be the ski - ground joint. The ski is considered as the first body of the system. The following facts will be taken into account during the formulation of the geometric restrictions:

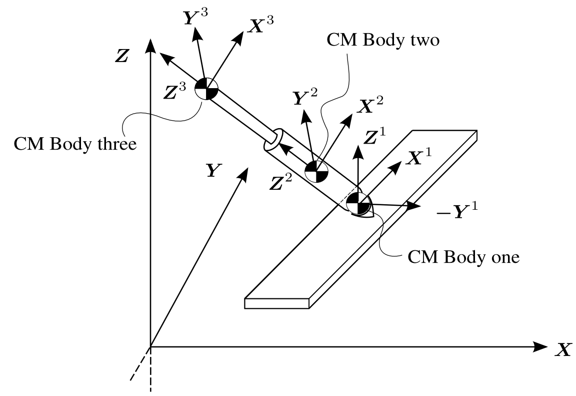

FIGURE 2. BODY REFERENCE SYSTEMS ORIENTATION [4].

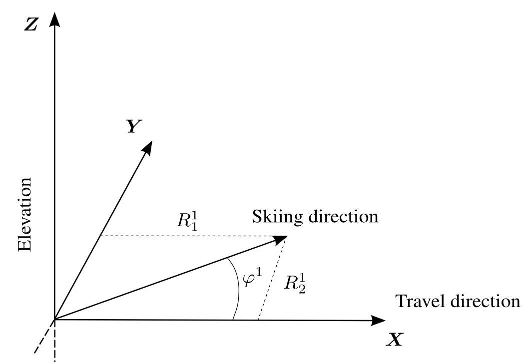

FIGURE 3. DESCRIPTION OF ACTIVE PHASE SIMPLE GEOMETRY [4].

- The ski may travel on a level or inclined plane.
- The direction of the travel of the ski does not change with respect to time.
- The orientation of the body reference system does not change during the active phase of the ski.

In order to understand the development of this first set of constraint equations, figure 3 presents the typical representation of the origin of the body coordinate system attached to the ski. This type of representation is also used for the rest of the bodies.

To constrain the movement to a level or incline plane, the relationship presented in equation (3) is imposed. This constraint guarantees the position level of the ski on the z axis.

$$C1 = R_3^1 - f_3^1(t) = 0 \tag{3}$$

where $f_3^1(t)$, is the time-dependent function that the body reference system has to follow on the z axis.

The assumed fact that the ski travels following a line orientated φ^1 degrees from the global **X** axis provides the necessary information to formulate the second constraint. This constraint is expressed in equation (4).

$$C2 = R_1^1 \sin\varphi^1 - R_2^1 \cos\varphi^1 = 0 \tag{4}$$

To represent the constant orientation of the body one reference system, the approach used to formulate the constraint equation $C1$ is used.

As the Euler angles representing the orientation at any moment of the body reference system do not change during the active phase of the ski, it may be said that the difference of the value of these angles with respect to a set of constant values $(c_{\varphi^1}, c_{\theta^1}, c_{\psi^1})$ does not change. These constraints are presented in equations (5) to (7).

$$C3 = \varphi^1 - c_{\varphi^1} = 0 \tag{5}$$

$$C4 = \theta^1 - c_{\theta^1} = 0 \tag{6}$$

$$C5 = \psi^1 - c_{\psi^1} = 0 \tag{7}$$

In constraint equations $C4$ and $C5$, the value of the constants used for this model is zero. In the remaining $C3$ equation, the angle φ^1 describes one of the most important parameters in the execution of the technique.

The second joint to be described is the spherical joint formed by the ski and the second body. When one analyzes the relative degrees of freedom that the spherical constraint allows between the two bodies, it can be concluded that because of the configuration of the joint, only the relative translation is constrained, leaving only three degrees of freedom of relative rotation.

The necessary condition to be fulfilled in the spherical joint is that two points, P^1 and P^2 on bodies 1 and 2, respectively, coincide throughout the whole motion. This condition may be written as

$$\begin{bmatrix} C6 \\ C7 \\ C8 \end{bmatrix} = \mathbf{R}^1 + \mathbf{A}^1 \bar{\mathbf{r}}_P^1 - \mathbf{R}^2 - \mathbf{A}^2 \bar{\mathbf{r}}_P^2 \tag{8}$$

In equation (8), $\mathbf{A}^1$ and $\mathbf{A}^2$ are the rotation matrices of bodies one and two, respectively, and $\bar{\mathbf{r}}_P^1$ and $\bar{\mathbf{r}}_P^2$ are the local position vectors of the point P.

The last joint to be described is the prismatic joint present between the second and third body. Figure 4 shows the configuration used to formulate the constraint equations.

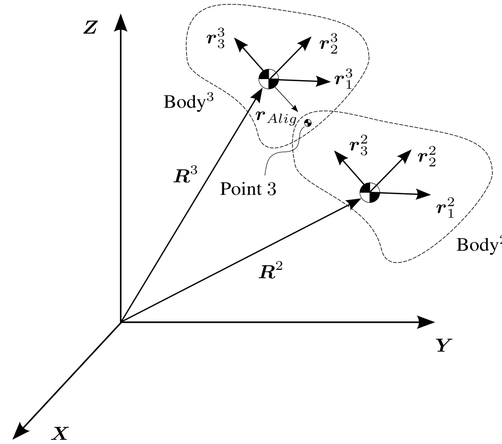

FIGURE 4. PRISMATIC JOINT CONSTRUCTION VECTORS.

A prismatic joint in three dimensions has one DOF and five relative movement restrictions comprised of two translations and three rotations. The use of this joint in the model is convenient for describing the vertical motion of the center of mass of the skier. Indeed, this effect has not been considered in an analogous research project carried out for the speed skater [6], but it is a very important consideration because of the close relationship with the force exerted by the skier during the push-off phase.

The five constraint equations that arise from this joint are based on the following assumptions:

- The vectors $\mathbf{r}_3^2$ and $\mathbf{r}_3^3$ are parallel and they are aligned.
- There is no relative orientation change between the two bodies.

Equations (9) and (10) represent the parallelism condition of the two body vectors positioned in the second and third body.

$$C9 = \mathbf{r}_1^{2\mathrm{T}} \mathbf{r}_3^3 \tag{9}$$

$$C10 = \mathbf{r}_2^{2\mathrm{T}} \mathbf{r}_3^3 \tag{10}$$

Constraints 11 and 12, represent the condition of alignment of the vectors $\mathbf{r}_3^2$ and $\mathbf{r}_3^3$.

$$C11 = \mathbf{r}_1^{2\mathrm{T}} \mathbf{r}_{Alig} \tag{11}$$

$$C12 = \mathbf{r}_2^{2\mathrm{T}} \mathbf{r}_{Alig} \tag{12}$$

The non-relative rotation condition is guaranteed if the vector $\mathbf{r}_1^3$ stays perpendicular to the vector $\mathbf{r}_2^2$ of the second body. In mathematical notation, this condition may be written in the form presented in equation (13).

$$C13 = \mathbf{r}_2^{2\mathrm{T}} \mathbf{r}_1^3 \tag{13}$$

Up to this point, the model contains 18 (generalized coordinates) -13 (constraints) $= 5$ degrees of freedom. It is necessary to specify additional constraints controlling the physiological parameters of leg extension and range of angles.

To restrict the extension of the leg during the active phase, the following length constraint is imposed. See figure 5 for an adequate interpretation of this constraint.

This geometric constraint may be written in terms of the position of the origin of the local reference system located in the third and first bodies (see equation (14)).

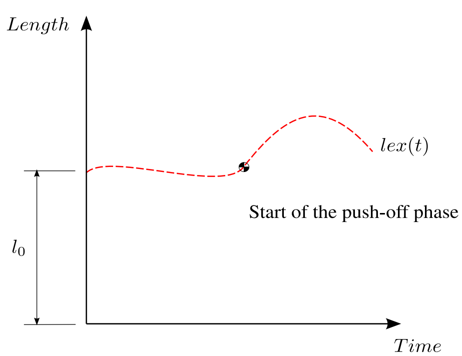

FIGURE 5. CONSTRAINT OF THE LEG EXTENSION IMPOSED IN THE MODEL [4].

$$lex(t) = |\mathbf{R}_3 - \mathbf{R}_1| \tag{14}$$

The next constraints to be imposed are those related to the angles that the leg covers while performing the movement during the active phase. This task can be achieved by using different approaches, one of which is constraining the relative orientation of the second and first body with respect to each other.

In this particular case, one can impose the constraints by formulating the trajectory of the origin of the local reference system of the second body with the introduction of an assumed movement. Figure (6) shows the description of this constraint.

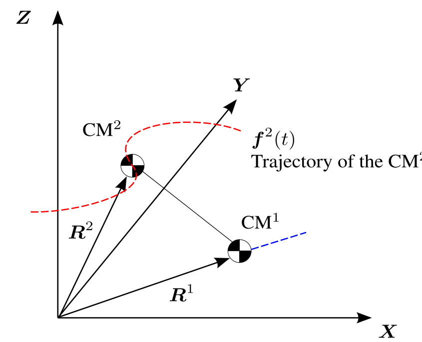

FIGURE 6. CONSTRAINED MOVEMENT OF THE ORIGIN OF THE LOCAL REFERENCE SYSTEM OF THE SECOND BODY [4].

This condition imposes three additional constraints to the model. These are shown in the following equations.

$$C15 = R_1^2 - f_1^2(t) \tag{15}$$

$$C16 = R_2^2 - f_2^2(t) \tag{16}$$

$$C17 = R_3^2 - f_3^2(t) \tag{17}$$

in which $f_1^2(t)$, $f_2^2(t)$ and $f_3^2(t)$ are, respectively, the $X(t)$, $Y(t)$ and $Z(t)$ components of the trajectory $\mathbf{f}^2(t)$ that follows this point.

Vector of Generalized Forces The forces originating from the work performed by the leg during the push-off phase are shown in figure 7. The push-off and the air-drag forces are

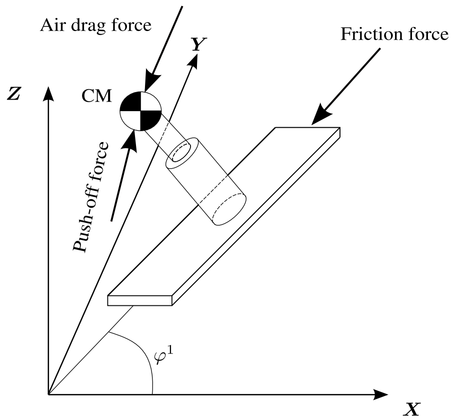

FIGURE 7. EXTERNAL FORCES APPLIED TO THE MODEL [4].

considered to be applied to the CM of the skier and, in the case of the friction force, this is applied to the CM of the first body.

The total vector of generalized forces consists of the following form:

$$\mathbf{Q}_e = \left[\mathbf{Q}_e^1 \ \mathbf{Q}_e^2 \ \mathbf{Q}_e^3 \right]^{\mathrm{T}} \tag{18}$$

where the terms $\mathbf{Q}_e^i$, with $i = 1 \ldots 3$ being the number of bodies, represent the individual vector of generalized forces applied to each one of the bodies.

In this specific case, vector $\mathbf{Q}_e^2$ is equal to **zero** because no forces are applied to this body.

To be able to define the push-off and friction forces on the model, it is necessary to describe the ski-snow interaction, which can be a fairly complex process. [8, 9].

Although many studies have been made in order to describe this phenomenon properly, there is still much to be done and discovered regarding the acting mechanisms in connection with the intervening variables.

One of the most general forms to formulate the ski-snow interaction is to consider the resistive force as a combination of the penetration forces and the friction force [10] produced. However, on the other hand, it is possible to include more complex formulations such as those presented in [8, 11, 12]. For the present case, no theoretical formulation will be used to model the ski-snow interaction; instead, these resistive forces and push-off forces will be taken from the force measurement system attached to the ski bindings that are used on the trial runs.

The specifications of the equipment used to obtain these data can be summarized as follows:

- Two custom-made small and lightweight (1070 g) force plate pairs built by the Neuromuscular Research Center, University of Jyväskylä

FIGURE 8. FORCE PLATES ON THE SKI BINDINGS.

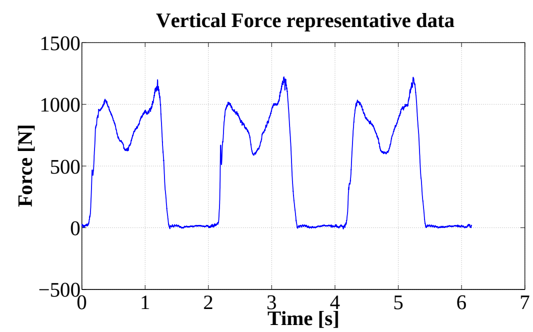

FIGURE 9. TOTAL FORCE EXERTED BY THE SKIER DURING THE ACTIVE PHASE [4].

- 12- channel ski force amplifier built by the Neuromuscular Research Center, University of Jyväskylä
- A/D converter with sampling rate of 1 kHz, model NI 9205; National Instruments; Austin, Texas, USA
- Wireless transmitter WLS-9163; National Instruments; Austin, Texas, USA. PC laptop with wireless receiver card and data collection software LabVIEW 8.5; National Instruments; Austin, Texas, USA.

The total weight of the measurement system was 2030g, being the data capturing and transmitting equipment the most representative part of this weight with 1050g.

In figure 8, the physical aspect of the ski binding system is shown. The output of the discrete force data to be handled is presented later in figure 9.

In order to use the discrete data generated from the measurement instruments, it is first necessary to transform the data into continuous functions, with the purpose of making them smooth (up to the second derivative). Because of the periodic characteristic of the force generation during the actives phases, the procedure employed to achieve this fitting is based on the use of the Fourier series. The set of data is fitted by the application of equation (19).

$$y(i) = a_0 + \sum_{k=1}^{m} \left(a_k \sin(k\,i) + b_k \cos(k\,i)\right) \tag{19}$$

In the Fourier expansion, m is the total number of Fourier coefficients employed to perform the fitting; a_0, a_k, and b_k are the Fourier series coefficients; y is the expected value of the unknown; and i is the known variable usually referred to as the time step size of the capture process. These last two terms are combined to form the set (y_1, i_1), (y_2, i_2), $\ldots$, (y_n, i_n).

To have an idea of how good the fitted function is, the Pearson correlation coefficient r_{xy} is calculated in the way specified in equation (20). If the result of this coefficient is closer to the value of one, it indicates that the function used to calculate the expected values might be used to explain the behavior of the captured data.

$$r_{xy} = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{\sqrt{n\sum x_i^2 - \left(\sum x_i\right)^2}\sqrt{n\sum y_i^2 - \left(\sum y_i\right)^2}} \tag{20}$$

The air friction or air drag will be accounted as presented in [6, 10]. The mathematical representation is given as

$$F_{air} = -\frac{1}{2} C_d A \rho v^2 \tag{21}$$

where C_d is the air drag coefficient. The value adopted for this simulation is 0.5 taking as a reference the modeling done by [10]. A is the projected area of the skier normal to the wind, ρ is air density, and v is the velocity of the air with respect to the skier. In this model, the air drag forces are assumed to be acting on the CM of the skier.

The projected area used for the inclusion of the air drag force was taken as a constant value similar to the rectangular dimensions of the upper body of the skier which was also considered to be facing the main axis of displacement at all times.

As in this simplified model, the forces are considered to be applied to the CM of the bodies, then the components of the generalized force vector acquire the form presented in equations (22) and (23).

$$\left(\mathbf{Q}_e^1\right)_R = \mathbf{A}^1 \bar{\mathbf{F}}^1_{friction} \tag{22}$$

$$\left(\mathbf{Q}_e^3\right)_R = \mathbf{A}^3 \left(\bar{\mathbf{F}}^3_{push} + \bar{\mathbf{F}}^3_{air}\right) \tag{23}$$

Form of the Equation of Motion The equation of motion is presented then as a set of DAE's index-1 system that can be integrated using the built-in MATLAB (Math Works, Inc., Natick, MA, USA) function ODE45 to obtain the velocities and positions of the interested points of the model during the simulation time. Equation (24) presents the matrix configuration of the equation of motions including the terms of the Baumgarte stabilization method.

$$\begin{bmatrix} \mathbf{M} & \mathbf{C}_\mathbf{q}^{\mathrm{T}} \\ \mathbf{C}_\mathbf{q} & 0 \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}} \\ \lambda \end{bmatrix} = \begin{bmatrix} \mathbf{Q}_e + \mathbf{Q}_v \\ \mathbf{Q}_d - 2\alpha\left(\mathbf{C}_\mathbf{q}\dot{\mathbf{q}} + \mathbf{C}_t\right) - (\beta)^2\mathbf{C} \end{bmatrix} \tag{24}$$

In equation (24), $\mathbf{M}$ is the mass matrix of the system, $\mathbf{C}_\mathbf{q}$ is the constraint Jacobian matrix, $\ddot{\mathbf{q}}$ is the vector of generalized accelerations, λ is the set of Lagrange multipliers, $\mathbf{Q}_e$ and $\mathbf{Q}_v$ are, respectively, the vector of external forces and the quadratic velocity vector, $\mathbf{Q}_d$ is the vector that arises after taking the second differentiation of the vector of constraints and, finally, α and β are the Baumgarte stabilization parameters.

In order to select a convenient value for these stabilization parameters, the method proposed in [14] is used.

EXPERIMENTAL PROCEDURE

In order to validate the simple model proposed, it is necessary to utilize the force and motion capture data. The specific purposes for which these data are used are, first, to provide the model with the real force data (push-off force and snow friction force) and, second, to know the other key input parameters such as ski orientation, leg extension and, leg rotation about the axis representing the travel direction.

The motion capture data equipment set used is the Vicon System MX manufactured by Vicon Motion Systems, consisting of 16 cameras to acquire the positions of the different interested points marked on the body of the skier.

The arrangement of the marker set attached to the test subject is presented in figure 10, and figure 11 illustrates the general location of cameras on test site.

Another important parameter to monitor while performing the trial runs is the travel velocity of the skier. This variable is controlled by using a set of light indicators which show the prescribed pace at which the skier has to perform these trials.

All the gathered information is then processed by using the Vicon Nexus software from which the data is exported as open source .c3d files to finish the post-processing on the own developed MATLAB code created for this purpose.

The main outputs of this own application are the resultant simulated position of the CM of the skier, its average travel velocity and the comparison of these previous parameters with those obtained from the motion capture data and, the post-processing of the representative push-off and friction forces. This

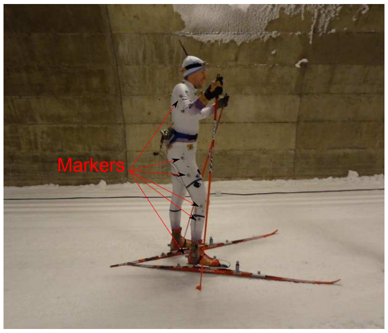

FIGURE 10. ARRANGEMENT OF THE MARKER SET [4].

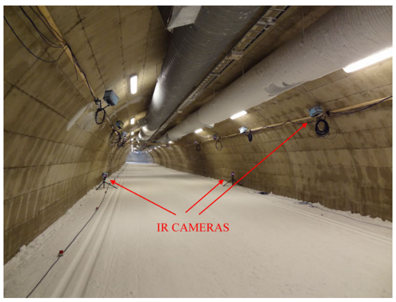

FIGURE 11. ARRANGEMENT OF THE CAMERA SET.

own application also has the capability of post-processing the information from the .c3d files obtained from the motion capture system.

RESULTS

In order to test the skier model, the following physiological parameters are needed to provide all the necessary data to run the simulation. These basic physiological input data are presented in table 1.

TABLE 1. MASS AND LEG DIMENSION OF THE SKATE SKIER

Variable	Value
Mass (Kg)	80
Upper leg (m)	0.6
Lower leg (m)	0.5

For the case of the parameters related to performing the technique, table 2 presents the basic parameters selected to perform the simulation.

TABLE 2. DEFINITION OF THE TECHNIQUE PARAMETERS

Variable	Value
Phase Time (sec)	0.59
Skating angle (deg)	$\pm$ 15
Initial velocity (m/s)	0
Number of strides	30

As the simulation is based on several strokes and changing the active leg (right or left) on every stroke, the skating angle can take a positive or negative value.

After simulating the simplified model with the aforementioned assumptions, the following results can be presented and discussed. Figure 12 illustrates the representative segment of the force chosen to act as a push-off force. This force is presented together with its equivalent fitted continuous function.

It is possible to see how the fitted curve seems to represent satisfactorily the behavior of the discrete data. The Pearson correlation coefficient found for this process was 0.9813.

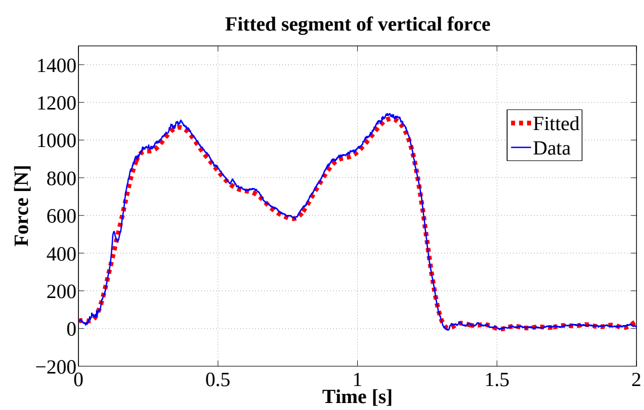

FIGURE 12. FORCE SEGMENT FITTED.

The same procedure was used to fit the representative friction forces and the discrete data from the motion capture system. The results of these fitting processes were similarly satisfactory as in the case of the push-off force.

Figure 13 presents the comparison of the skier's CM position with the data obtained from the motion capture system (dotted line) and the results of the simulation after the velocity of the skier model remained constant (continuous line).

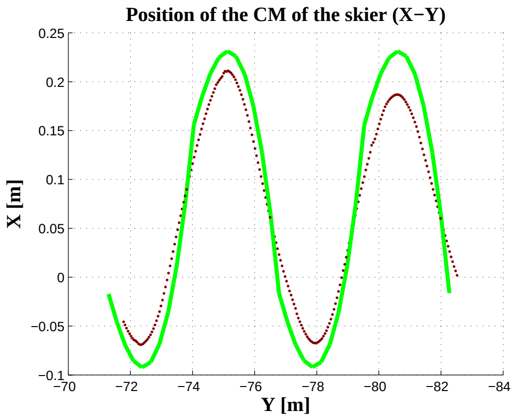

FIGURE 13. CM POSITION COMPARISON.

Table 3 shows the differences found between the trial run, motion capture, and simulated velocity.

TABLE 3. RESULTANT AVERAGE TRAVEL VELOCITY

Velocity (m/s)	Value	Diff. (Rel.)
Trial run set up velocity	5	–
Motion capture system travel velocity	4.83	3.52%
Simulation travel velocity	4.36	10.78%

CONCLUSIONS

The use of multibody dynamic models in real-life applications has been undoubtedly useful for enhancing the processes in which they are involved. The ski-skating technique does not escape from this trend.

When developing this research, it became evident during the literature review about ski-skating multibody models [4] that the amount of studies related to multibody dynamic models of this technique, and even more so in the case of forward dynamic modeling. This leaves an interesting open area to be addressed in the near future.

One of the most interesting results is the reproduction of the skier's CM position achieved after performing the simulation. As the movement of the legs is introduced as a combination of data derived from the motion capture system and the capabilities of the constraints imposed on the model, it is possible to refine the study even further in order to get much closer results without converting the model into an inverse dynamic model.

In the case of velocity, the selection of the active forces acting on the model have proven to be of importance. In the case of the present model and the environment where the trial runs were performed, more precise values of air drag coefficient should be considered. It is also important to expand this model to re-create closer ski-snow interactions.

The form of comparison of the results was limited by the physical capacity of the Vicon Motion Capture System. This system is well recommended to perform studies such as gait analysis in closed spaces. However, due to the characteristics of the skiing technique, only a capture of 12 to 14 meters of trial-run length was achieved.

The authors consider that this model can be used as step towards understanding the ski-skating technique and its integration with the multibody dynamics.

Further development of multibody dynamic models may support the research on muscle actuation, energy consumption, and stresses affecting bones. Also, the possible impact of the ski-skating technique on atheletes' lower limb joints can be assessed, and common injuries that top competitive athletes may develop with the continued practice of this sport discipline can be studied better.

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