A review on bicycle and motorcycle rider control with a perspective on handling qualities

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This paper is a review study on handling and control of bicycles and motorcycles, the so-called single-track vehicles. The first part gives a brief overview on the modelling of the dynamics of single-track vehicles and the experimental validation. The second part focusses on a review of modelling and measuring human rider control. The third part deals with the concepts of handling and manoeuvrability and their experimental validation. Parallels are drawn with the literature on aircraft handling and pilot models. The paper concludes with the open ends and promising directions for future work in the field of handling and control of single-track vehicles.

1. Introduction

Bicycling has become the most dangerous mode of transport in the Netherlands.[1] During the period of study, 2004–2009, there was a steady 3.6% decrease in the number of deaths due to traffic accidents, from 881 deaths in 2004 to 720 in 2009. The safety improvement was not found for cyclists. The number of cyclist deaths did not decrease but remained constant during this period at roughly 185, accounting for a quarter of all traffic-related deaths in 2009. Most of the deaths in the cycling category occurred as a result of an accident between a cyclist and a motorised vehicle, only 15 deaths occurred as a result of single party cycling accidents.

The number of serious injuries as a result of traffic accidents (all modes) alarmingly has increased from roughly 15,400 serious injuries during 2006 to over 18,500 in 2009. Even more alarming is that more than 50% of these serious injuries were incurred in an accident without a motorised vehicle (bicycles and pedestrians only). Furthermore, 10,810 serious injuries, roughly 58% of all serious injuries, were cyclists, of which the vast majority (9240 in 2009) occurred in single party accidents.

Decreasing the number of bicycling accidents and the seriousness of the accidents has, therefore, become a priority for the Dutch government. Possible methods include: improving road infrastructure and roadside furniture; changing traffic laws; adding passive and active safety features to the bicycle, rider and other road vehicles; (refresh) rider training and riding courses specifically tailored for groups such as young children, teenagers and the elderly (the

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elderly are a significant group which stands to benefit from improved safety [1]); adjusting the bicycle's handling qualities to be better suited to the rider. This latter method implies improving the rider–bicycle system to reduce the number of accidents. Considering that the vast majority of the cycling accidents are single vehicle accidents, improving the rider–bicycle system could yield a significant reduction in accidents.

Handling qualities can be viewed as vehicle performance indicators. The handling qualities of a bicycle reflect on control aspects related to the use of the bicycle during cycling, where good handling qualities indicate easy use by the rider. The bicycle handling qualities, therefore, are related to both the bicycle and the rider. To be able to predict bicycle handling qualities, therefore, requires models of both the bicycle and rider. There are validated models of the bicycle,[2] but no validated models of the rider exist at present.

The field of bicycle handling qualities is still in its infancy. The same is true for the other single-track vehicle, the motorcycle. In other fields, most notably in the field of aircraft flight dynamics this is not the case. Enormous interest was generated for aircraft handling qualities, also called flying qualities, due to the serious safety issues that badly flying aircraft present.[3,4] There are, however, some important and fundamental differences between modelling the pilot–aircraft and rider–bicycle interactions, making a direct implementation of aircraft handling qualities in the field of cycling unlikely: In cycling, the rider is not only the controller but the rider also contributes significantly to the mechanical system (up to 90% of the mass of the complete bicycle–rider system); the rider has a two significant control mechanisms to steer the bicycle: turning the handlebar to steer the bicycle, similar to the manner in which a pilot controls an aircraft by moving the control stick, and the application of a lateral lean (roll) torque to steer the bicycle. These differences make the rider–bicycle interaction challenging and require different modelling and simulation approaches.

This paper aims to provide a review of research efforts that have gone into the handling and control of bicycles and motorcycles. Additional material on bicycle dynamics and control can be found in [5]. The paper here, is split into three parts: vehicle dynamics, rider control and handling qualities. It starts by briefly summarising, in Section 2, the most important work on the modelling of the dynamics of single-track vehicles and the experimental validation there of. Rider control, Section 3, starts by recounting experimental rider control observations before focussing on single-track vehicle specific rider control models. The rider control section ends with a review of the experimental validation of many of the rider control models. Then, in Section 4, handling qualities and in particular handling qualities in relation to three vehicle roles: safety, normal riding and racing, are discussed. Definitions of concepts such as manoeuvrability and handling qualities are reviewed. The paper ends with conclusions and a discussion on open ends and promising directions for future work in the field of handling and control of single-track vehicles.

2. Dynamics of single-track vehicles, modelling and experiments

There have been a number of review papers on the subject of the dynamics of single-track vehicles. Notably Meijaard et al. [2] carry out a comprehensive review on all those that investigated bicycle dynamics. Sharp [6] recently reviewed the dynamics and control of bicycles and earlier on motorcycle steering behaviour.[7] In this section, only some of the highlights will be presented.

The actual physical development of the bicycle was an evolutionary process as described in [8]. Starting as a walking machine with vertical steering axis in the early nineteenth century, the bicycle transitioned through a period as a high-wheeler for greater speed but less braking

Figure 1. The safety bicycle, around 1890, from [9].

stability, to the standard 'safety' bicycle by the end of the nineteenth century, see Figure 1. This evolutionary process took many years and was based on a system of trial and error. Since 1900, the bicycle has been optimised further, however, the geometry did not change much and most of these optimisations were cosmetic.

As early as 1869,[10] it was already noted that a bicycle and rider in forward motion balance by steering towards a fall. By 1899, Carvallo [11] and Whipple, [12] independently of one another, were the first to develop the equations of motion for a bicycle with which they could predict the stability of the lateral motions. In the following, 100 years over 50 other authors independently modelled the bicycle until in 2007 Meijaard et al. [2] benchmarked Whipple's bicycle model and compared them all.

The first to develop the equations of motion for a motorcycle was Döhring [13] in 1953. He also carried out experiments with a motorcycle to validate his model. His model was essentially the same as that of Whipple [12] and did not take tyres or suspension into account. The first to investigate the motorcycle's stability using a proper tyre model was Sharp.[14] Since then practically all research into single-track vehicle dynamics and modelling has been focussed on motorcycles, see the recent review by Popov et al.[15]

The most significant (complete) multi-year, multi-researcher scientific programme focussed on single-track vehicle stability and control was carried out in the early 1970s by the Cornel Aeronautical Laboratory (later renamed Calspan). Much of the work on bicycles, however, was carried out for the Schwinn Bicycle Company and has only just become publicly available. The work consisted of both modelling and experimentally measuring bicycles and motorcycles (with tyres) [16,17] and their control [18,19] and the comparison of experimental manoeuvres with time series simulations with the computer. This was quite a feat considering the (analogue) computer system technology available at the time. Some of the results of the Calspan experimental work on bicycle tyres [16] which was done to develop their tyre models has been doubted by some with respect to their camber thrust coefficients, but 40 years on their work is still the only complete bicycle tyre data set available.

Most experimental work for measuring bicycle and motorcycle parameters, the necessary input for bicycle models, has been performed in a similar manner to Döhring; where the most difficult part of measuring the mass moment of inertia has been done by using a torsional pendulum.[16,20–22] In performing model validation, Roland and Massing [16] at Calspan developed an interesting method for applying a known lateral (perturbation) force to the

bicycle: a calibrated fireworks style rocket attached to the bicycle! They concluded that this method was not ideal as it did not cause significant lateral dynamics. Instead, the bicycle rolled and steered slightly to one side which caused a large (and undesired) yaw motion, but did not dynamically excite the roll and steer. For the recording of data during experiments, an ingenious boom system was used at Calspan to relay the measured signals of the wired sensors on the bicycle via cables to large (bulky) ticker-tape recorders that were placed in a chase car. This method was also used by Eaton [20] to carry out experiments to validate the motorcycle model developed by Sharp [14] and theoretical rider control and motorcycle handling work by Weir.[23] With modern electronics, this has become much easier. Kooijman et al. [22] placed all the required measuring equipments (power supply, sensors, digital to analogue converter and laptop computer) on the bicycle to experimentally validate the benchmark bicycle model for a range of speeds.

Recent experimental investigations into the benchmarked bicycle model include Stevens, [24] who used a variable geometry bicycle to validate the model for a wide variety of bicycle geometries. Tak et al. [25] investigated both theoretically and experimentally the effect of various parameters on the stability of a bicycle, while Moore et al. [26] comprehensively measured the parameters of six common bicycles to compare their uncontrolled stability based on the linearised equations of motion. In order to carry out rider control experiments on a treadmill, Kooijman and Schwab [27] experimentally investigated the dynamics of the bicycle on the same treadmill to validate that no significant slip takes place at the wheel–treadmill contact.

Recently and based on the work by Meijaard et al.,[2] Kooijman et al. [28] showed both theoretically and experimentally that the long-cited myths, essential for bicycle self-stability: trail and gyroscopic effect of the front wheel, are neither required nor necessary for bicycle self-stability. They also showed that a third set of parameters, the mass and mass moments of inertia, can be used just as effectively for stabilising and destabilising a bicycle. This discovery radically changes the available design space for self-stable bicycles. For example, in the same work, they present a design for a self-stable rear-wheel-steered bicycle.

3.   Rider modelling

How rider control is modelled depends on the control actions carried out by actual riders and which of these rider control outputs are expected to be of influence according to the author. For example, if the rider's motion is considered negligible, i.e. the rider does not move relative to the vehicle, then the rider is often modelled as a single rigid body that is rigidly connected to the rear frame, as done by Whipple.[12] On the other hand, if the rider is perceived to move laterally, the rider is often modelled as a two-piece body. The lower part is then usually rigidly attached to the rear frame and the upper part is modelled as an inverted pendulum and between the two, a spring and damper is sometimes placed.[29–32] For some applications, such as recumbent bicycles, the lower body motions are considered important and thus the upper legs and lower legs are modelled as moving separately whilst the upper body is fixed to the rear frame.[33] Finally, some authors have tried to make more accurate statements about the rider's control actions by applying multi-body approaches, including full muscle–skeleton models.[34,35]

A second important aspect is the type of steer control that a rider performs; position (angle) or force (torque) control. For automobiles, it has generally been accepted that the driver performs position (angle) control.[36,37] However, recent research has shown that stiffness control is implemented by the driver in throttle control (using the ankle) and that the impedance can

also vary for steering control.[38,39] The automobile driver performs large rotational motions of the steering wheel during normal driving situations. This is not the case on bicycles and motorcycles, where the handlebar is only turned through a very small angle. Furthermore, to enter a corner counter-steer, the turning of the handlebar in the opposite direction to the desired direction is initially applied to the steering assembly for all single-track vehicles. It is, therefore, of interest to know whether the rider uses steer torque or angle control, or maybe something in between such as impedance (steer-stiffness) control.

The third important aspect in rider modelling is the control task. Authors distinguish between stabilising control and path following control. Stabilising control is generally implemented when the goal is to understand the machine to be controlled, i.e. the effort required to keep the machine from falling over. Path following control is implemented when the goal is to follow a set course. There are generally two path following approaches: the first is compensatory, where the current position is compared with the pre-determined desired position and the control output adjusted accordingly to compensate for a miss-match. The second path following control approach uses look ahead (preview). By applying some form of weighting function, a required control output based on the oncoming path and the current location is determined. Preview control has been implemented in lap time optimisation,[40] but also for more general purposes,[41] to compare different control algorithms, vehicles or both such as with the optimal manoeuvre method.

This section will first review those that have investigated rider control through observations, before reviewing proposed theoretical rider models. The section will close with reviewing the experimental validation of rider models and some discussions.

3.1. *Rider control observations*

Many authors have observed which rider actions or motions actually take place. Most have done so in a qualitative manner, without actually measuring any parameters. A smaller group has performed quantitative observations of which the majority of authors have focussed on motorcycle riding.

3.1.1. *Qualitative analysis of rider control*

The inventor of the bicycle, Karl von Drais in 1820 was already well aware of the counter steer mechanism used to balance a Draisine (velocipede) [42][1]:

> Alsdann mache man, mittelst leichten Aufsetzens der Füße, große, aber Anfangs langsame Schritte in paralleler Richtung mit den Rädern, und halte die Absätze dabei nicht einwärts, daß man nicht mit denselben unter das hintere Rad komme, und wann man nachher in dem Schuß ist, und aus Versehen die Balance etwas verloren hat, kann man sich gewöhnlich mit den Füßen helfen, oder durch das Leiten, wenn man ein Bischen gegen die Richtung leitet, auf welche der Schwerpunkt des Ganzen sich neigte, und wenn man eine Schwenkung machen will, richte man unmittelbar vorher den Schwerpunkt etwas auf die innere Seite und lenke gleich darauf hin. [42, 1820, p.375]

> Then one makes, by means of lightly putting the feet to the ground, big but initially slow steps in the direction parallel to the wheels, and keeps the heels not inward, in order that one does not come with them under the rear wheel, and when one has later got some speed, and has lost by accident the balance a bit, one can usually help oneself with the feet, or by steering if one steers a little towards the direction in which the centre of gravity of the whole leans, and if one wants to take a turn, one directs the centre of gravity immediately before a little to the inside and steers right after that to that side. (Translation by Meijaard [150])

Prior to the invention of the safety bicycle ($\approx$ 1890), balance by rider steering control had also been described by many others.[8] In 1869, Rankine [10] already described how a leaned forward-moving high wheel bicycle is primarily righted by the lateral acceleration of the support line due to steering. To balance a bicycle that initially is falling to the left, it is steered

to the left, causing the wheels to move on curved paths to the left. These leftward curved paths lead to a leftward acceleration of the support line and thereby bring the support back under the centre of mass. Rankine also compared bicycle balance to that of the motion of an ice skater who, similar to a bicycle, cannot exert a lateral force without rolling over due to the single line of contact. To manoeuvre, riders manipulate this falling: to turn right they first counter-steer left, inducing a lean to the right, and then later steer right in the direction of the induced fall. Rankine illustrated this in Figure 5.[10, p.153]

Since then, many others have qualitatively investigated bicycle stability and control. One of the first to investigate the rider's actions was Wilson-Jones,[43] who investigated motorcycle corner entrance. He installed a torque indicator on the handlebar to indicate if a lean torque, also known as a roll–steer torque, was applied. The force applied to the handlebar by the rider can be decomposed in forces perpendicular to the steering axis and a force parallel to it. The torque generated as a result of the force parallel to the steer axis is called the roll–steer torque. Note that due to muscle stiffening, a rider does not have to lean to apply a roll–steer torque. Wilson-Jones found that the rider applies a lean torque on the handlebar in the direction of the desired roll when entering the corner, and that the rider simultaneously applies a negative torque to steer the handlebar, where a negative torque is a moment applied that opposes the direction of steering (counter-steer). Furthermore, he found that the rider applies a positive steer moment (in the desired direction) in the corner. Exiting corners is achieved by applying a lean torque on the handlebar to roll the bike towards the upright, and that the steer torque is increased further to decrease the radius of curvature of the bike, increasing the centripetal acceleration and thereby righting the bicycle.

The qualitative analysis of rider control actions is an essential element in the bicycle training programme for children with physical and/or mental disabilities developed by Richard Klein and colleagues.[30,44] The programme uses bicycles with different levels of stability augmentation and which have slower dynamics than normal bicycles. The children that participate in the programme generally have slower neuromuscular sensorial feedback loops. The slower dynamics of these bicycles enables the children to learn the non-minimum phase control (turn left to go right) by giving them more time to feel and adjust to the tipping bicycle and to learn to apply the (correct amount of) counter steer. Each rider progresses through a number of bicycles where each new bicycle has less stability augmentation and feels more like a 'normal' bicycle. The first bicycles that each rider uses have so much static stability (very large tyre crown radius) that the rider has to actively 'lean' into the corner for the bicycle to follow suit. The 'teacher' observes whether the rider is leaning in the correct direction when riding in a curve, i.e. leaning into the curve not out of it such as they would if they were relying on an extra support like when using a tricycle or trainer wheels (or a very large tyre crown radius). When this is the case, the pupil progresses to the next bicycle with less augmentation.

3.1.2. *Quantitative analysis of rider control*

The authors that have measured rider control have done so mostly for motorcycles,[17,19–21, 34,45–55] whereas only a few investigated bicycle rider control.[16,18,56–58]

To quantitatively analyse rider control, vehicle and rider states are measured, the most widely measured vehicle states are the steer angle and steer torque (many authors). More elaborate (complete system) state measurements have been carried by Rice [17,45,59] and Aoki [46] with motorcycles and Roland [16,18,57] for bicycles, who measured rider lean, steer torque, steer angle, roll angle,[2] yaw rate and lateral acceleration. Eaton [20] measured steer angle, steer torque, roll angle and roll rate, but not the rider lean, in order to validate Weir's rider model.[23] Katayama et al. [47] measured steer torque and built a device for measuring rider

Table 1. Research questions and authors in quantitative rider control investigations.

Research question	Authors
Control in general	Aoki,[46] Katayama et al.,[47] Bocciolone et al.,[60] Rice et al.,[19,59] Prem and Good,[21]
Experience	Rice,[19,45] Prem and Good,[21] Evertse [52]
Physiological limits	Yokomori et al.,[49] Cossalter et al.,[51] Pierini et al. [50]
Bicycle rider control	van Lunteren et al.,[61,62] Doyle,[58] Kooijman et al. [63,64]

upper body lean, yaw and pitch angle and the rider's lower body lateral motion which they used to validate a motorcycle control model. Rider lean has generally been measured using a rod with one end attached to the rear of the rider's torso near the arms, and the other end to an angular potentiometer based near the saddle. Steering torque has been measured in a number of ways, mostly by incorporating strain gauges. More recently, Evertse [52] measured all the forces applied by the rider on the motorcycle by measuring the forces along and perpendicular to the steering axis on the handle bars, the force applied in the lateral direction on the tank, and in the vertical direction on the pegs and saddle (left and right sides). With these measurements, he calculated rider roll–steer torques and rider lean torque.

Most research has been initiated with a specific scientific question in mind and they can be grouped as: rider control in general; rider experience; rider physiological limits; bicycle rider control. Table 1 gives a brief overview of these rider control research questions. What follows is a discussion of these different groups.

3.1.2.1 Rider control in general. To understand the type of control that is applied by riders, authors have investigated the number of control inputs, their magnitudes, and phases and compared them to the vehicle state outputs.

Transfer functions for rider lean and steer torque control have been experimentally investigated by Aoki,[46] who did this with four heavy motorcycles at moderate to high speeds in four tests (steer pulse, lane change (LC), entering a curve and slalom). First, the rider control was investigated at various speeds, then for the different motorcycles and finally for the different manoeuvres. Based on the yaw velocity transfer function, Aoki concluded that only steer torque has to be considered as an input to the system. Aoki, therefore, concluded that motorcycle rider control can be treated as a single input system. Similarly, Katayama et al. [47] found, by carrying out single-LC experiments at 60 km/h with 12 expert riders fitted with a device to measure the rider upper body lean and lower body's lateral shift, both measured relative to the frame, and a steer torque sensor, that steering torque is the dominant control. However, Katayama found that also lower body torque (as the result of a lateral shift) assists and that upper body torque (rider lean) is such that the upper body is kept vertical and does not really contribute to the control. Katayama hypothesises that keeping the upper body upright is probably only performed as a comfort measure by the rider.

An instrumented motorcycle to measure the motorcycle motions together with a method to measure the relative motion of the rider was made by Bocciolone et al.[60] With this setup, they demonstrated that not only does the rider lean, but also shifts inwards (into the curve) on the saddle during tight manoeuvres. They hypothesise that this is because the rider prefers a more upright position during tight manoeuvres.

Rice,[59] on the other hand, found that riders only lean during transient situations (such as during the LC); once the vehicle is in steady state, the upper body returns to the motorcycle's plane of symmetry.

Differences in rider control actions for successfully and unsuccessfully completed manoeuvres were investigated by Rice and Kunkel [19] and Prem and Good.[21] They carried out LC

experiments with both experienced and novice riders and found that there are no discernable differences between the magnitude of the steer or lean control for successful and unsuccessful runs for the same rider. This indicates that timing or phase differences are likely to be the most significant factor for determining whether a LC will be successful or not.

3.1.2.2 Rider experience.   These studies are aimed at finding differences in rider control actions amongst riders with different levels of experience.

For two experimental manoeuvres, Rice [45] and Rice and Kunkel [19] found different rider lean control strategies amongst the different rider groups. In the single-LC manoeuvre,[19] all riders (novice, medium and experienced) initiated the manoeuvre by applying a steer torque. However, they found significant differences amongst the rider groups with regard to leaning action. The novice riders appear to use only upper body lean as a reactive rather than deliberate control, unlike more experienced riders. With the second experiment: entering a constant radius turn,[45] Rice found that novice riders only use steer torque control to initiate the turn while experienced riders use their upper body to get the motorcycle in the desired lean before they apply a steer torque in the desired direction. It was recently shown with multi-body software by Balleti et al. [65] that this type of control, where the rider uses upper body lean to roll the bike in the desired direction and then applies a steer torque, allows a quicker and more precise manoeuvre at a given speed.

Prem and Good [21] also carried out a single (emergency) LC and a constant radius curve test to investigate differences between novice and experienced riders. They concluded that skilled riders have a shorter reaction time, achieve larger maximum steer angles, and apply a reverse steer angle for a shorter period of time than the less skilled riders. They also concluded that short reaction time is important for the success of the manoeuvre. Interestingly, and as could be expected, the less skilled rider group showed more inter-rider variability than the experienced rider group.

Evertse [52] investigated the difference in riding 'styles' between novice, expert and racing motorcycle riders for a 90° corner and a single-LC manoeuvre. Similar to Prem and Good, he found that there was less inter-rider variability for the expert and racing riders than for the novice riders. Furthermore, for both transient manoeuvres, he found that novice riders apply large roll–steer torques to initiate a corner whilst expert and racing riders do not. From the LC manoeuvres, Evertse concluded that the difference between novice and experienced riders is that the novice riders are unable to apply sufficient steer torque at higher speeds. He also found that the shape of the applied steering torque becomes more homogenous across the groups at higher speeds as both novice and expert riders resort to applying a pulse torque to initiate the LC and another pulse to exit the manoeuvre. This suggests that force control is taking place. This is likely to be caused by the large inertial force of the front wheel that has to be overcome to change the direction of the front frame at higher speeds as described by the handling quality index by Cossalter and Sadauckas [48] (Section 4.2 and Table 5). The size of the required force is so large that the rider can no longer accurately control the position of the handlebar and thus applies a more 'bang-bang' type of force control.

3.1.2.3 Rider physiological limits.   Physiological limits are human limitations such as the time required to sense a change in a state (time delay), the maximum force a rider can apply, the maximum speed a limb can move at and its maximum acceleration. Rider physiological limits have been investigated in order to limit values for rider model controller feedback gains, etc.

Yokomori et al. [49] carried out straight line experiments with an instrumented motorcycle to study low-speed control. They measured steer angle and motorcycle roll and rider lean angular rates. The riders either had hands on or off the motorcycle during the experiments

at multiple speeds between 3 and 25 km/h. The experiments were carried out by only two riders – one experienced and one novice. The time delay between the rider lean angle and the motorcycle roll angle was investigated and despite the large standard deviation, a slight general trend of decreasing lean and roll angles for increasing speed was found. The power spectra was also investigated and found that these remain constant up to approximately 1 Hz and then fall off.

Cossalter et al. [51] carried out experiments on a simulator setup [66] with five test subjects (with various riding experience) in order to investigate the effect of rider impedance on motorcycle stability. Only the steer and roll actuators of the simulator were used to carry out two experiments, one to measure the effect of a passive rider on steer motion, the other on lean motion. To investigate the passive steer behaviour, they applied a stepped sine with constant amplitude of 4° for a frequency range of 0.5–8.5 Hz to the handlebar of the simulator and measured the steer torque. They then used the data to identify mass, spring and damper constants by curve fitting of the experimental frequency response function (FRF) to the FRF of two multi-body models. The first model has rigid links connected by hinges for the upper and lower arms whilst in the second the arms are modelled by spring–dampers only. The torso rotates about a fixed axis in both models. The results compare well with those from the literature [34] showing a peak at 2 Hz and an increase in response at about 6 Hz, which they suspect may influence wobble instability. For the passive lean behaviour, they experimentally determined the FRF of the head, chest and waist by placing three accelerometers on the test subject to measure the lateral accelerations. They found that the passive rider roll response shows a peak at about 1 Hz and then drops off, similar to what Yokomori et al. [49] found. They note that the rider's motion relative to the motorcycle tends to minimise the rider's lateral acceleration. More recently, Cossalter et al. [53] and Massaro et al. [54] investigated, both theoretical as experimental, the effect of passive rider effects on motorcycle stability. The numerical models shows that the passive response of rider's body causes a significant stabilisation of the wobble mode, but may destabilise the weave mode at high speed. This effect is similar to the one caused by the steering damper. These results are confirmed by the experiments from Massaro et al.[54]

To investigate the maximum effect that rider lean can have on the motion of a motorcycle, Pierini et al. [50] investigated the maximum acceleration and the maximum displacement that riders could move their upper body on a motorcycle. They investigated how five riders, leaned left then right before coming back to the upright position as fast as they could and as extreme as they could in a single motion. The experiments were carried out on two stationary upright motorcycles and each rider repeated the experiment six times. The motions were recorded using motion capture equipment with 16 markers placed on the upper body of the rider. With the measured data, they found that the seating position affects the maximum values of acceleration and displacement of rider's centre of gravity. They also found two different styles among the riders for which the maximum values in terms of rider lateral acceleration and displacement were quite different. Therefore, they concluded that even defining the rider's physiological limits is difficult.

3.1.2.4 Bicycle rider control. While most authors have investigated motorcycle control due to the increased safety risks involved, a few have investigated bicycle rider control.

In the late 1960s and early 1970s, van Lunteren et al. [61] were interested in modelling the human control actions, and chose to do this using a bicycle simulator. They had modelled the rider under normal circumstances and validated the model by system identification techniques as discussed here in Section 3.2.1. Even though the correctness of their simulator dynamics was debated, they used it to determine the effect that four different drugs have on human control behaviour for stabilising a bicycle (simulator). Two drugs, secobarbitali natricum

(Seconal Sodium) and aethyl alcohol (Vodka), showed a marked effect, increasing the time delays between the input and output of riders and strongly acting on the remnants, increasing the upper body motion and decreasing the handle bar action. The two other drugs tested: chlordiazepoxydi hydrochloridum (Librium) and perphenazinum (Trilafon) did not have a marked effect on the riders' control actions. Interestingly, they noticed that the time delay of the handlebar action was always about one and a half times that of the upper body action (handlebar control was found to have a time delay of 150 and 100 ms for upper body control) suggesting that the upper body control is governed by hierarchically lower centres of the central nervous system than those which are involved in the control of the handlebar action.

Stassen and Van Lunteren [62] also performed experiments where they restrained the upper body of the rider (similar to Eaton [20]) and experiments where the upper body was free to move. By comparing the two, they concluded that rider body motions are important in normal bicycle riding, however, 'perhaps they are not consciously intended as a contribution in the stabilisation of the bicycle', but rather are intended to control the rider's head position and orientation in space.

Almost two decades later, Doyle [58] investigated bicycle rider control to understand to what extent motor skills necessarily involve higher functions of the cerebral cortex. Doyle investigated balance control during normal cycling for two situations: first on a normal bicycle, and second on a bicycle where presumed self-stability factors such as gyro, trail and head-angle had been removed (destabilised bicycle). The roll angle and steering angle were recorded on both bicycles.

To get such a bicycle with no self-stability factors, he followed Jones' [67] reasoning for an 'unrideable bike' and constructed a machine that had the presumed front frame stabilising factors removed – a vertical head angle and no trail, no gyroscopic effect (counter rotating wheel) and no mass offset from the steer axis (counterweight added). Doyle reasoned that 'without these, all movements of the front wheel come exclusively from the human control system'. More specifically, he reasoned that it eliminates the lean to steer coupling, thus on the destabilised bicycle body movements have no effect on the overall motion (the system becomes a single (steer) input system).

By comparing steer and roll angle, rate and accelerations for entering and exiting a circular path at about 13 km/h for the normal bicycle, Doyle found that there was a 120 ms lag between the roll and steer action, indicating that steering follows rolling. Doyle was not sure if the control is achieved through control of the riders arms or through the bicycle's self-stability and coupled upper body motion. Therefore, he continued with experiments on the destabilised bicycle.

The experiments with the destabilised bicycle were carried out at about 7 km/h. The riders were told to simply stabilise the bicycle and not to track a path. To assist in this, the riders were blindfolded, yet all riders tended to remove any turns automatically so that the general direction of the start was maintained. The recorded data showed a 0.2 Hz signal and a 1 Hz signal present in the roll angle. The steering signals follow the roll signals with a mean 120 ms delay. In particular, the steering acceleration signal follows the roll acceleration signal. Thus, Doyle concluded that the basic rider control mechanism feeds the roll acceleration back, multiplied by some constant (gain), as an angle independent force at the handlebar. Interestingly, the recorded data indicate that the rider 'pumps' energy into the system regardless of the control requirements, which Doyle suggests is to increase the system output values such that they go above a threshold below which the rider cannot detect the value. He concluded that:

> Because the system delay in the roll rate is so short it is evident that the output from the vestibular system must go almost directly to the controlling muscles making little or no demand on higher cortical processes for this part of the system.

Two decades after the research by Doyle, understanding what rider control actions are performed, in particular for stabilising without a significant tracking task but also during normal cycling, was explored by Kooijman et al.[63] They used an instrumented bicycle and carried out initial experiments on the open road amongst traffic. Extracting good data from these trials proved rather difficult due to all the external factors that were influencing the control such as wind, (speed)bumps, traffic and dogs. Therefore, they also carried out their experiments under controlled conditions (indoor) on a large treadmill ($3 \times 5\,\text{m}$). The bicycle was ridden by two averagely skilled riders at various speeds, each rider was given enough time to adjust to riding on the treadmill before the measurements started. Three riding cases were considered: normal bicycling, towing and normal bicycling with lateral perturbations. These latter experiments were carried out to identify the effect of the pedaling motion and the effect of upper body motion on the control. The bicycle was equipped with a camera system facing the rider and connected to the rear frame, making it possible to qualitatively investigate rider motion on the bicycle. They concluded that very little upper-body lean occurs and that stabilisation is done by steering control actions only. However, they also found a second control action at very low-forward speed: knee movement. Moreover, they note that all control actions, except for those at very low-forward speeds, are performed at the pedaling frequency, and that the amplitude of the steering motion is inversely proportional to the forward speed. Moore et al. [64] then repeated the treadmill experiments with a motion capture system and quantitatively confirmed the qualitative conclusions from Kooijman et al.[63]

3.2. *Theoretical controller design*

In order to understand and predict the stability and handling of a bicycle–rider system, a model for the complete cycling system is required. In other words, a model of both the bicycle and rider is required. The bicycle is well described by the Whipple model as benchmarked and reviewed by Meijaard et al.,[2] this section, therefore, focusses on the proposed rider models. The rider's influence on the system can be split into two aspects: a controller; and added system dynamics. The added system dynamics, caused by, for example, the rider moving relative to the bicycle, could require the vehicle model to be expanded to include these extra dynamics, such as adding a (controlled) pivoted point mass pendulum to the vehicle to simulate upper body lean.

For the modelling of the human controller, authors have followed three roads for the design and development thereof. First, there is the classical control approach which has been extensively applied to pilot aircraft modelling. This approach is based on observations and the control is determined using system identification techniques which include rider time delays. At the cross-over frequency (the frequency at which the magnitude of the transfer function is unity), the gain roughly has a 20 dB drop-off per decade. Continuous feedback control systems with human neuromuscular properties (dynamics) are usually included in these models. The second road that authors have travelled down is the optimal control framework, where the rider is assumed to be an optimal controller. The method uses optimal control criteria by weighing control effort against the error in the control task. The third road is a collection of 'other' control strategies including fuzzy logic, neural network and very simple 'intuitive' controllers. Authors for both the optimal control and 'other' control strategies have not limited their research to mimicking a human rider, but have also taken advantage of these control strategies to develop 'autopilots'. Table 2 lists the authors that have applied the different control routes and for which situation, tracking or stabilisation, they are used.

All three routes have been reviewed by other authors. These include Guo and Guan [36] and Macadam,[97] who reviewed the driver models for general road going vehicles (mostly

Table 2. Types of single-track vehicle control models.

Type of model	Stabilising	Path following
Classical control	van Lunteren et al.,[61,68,69] van Zytveld,[70] Ruijs and Pacejka,[71]* Yokomori et al.,[72]* Åström et al. [30]	Roland et al.,[16,18,57] van Lunteren and Stassen,[73] Weir,[23]* Rice,[45]* Prem and Good,[21]* Nagai,[74] Katayama et al.,[47]* Levandowski et al.,[75]* Yi et al.,[76]* Tanaka and Murakami,[77] Hess et al.[78]
LQR/LQG optimal control	Schwab et al.,[79] Connors and Hubbard [33]	Katayama et al.,[47]* Sharp,[6,41, 80,81]*, Cossalter et al.,[40,82]*, Bertolazzi et al. [83]*
H_∞ optimal control	Mammar et al.,[84]* Nishimura et al.,[85]* Thanh and Parnichkun [86]	
Intermittent control	Doyle [58]	
Intuitive control	Schwab et al. [79]	
Neural networks		Cook [87]
Fuzzy logic		Fujii et al.,[88]* Chen and Dao,[89–91] Levandowski et al. [75]*
Inverse dynamics		Getz [92–94]
Forward dynamics	Cook [87]	von Wissel [95,96]

Note: A star (*) indicates the model is for motorcycle control.

automobiles). Popov et al. [15] reviewed the modelling of the control of single-track vehicles and in particular the control of motorcycles, while Sharp [6] reviewed the work on the control of bicycles. Here, an updated broad overview of all three routes is given with particular attention paid to the modelling of bicycle control.

3.2.1. *Classical control system design*

In classical control, feedback of the states is used to create a closed-loop controlled system. The systems are usually multiple input multiple output (MIMO) and linear or linearised about a given state. Sometimes, nonlinearities like time delays are introduced. McRuer was the first to develop the classical control systems approach for modelling human control. He applied it successfully to pilot aircraft control.[98–101]

Such a classical feedback control system is shown in Figure 2. According to McRuer, experimental data for a wide variety of single- and multi-loop situations show that the operator (i.e. pilot, driver or rider) adjusts his/her transfer function, Y_e^c, in each feedback loop such that the open-loop function, $Y_e^c Y_c^m$, comprises the effective vehicle dynamics, Y_c^m, and the operator, in the vicinity of the gain cross-over frequency (the frequency at which the gain is unity), ω_c, for that loop has the approximate form:

$$Y_e^c Y_c^m = \frac{\omega_c}{j\omega} e^{-j\omega\tau},$$ (1)

where τ is an effective pure time delay that includes rider neuromuscular dynamics as well as any net high-frequency vehicle dynamic lags and j is the imaginary unit. The cross-over frequency (ω_c) is the product of the rider and vehicle gains. The form of Equation (1) emphasises that the rider's characteristics are optimised to the specifics of the control situation and the vehicle. However, McRuer also found that the human controller is limited in its physical
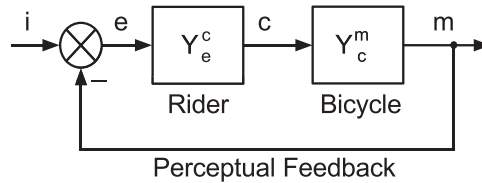
Figure 2. Single-loop control. *i* is reference control value; *m* is actual control value; *e* is control value error; Y_e^c is rider transfer function; *c* is rider output control variable; Y_c^m is machine transfer function.

control capabilities amongst others by the muscle dynamics and neural transport resulting in a delay. He found that a human controller can be described by

$$Y_e^c = K_p \frac{(T_L j\omega + 1)}{(T_I j\omega + 1)} e^{-j\omega\tau}, \tag{2}$$

where T_L is a lead time constant, T_I is a lag time constant, K_p is a static gain and τ is an effective time delay. This in essence makes the human controller a lag–lead system with time delay and limits the systems that a human operator can control.

Although the classical control method is very promising with respect to determining the performed rider control in individual control loops, it is mathematically less well suited for performing/determining multi-loop control. As bicycles and motorcycles require multi-loop control (stabilisation and path following), only a few authors have delved into this method.

Two major classical control rider projects took place in the early 1970s. The first was a study on the differences in rider control under specific circumstances (effect of drugs, alcohol, etc. on rider control) by Stassen and van Lunteren.[56,61,62,68,69,73] The other was a comprehensive theoretical exploration of the manual control of a motorcycle by Weir [23,102] and experimental validation thereof by Eaton.[20] The major topic of discussion amongst authors is the nature of the applied control: position (angle) control or force (torque) control.

Stassen and van Lunteren assumed position control and carried out system identification experiments on a self-developed bicycle simulator. The simulator was a device that could roll, steer, and pedal, and initially had no visual feedback. With the simulator, they experimentally determined the rider control parameters that fit their position (angle) controlled rider model. Unfortunately, they only simulated and measured at one fixed forward speed, namely 15 km/h. They measured the simulator's roll angle, steer angle and the rider's lean angle. Stassen and van Lunteren concluded that the human stabilising control can be described by a proportional and derivative (PD) controller with a time delay for which the input is the frame roll angle and the outputs are the steer angle and upper body lean angle.

The simulator was later extended with visual feedback with which they showed the rider's deviation from a pre-specified path. Further experiments then led Stassen and van Lunteren to conclude that the tracking task does not significantly alter the stabilising task controller parameter values.

Weir,[23,102] who was aware of the work by Stassen and van Lunteren, found, based on determined transfer functions, that it is unlikely that position control is used by a rider and that torque control is far more likely. The transfer functions for motorcycle rider control actions were theoretically investigated by Weir [23] using a theoretical model of the motorcycle by Sharp [14] and a McRuer style rider model based on transfer functions from the literature for the different sensory organ control loops. Weir searched for the best single input to single output transfer functions for rider control in the frequency domain. He found that the best vehicle stabilising transfer function is that of the roll angle to steer torque due to the relatively high cross-over frequency of roughly 1 rad/s (therefore, easy to detect) and the accompanying large
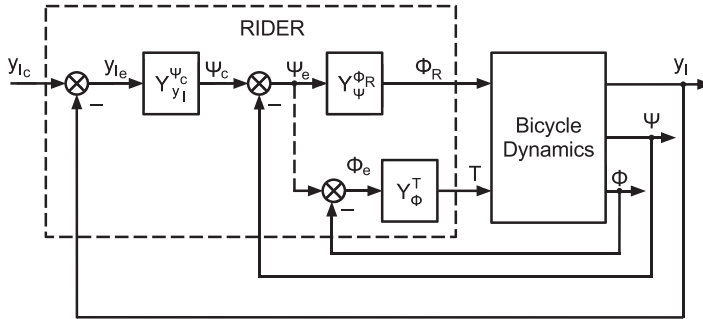
Figure 3. Weir's proposed multi-variable control loop. ϕ is roll angle; ψ is heading angle; y_I is lateral position with respect to the track; ϕ_R is upper body lean angle; T is steer torque and loop-specific transfer functions Y_j^i.

gain and phase margins. The only other good transfer functions Weir found were roll angle to rider lean angle and yaw rate to rider lean angle. None of these three transfer functions, how-ever, is suitable for tracking purposes and lateral position (the lateral deviation from the desired position) must be added to the loop for this to be the case. To achieve good stabilisation and tracking performance with minimal attention and control workload, Weir proposed the rider control model shown in Figure 3. It consists of an inner loop for roll stabilisation ($\phi = 0$), a mid-dle loop controlling the heading (ψ) and an outer loop for the lateral position control (y_{I_c}). The desired lateral position is expressed in a desired heading angle ψ_c from which the actual head-ing angle ψ is subtracted, resulting in the heading error ψ_e. This heading error is then used to control the rider upper body lean angle ϕ_R and steering torque T, which are the human outputs.

Also in the early 1970s, Roland, Massing and Lynch [16,18,57] developed a bicycle (includ-ing tyres) and rider model to study the effect of design parameters on bicycle stability and control where the end goal was to be able to perform simulations of bicycle manoeuvres. They developed a rider model that incorporated a steer and lean torque, delayed proportional and integral and derivative (PID) controller. It was implemented as a simplification for a human lead–lag controller model based on the literature.[103] The developed controller was not well documented but it had both tracking and stabilising control loops.[18] The rider lean torque and steer torque were the outputs for both the stabilising and tracking controller. The stabilis-ing controller inputs were the roll: angle, rate and acceleration. For tracking control, similar to Weir, the vehicle path and heading error information were also required. The tracking con-troller predicted the path based on the current state and comparing this with the desired path and generated an additional roll angle that was added to the desired roll angle. Roland tuned the coefficients of the stabilising controller by investigating the system's response to driving straight ahead and applying a 20° command roll angle (simulating driving straight ahead and going into a constant radius curve). However, even for the best controller, he had an offset between the desired and obtained roll angle.

To our knowledge, Roland never used the rider model that he developed for comparing real manoeuvres with simulations, but Rice [45] later used the controller for the simulation of the same manoeuvre with a motorcycle and riders of different levels of experience. The model, however, did not compare well with experiments, except for in the transient stage of the manoeuvre.

Experimental data, certainly for novice riders, collected by Prem and Good,[21] suggest that there is a strong coupling between steering and rider upper body leaning inputs. Therefore, Prem and Good used Weir's motorcycle and rider models and parameters to analyse the transfer function for roll angle control by rider lean with lean torque to steer torque coupling (Figure 4). They found that the extra coupling increases the gain of the transfer function, allowing the large
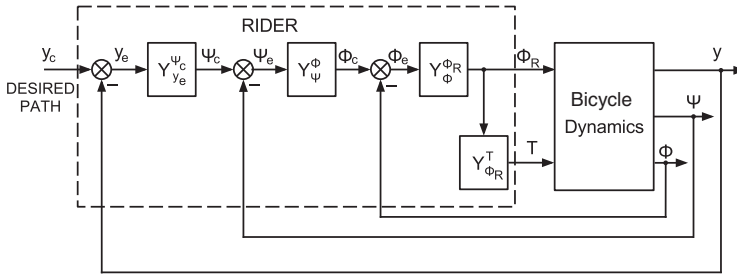
Figure 4. Prem and Good's proposed multi-variable control loop for a novice rider. ϕ is roll angle; ψ is heading angle; y is lateral position with respect to the track; ϕ_R is upper body lean angle; T is steer torque and loop-specific transfer functions Y_j^i.

rider lean to roll angle error gain values (5.5 deg/deg) found by Weir to be decreased to more realistic values that novice riders can achieve (1.0 deg/deg). They also found that this lean to steer coupling applied in the multi-loop situation to have comparable system performance for tracking capabilities. They conclude that the proposed 'unskilled rider' model may have lower stability margins, but requires physically less extreme upper body motions.

Upper body lean control but now for the hands-free situation was investigated by Yokomori et al.[72] They were particularly interested in the effect of time delays. Their rider model consisted of a delayed proportional controller with constant gains. By varying the time delays and the forward speeds (5–25 km/h), they determined the stable region for the time delay. This time delay stable region was found to increase with increasing speed, and the time delay can even become negative (feed forward!) for speeds above 12 km/h. Yokomori et al. did carry out physical experiments to validate their findings but practically all results were within the stable region, and therefore rather inconclusive.

Finally, in a recent theoretical study to introduce a task independent handling qualities metric (HQM) to bicycle control, Hess et al. [78] directly applied a HQM from aircraft handling studies using the classical control method to bicycling, see model in Figure 5. They propose that handling qualities of bicycles can be reflected in the maximum magnitude of the transfer function between the inner-loop rate feedback of a variable (U_M) and the command input (C). To remove the effects of control sensitivity, they normalise the equation with the magnitude of state feedback gain K_P.

$$\text{HQM} = \left| \frac{U_M}{C}(j\omega) \right| \frac{1}{|K_P|} \quad \left[\frac{1}{s} \right]. \tag{3}$$

Hess et al. directly import (highly skilled and trained) pilot properties from aircraft handling research into the cycling situation including pilot gains and time delays. In the study, different bicycle models were evaluated on handling for a double-LC manoeuvre, but no significant differences were found amongst them. It is unclear if such a direct implementation of the pilot is possible in the cycling situation as there are definitely differences between the tasks of a bicycle rider and those of an aircraft pilot. Certainly, turning the handlebar is different from controlling the (joy)stick. However, the methodology is encouraging and will hopefully be validated experimentally in the near future.

3.2.2. *Optimal control*

Optimal control deals with finding a control law for the system such that it optimally fulfils certain criteria. The control problem minimises a cost function, which is a function of the
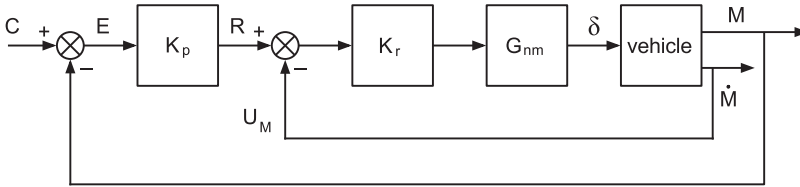
Figure 5. Proposed bicycle rider model by Hess et al. [78] for a single-axis tracking task. M and $\dot{M}$ are the bicycle output and output rate response for the variable being controlled, and C is the desired value of M. G_{nm} represents the rider neuromuscular dynamics (highly simplified). The gains K_p and K_r are chosen such that a specific bandwidth (K_p) and a specific level of damping at an oscillatory mode (K_r) are achieved.

state and control variables. The optimal concept is capable of treating multi-variable systems within a single conceptual framework using state-space techniques.[104] The optimal linear quadratic regulator (LQR) method can be used for MIMO systems described in state space form by

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t), \tag{4}$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t). \tag{5}$$

where $\mathbf{x}$ is the state vector, $\mathbf{u}$ the input (or control) vector, $\mathbf{y}$ the output vector, $\mathbf{A}$ is the system dynamics matrix, $\mathbf{B}$ is input gain matrix and $\mathbf{C}$ is the observer matrix. The linear quadratic Gausian (LQG) method can be used for systems described in state space form by

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{w}(t), \tag{6}$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{v}(t), \tag{7}$$

where $\mathbf{w}(t)$ and $\mathbf{v}(t)$ are uncorrelated Gaussian system and observation noise, respectively. A linear feedback control law of the form

$$\mathbf{u}(t) = -\mathbf{K}\mathbf{x}(t), \tag{8}$$

is assumed, with feedback gains $\mathbf{K}$. For both systems, the optimal linear feedback gains $\mathbf{K}$ are determined by minimising a cost function J which is a function of the states $\mathbf{x}$ and the control inputs $\mathbf{u}$, weighted by, respectively, the matrix $\mathbf{Q}$ and the matrix $\mathbf{R}$. For an infinite horizon continuous system, the weighting function is described by

$$J = \int_0^\infty (\mathbf{x}^\mathrm{T}\mathbf{Q}\mathbf{x} + \mathbf{u}^\mathrm{T}\mathbf{R}\mathbf{u}) \, \mathrm{d}t. \tag{9}$$

A drawback to this method can be the consideration required for determining the weighting aspects for the desired input and output signals and the formulation of the cost function, therefore, the objective optimal control method can have a subjective nature.

Kleinman, Barron and Levison [105,106] were the first to fully develop the idea of optimal control for describing human (manual) control back in 1970. They hypothesise that the human operator works in an optimal manner when carrying out a compensatory control task, but that the actions of the human are bounded by human limitations such as time delay and neuromuscular lag. They derived linear feedback for MIMO human operator models based on the gains calculated from minimising the cost function J and comparing these with actual measured tasks performed by aircraft pilots.

Interestingly, the skateboard model by Hubbard [107] shows the similar dynamics to that of the bicycle, with a coupling between the lean and steer and a dynamically unstable speed

range. Hubbard [108] applied full-state feedback LQR to the stabilising and tracking control of the skateboard. The human rider skateboard control is modelled by body lean relative to the skateboard. No human limitations are set and the dynamics of the skateboard itself are neglected. The roll angle of the skateboard is taken as the control input. The analytically derived results were compared to some experiments which show qualitative agreement in the time series. Future plans are to apply system identification techniques in order to determine the feedback gains.

Only a few optimal investigations for bicycles have been performed. Schwab et al. [79] used a similar LQR controller as Hubbard [108] with full-state feedback which was implemented in two different situations to investigate the effect of a leaned upper body on the control required to stabilise a bicycle. In the first situation, they investigated a rider rigidly attached to the frame of the bicycle and they show that the system can be stabilised easily through steer torque control but that at low speeds the roll feedback gains become unrealistically large. In the second situation, the rider is modelled with a leaning upper-body (inverted pendulum). They find that adding a pivoted upper body does not greatly affect the uncontrolled system eigenvalues or eigenmodes. However, at low speeds, the upper body lean requires large upper body lean feedback gains and similar to the rigid rider case, large roll feedback gains are required for the steer torque. Unlike the rigid rider case, they find that at high speeds significant steer and rider lean feedback gains are required for both the upper body lean and steer torque control. Furthermore, they find for the situation where the stabilisation only takes place by the upper body (hands free situation) that hugely unrealistic feedback gains (all states) are required at low speeds, suggesting that lean is unlikely to be used when steering is possible (hands on situation).

Connors and Hubbard [33] investigated the effect of pedaling on the steering control torque for a recumbent bicycle and modelled the rider's control to balance the bicycle as LQR steer torque optimal control. They found that for a recumbent bicycle, the oscillating legs can drastically increase the roll angle sensitivity and the steer torque required to balance the bicycle. Based on their findings, they devise a gear-shifting strategy (to reduce pedal cadence at higher speeds) to reduce the control effort at very high speeds (> 15 m/s).

One of the first to apply LQR optimal to a motorcycle were Katayama et al.[47] They used optimal in a tracking with preview problem to investigate the rider control applied during single-LC manoeuvres. They modelled the rider as a double pendulum with a lower and upper body connected to the frame and lower body, respectively, via passive spring–dampers. They use the roll angle and average heading error as the control inputs and the steering torque, upper body lean torque and lower body lean torque (offset centre of mass) as control outputs. Oddly they use a car-like optimal strategy where they take the average heading error as the lateral separation of the desired path and the straight line predicted motorcycle path (not curve!), weighted around a preview point. The LC manoeuvre studies indicate that steering torque is the dominant rider control mode, lower body torque assists and that upper body torque is such that the upper body is kept vertical and does not really contribute to the control.

Sharp also showed with LQR tracking with preview optimal that for motorcycles steer torque is dominant over body lean. Sharp [41,80] investigated the situation where the desired route is a curve and the control inputs are steer torque and upper body lean torque. The used cost function weighted the tracking errors for different preview lengths and the required control power. It was found that much larger preview distances are required compared to cars; however, extending the preview beyond a certain distance becomes pointless as the gains associated with these preview points reduce to zero. The preview distance, however, increases more than proportionally with the speed, meaning that the required preview time increases with increasing speed. A single-LC as prescribed by Rice [45] and an S curve according to Frezza and Beghi [109] were simulated. Qualitative agreement with the LC experiments

performed by Rice,[45] Katayama et al. [47] and Zellner and Weir [110] were found. Sharp notes that the higher the weighting of the controller power, the more the corners are 'cut'. He also notes that there is a strong relationship between the optimal tracking steering control and the motorcycle oscillatory weave mode, giving rise to the idea that if the oscillations of the motorcycle are within the rider's control bandwidth, and if the rider is skilled enough, then the rider will perform control at the eigenfrequency to get good response with little control power.

Sharp also theoretically investigated LQR optimal tracking with preview for bicycles.[6,81] This was based on the motorcycle research [41,80] but now implemented on the benchmark bicycle.[2] For the path following simulations, he looked at two different tests: a random road; and a straight section into a circular path (90°) followed by a straight section again. Different weighting factors for tracking errors against control power were investigated. The feedback gains are clearly speed dependent but again become unrealistically high with reducing speed. Sharp concludes that the necessary preview time, as opposed to the motorcycle case, depends very little on speed. Therefore, for bicycles, the preview distance is roughly proportional to the speed. Furthermore, he concludes that tight (precise) control requires about 2.5 s of preview independent of the forward speed.

Cossalter et al. [40,82] apply optimal not to model a rider but as a method to solve the problem of the 'optimal manoeuvre' for assessing the intrinsic motorcycle handling and manoeuvrability properties (for a so-called perfect driver). They point out that the optimal control for a specific manoeuvre can be vehicle dependent. The optimal manoeuvre method has handling and manoeuvrability as part of the cost function, where handling is defined as the 'ease to drive' (control effort) and manoeuvrability as 'ability to perform complex manoeuvres fast' (performance). First in [82], they apply optimal to solving the problem of the most efficient trajectory for each specific motorcycle, with its specific parameters, within prescribed boundaries (the road) and between two endpoints. They apply this optimal to three manoeuvres: slalom, LC and U-turn. The optimal manoeuvre is investigated for different motorcycle configurations by carrying out parameter changes to the wheelbase, centre of mass position, gyroscopic effect and tyre adhesion. For significant tyre adhesion differences, they find that for a U-turn manoeuvre the trajectory performed during the optimal manoeuvre also differs significantly. Then, in [40], they compare results with experimental data and find good comparison in a racing situation for a set of corners in S configuration. Bertolazzi et al. [83] used this optimal manoeuvre method to investigate how a manoeuvre changes with increasing upper body lean movement. For a LC manoeuvre, they show that when upper body lean is used it is possible to increase the performance as it is possible to carry out the manoeuvre more quickly.

Massaro et al. [111] developed a virtual motorcycle driver that mimics real drivers experimental behaviour from a predictive optimal control strategy. They find good comparison between the virtual rider inputs (the steering torque, the rear-wheel torque and the front-wheel torque) and real rider inputs. However, upper body motions of the virtual rider are not used to control the vehicle; which may become significant for replicating extreme manoeuvres on track.

Another form of optimal control is H_∞ control, where the feedback gains (for a multiple input to multiple output system) are chosen such that the peak values in the frequency responses (transfer functions) across each complete frequency range is minimal.

Nishimura et al. [85,112] performed experiments with a motorcycle at 30 km/h to identify the motorcycle dynamics. For both the identified fifth-order model and reduced third-order model which discarded the small, fast steering mode, they developed an H_∞ optimised steer torque controller that uses the roll angle as input. The simulations performed verified that the models were capable of stabilising a roll disturbance and that the reduced-order controller exhibits efficient stabilisation performance in comparison with the full-order controller.

For a motorcycle model with a leaning rider, Mammar et al. [84] synthesised a PID steer torque controller with feedforward and feedback capabilities using H_∞ optimal for the stabilisation. Once again, the roll angle was used as the input for the steer torque controller. The developed controller is shown to stabilise the motorcycle model, to be able to enter a constant radius corner, and to be robust to parameter variations for this manoeuvre.

Finally, we mention the mixed H_2/H_∞ controller design for the stabilisation of a bicycle robot using gyroscopic precession by Thanh and Parnichkun.[86] They chose to apply the control method to a flywheel to ensure that the bicycle can balance at all speeds (also stationary). The decision to apply an H_2/H_∞ controller was for the good robustness of the optimal controller for systems with uncertainties and not to mimic a rider in any way. However, optimisation of such a controller ends up as a complex non-convex problem and for this reason, they apply a particle swarm optimisation algorithm as it enables fast and structured optimisation routes. The control was implemented on a real bicycle and was shown to be stable when the bicycle was stationary and moving slowly in both forward and backward directions.

3.2.3. *Other control*

A variety of different control methods have been proposed by authors to model riders. These are discussed below.

3.2.3.1 Intermittent control. Doyle [58] used bicycle stabilisation observations to develop a stabilising bicycle rider model with intermittent control (as discussed here in Section 3.1). He developed classical style controllers for the dynamic model of the bicycle and applied numerical integration to get the bicycle state solutions in time. He found that the stabilisation of the bicycle performed in a similar manner to that observed with real riders can be achieved by steer torque control with continuous feedback of the roll rate and acceleration, in combination with intermittent roll angle feedback. The intermittent feedback takes place in the form of a pulse torque that is triggered when the roll angle exceeds a certain threshold roll angle value, which he finds to be 1.6° in his observations.

3.2.3.2 Intuitive control. An intuitive bicycle controller was developed by Schwab et al. [79] for balancing a bicycle using the 'steer into the undesired fall' principle. They investigated two situations, the first was a rigidly attached rider and second a rider with a moveable upper body modelled as an inverted pendulum connected to the frame with a passive torsional spring. In both cases, they apply a simple steer torque control law: at low speed, they apply proportional feedback of the roll rate with the gain increasing with decreasing speed and above the stable speed range proportional feedback of the roll angle with the gain increasing with the speed. For both situations, they show through an eigenvalue analysis that the system can have marginal stability for almost the complete forward velocity range. Furthermore, they find that the controllers in both situations require far more realistic steer torque feedback gains (K_c and K_v) than for the same models using an LQR optimal controller for determining the feedback gains.

3.2.3.3 Fuzzy logic. For the low-speed tracking control of a motorcycle, Fujii et al. [88] develop a fuzzy PD controller to control the roll angle with respect to some desired roll angle. The gains of the fuzzy controller were determined using a genetic algorithm applied to constant speed, constant roll angle situations for forward speeds ranging from 1 to 15 m/s. To evaluate the controller a 90° corner, a LC manoeuvre and a set course were evaluated at multiple speeds. Initially, the controller was unable to track the desired path at low and high speeds.

The genetic algorithm is evaluated using a fitness function and considerable effort had to be invested in developing (altering) the fitness function such that the path tracking capabilities of the fuzzy controller became satisfactory at all speeds. They were able to compare their model simulations with real experimental results and found good agreement with the vehicle states but the applied steer torque only roughly follows the experimental data.

Chen and Dao [89–91] developed a number of fuzzy logic-based steer torque controllers of increasing complexity for a bicycle. First, they developed a PID steer torque controller for stabilising the bicycle where the PID gains are set but the control values are determined via the fuzzy logic controller placed in series. Then, they investigated roll angle tracking by introducing a second fuzzy logic controller placed in parallel to the stabilising fuzzy logic controller. Finally, in [91], they optimised the fuzzy logic controllers using a genetic algorithm. They propose a strategy to optimise the fuzzy logic controllers by keeping the rule table fixed but tuning their membership functions and by introducing scaling factors and deforming coefficients. In this way, the number of parameters to be trained can be reduced to speed up the learning process. They verify their control schemes with simulations and find good correspondence.

3.2.3.4 Neural network.   Cook [87] devised a neural network controller with only two neurons as an example of a simple human bicycle tracking controller. The first neuron is a proportional controller on the heading with a threshold function. It outputs a desired roll angle, which is an input for the second neuron which in turn outputs a steer torque based on PD control. The desired heading is set using way points enabling the bicycle to perform complex tracking tasks. He finds that the controller is relatively robust as (gain)values are not very speed dependent and do not have to be perfectly adjusted to the specific bicycle. The controller works at a range of velocities but it fails at low speed.

3.2.3.5 Inverse dynamics.   Controllers with a state observer can be used to predict the future motion of the vehicle based on the current state and inputs. Inverse dynamics is used to determine the forces required to pursue a desired course based on the current state.

Getz [92] developed an inverse dynamics method he calls dynamic inversion which he applies to bicycle control.[93,94] The controller determines, based on the desired path or roll angle of the bicycle as a function of time, the forces that have to be applied to the steering system as a function of time. Getz illustrates the potential of his controller on an oversimplified bicycle model in a number of examples such as a straight path at constant speed, a sinusoidal path, a circular path at constant speed and a figure eight trajectory. Each example starts with an offset from the track and all show counter-steering effects. Such an inverse dynamic method is certainly of interest for determining the performed control by a rider based on a traversed path, but it is probably less well suited for determining the control a rider will perform based on a current state and some roughly described desired path.

3.2.3.6 Forward dynamics.   In forward dynamics, the motion of the system is determined from the applied forces and the equations of motion, which then include the controller. Von Wissel and Nikoukhah [96] applied forward dynamics to investigate the control of a bicycle with a multiple stage obstacle avoiding optimisation methodology. They find trajectories for a bicycle in a complex space (a grid with ordinary cells, forbidden cells and end cells). The bicycle has a constant forward speed and a number of discrete steer torque manoeuvres can be applied. The selection of the manoeuvres is multiple stage. Therefore, large tree structure path possibilities evolve. The manoeuvres that make the bicycle unstable are discarded first; after this, manoeuvres that cause the bicycle to come into forbidden cells are discarded. Finally, all but the trajectories that penetrate in the end cells with the lowest cost function are

discarded. The method uses a moving window methodology for the path tracking to reduce the computational power required by moving the end cell(s) through the complex space. This speeds up the computing as branches can be deleted along the way and then the simulation can be restarted. They give interesting examples implementing the method and showing how an optimal path changes with the movement of the end cells. In the given example, the method is applied at a high (stable) forward speed of 8 m/s removing the need for a lateral feedback controller. While it is not clear if the method is directly applicable to a human rider as a rider most likely does not compute all possible paths when determining which route to take, it does give interesting insight into possible route choices and is certainly interesting for automatic vehicles.

A similar approach was used by Cook [87] for determining the stabilising path of a bicycle. He did this for an unstable forward speed, beneath the weave speed. At each time step, the effect on the trajectory of a handlebar push to the left, right or no push is calculated. The process is repeated at each time step for each path. Each path is evaluated until the bicycle has fallen over and that path ends. This leads too to a large tree of possible paths. The control applied in the example that Cook gives, however, was unable to stabilise the bicycle over a long distance.

3.3. *Controller design: experimental validation*

The experimental validation of controllers can be split into two groups, namely rider control model validation and machine control model validation, where the latter group is usually interested in creating some form of autopilot function for the machine.

3.3.1. *Rider model validation*

While many have developed rider models, only a few authors have gone to the expense of actually validating them. Stassen and van Lunteren [56,62] were the first to carry out many experiments with riders (on a bicycle simulator), but they never explicitly validated their models. However, as they used the experiments to identify the model parameters, they thereby implicitly validated it. Nevertheless, a number of authors including Eaton,[113] Roland and Lynch,[18] and Koenen et al. [114] were critical of the work. In particular, the used simulator and the steer *angle* (opposed to torque) control models were doubted. In [113], Eaton writes:

> While the research represents a pioneering effort in obtaining transfer functions experimentally (with a bicycle simulator), it should be pointed out that van Lunteren's major interest was the performance of the human operator under various conditions (drugs, etc.) and not the dynamics of the bicycle. Thus, the accuracy of the simulator dynamics with respect to real bicycles and the validity of the assumption of steering angle control (rather than steering torque) are questionable.

Stassen and van Lunteren deemed the simulator sufficient for the intended purpose, but the fact that riders had to learn to ride the bicycle simulator is an indication that the control of the simulator was probably not the same as on a real bicycle. Furthermore, de Lange [115] discovered some sign errors in the work of Stassen and van Lunteren and that the complete model of the bicycle with the identified rider model is unstable, even after having corrected sign errors in their equations.

Doyle [58] too developed bicycle rider models based on experiments. However, his experiments did take place on real bicycles. He compared the state time series results of a number of control models with measured data, and found that it is insufficient to use an average or filtered angular roll velocity as a feedback signal, the actual roll velocity has to be fed back.

Furthermore, he found that continuous feedback of the roll angle gives very different simulations from what is seen in real life: instead intermittent torque pulses are required to stabilise the bicycle. He observed that impulsive control is applied when the roll angle exceeds the threshold of 1.6°. Doyle also found that

> if an attempt is made to control the system by responding to absolute angle without any velocity feed-back then after one or two reversals the velocity reaches such a high value that excessive lean angles are generated before control takes effect.

This was also found by de Lange,[115] who made a simple desktop computer game style bicycle simulator. The game player had a gamepad to apply steer torque and pedalling force. He based the dynamics of the bicycle on the benchmarked linearised equations of motion from [2] and for the visual feedback showed a first or third persons view of the bicycle moving on a flat surface. He found that it was impossible to stabilise the bicycle except by applying impulsive steer torque inputs triggered by an extra roll rate indicator on the screen.

The first to do an actual validation of a rider–vehicle model was Eaton,[20] who carried out experiments to validate the rider control cross-over model by Weir [23] together with the motorcycle model (including tyres) by Sharp.[14] Eaton investigated the stabilisation of the motorcycle roll angle by means of steer torque applied to the handlebar. Path following tasks and control by body movements were not studied. The rider's body motion was, therefore, restricted by a rigid brace during the experiments. For low speeds (< 64 km/h), significant differences between the motorcycle model and experimental results were found, but at 64 km/h they match reasonably well. The identification method that Eaton applied, the Wingrove–Edwards method,[116] has had many critiques. This method applies no external excitation, all excitations are assumed to be a result of rider remnant. However, the time delay and signal are both small and can vary in size hindering the correct identification of the rider.

While Eaton prevented rider motion relative to the motorcycle, other authors have investigated the influence of rider motion on manoeuvres. Interestingly, though none have looked at stabilising, all have looked at manoeuvring. Capitani et al. [117] found that ignoring rider motions leads to incorrect steer torques and steering angles. Roland et al. [16,18,45,47,57] tried to experimentally validate models of a non-rigid rider. Both Roland, for bicycles, and Rice, for motorcycles, modelled the rider upper-body as an actively controlled pendulum and the lower body as fixed to the machine. Katayama et al. [47] in essence extended this to include rider lateral motion on the saddle by modelling the rider as an actively controlled double pendulum. All three had issues.

Roland [16,18,57] developed the rider control model that was also used by Rice. The model actively controls the upper body in a closed-loop manner in which the rider model feeds steering torque and lean torque inputs to the vehicle dynamics model in response to vehicle roll motion information (for stability) and to vehicle path and heading error information (for guidance). Roland, however, did not use the algorithm himself, instead he used a very simple guidance control algorithm for a slalom manoeuvre: the sign of the command roll angle (set at 20°) is opposite to the sign of the current steer angle. This gave very similar qualitative results between model and experiment. However, it is unclear if this was a 'lucky shot' that the actual slalom manoeuvre looks similar, or if this really is a good model for the control carried out by a rider.

In a number of motorcycle rider control observations, Rice [45] found significant differences between a novice rider and an expert rider. The expert riders actively use upper body motion as a manner for (feed-forward) control, whilst the novice riders only show steer torque and compare well to the model. On the other hand, Rice found that during the transient stage of a manoeuvre both novice and expert riders show similar motion to that of the rider model. This suggests that the manoeuvre initiating body motion by the expert rider is a result of rider

Table 3.   Machine controllers used in experiments.

Control outputs	Author	Stabilising task	Tracking task	Control inputs	Successful
Steer torque	Ruijs and Pacejka [71]*	ϕ		$\dot{\delta}, \dot{\phi}, \phi$	Yes
	Saguchi et al. [118,119]	ϕ		$\dot{\delta}, \delta, \dot{\phi}, \phi, \alpha, \gamma$	Yes
	Michini and Torrez [120]	ϕ		$\dot{\phi}, \phi$	Yes
	Andreo et al. [121,122]	ϕ		$\dot{\phi}, \delta$	Yes
Steer torque and moving mass torque	Iuchi et al. [123]	ϕ	ϕ	$\delta, \phi, \dot{\phi}, \beta$	Yes
Steer torque and gyroscope	Levandowski et al. [75,76]*	ϕ	y	Unclear	Yes
Steer angle	Miyagishi et al. [124]*	ϕ	y	$\phi, \dot{\phi}, \delta, \gamma$	Yes
	Tanaka and Murakami [77]	ϕ	y, γ, k	$\phi, y, \gamma, k, \delta$	Yes
	Yamaguchi [125]	ϕ		$\phi, \dot{\phi}, \ddot{\phi}$	Yes
Steer torque and angle	Lenkeit [126]*	ϕ		δ or τ_δ	Unclear
Steer angle and moving mass position	Nagai [74]	ϕ	y	y, δ, ϕ, β	Yes
Moving mass torque	van Zytveld [70]	ϕ		$\phi, \dot{\phi}, \beta, \dot{\beta}$	No
Moving mass position	Yamakita et al. [127–130]	ϕ, β	y	δ, ϕ, β, y	Yes
Gyroscope orientation	Thanh and Parnichkun [86]	ϕ		ϕ, φ	Yes

Notes: A star (*) indicates the control is for a motorcycle. ϕ is roll angle, δ is steer angle, α is slip angle, γ is yaw angle, y is lateral position, k is the curvature of the path, τ_δ is steer toque. β is the lean angle of a moving mass and φ is the orientation of the gyroscope.

–forward control, while during the transient phase the forces are too high for the rider who, however, can contribute little more than in a passive manner. Different rider models should, therefore, be used for novice and expert riders as they apply different (initiating) manoeuvre control sequences but during transient stages of a manoeuvre the motion of the complete system is similar to that of a rider–vehicle model with a controlled upper-body.

The theoretical LC studies that Katayama et al. [47] performed on a double-pendulum rider model were confirmed by his single-LC at 60 km/h experiment: steering torque is the dominant rider control method; lower body torque (lateral shift on the saddle) assists in the process; upper body lean torque is only such that the upper body is kept vertical and does not really contribute to the control but is used for comfort of the rider. Therefore, on those motorcycles where lateral motion of the rider takes place, it appears that this motion should be accounted for. Katayama et al., however, did not test stabilisation control (at low, unstable, speeds), nor did they carry out high-speed or low-speed tracking experiments.

3.3.2.   *Machine model validation*

In general, there are four main methods that authors have used to stabilise and control single-track vehicles: steer control; a moving mass; a gyroscope; a combination of the above. An overview of the projects carried out using the different control methods is given in Table 3.

3.3.2.1 Steer control.   Stable control of single-track vehicles has been achieved using either steer torque or steer angle control for both bicycles and motorcycles. However, the vehicle states that were used in the feedback loop and the feedback gains' speed dependency were different for the different approaches.

The first to develop a robotic motorcycle using only steer actuation were Ruijs and Pacejka,[71] who used steer torque control based on a Sharp [14] motorcycle model with tyres and leaning rider (but they did not include a leaned rider in their hardware). Others that have used steer torque control for robotic single track machines include Saguchi et al.,[118,119]

who based their bicycle rider robot on a Getz [92] style bicycle model with added tyre slip. Michini and Torrez's [120] and Andreo et al.'s [121,122] bicycle robots were based on the benchmark bicycle model. Out of these robots, only Saguchi et al. investigated tracking control (straight ahead running and constant curve motion), the other three only investigated stabilisation control. Ruijs and Pacejka, however, were able to set the roll angle by a remote link and thereby make the motorcycle able to follow a path.

While at least three of the four used velocity-dependent feedback gains (it is unclear if Michini and Torrez calculated feedback gains for multiple speeds or used the same feedback gains for the two speeds that they tested at), each used a different combination of a set of feedback signals and control strategy: Ruijs and Pacejka used pole placement for proportional control on the roll angle, roll rate and steer rate. Similarly, Michini and Torrez also used proportional control but they only used roll angle and roll rate and it is unclear if they used pole-placement in determining the gains or some other method; Andreo et al. used linear-parameter-varying (LPV) state feedback control for which they measured the forward speed, roll rate and steer angle and calculate the roll angle through integration; Saguchi et al. implemented roll angle tracking by optimal control on the difference between the desired and the actual roll angle, and implemented stabilising control, using proportional feedback of the roll angle and rate, steer angle and rate, yaw angle (ω) and slip angle (β), where ω and β are estimated using a Kalman filter.

Despite these major differences, all four projects achieved very encouraging results. Ruijs and Pacejka's motorcycle robot was shown theoretically to be stable between 5 and 60 m/s, experimental tests proved that motorcycle is, in fact, stable from 10 up to at least 110 km/h (2.8–30.6 m/s). The robotic bicycle by Michini and Torrez was shown to stabilise the uncontrolled motion at both an unstable speed ($\text{Re}(\lambda_{\text{weave}}) > 0$) and a neutrally stable speed (($\text{Re}(\lambda_{\text{weave}}) = 0$) despite the fact that they calculated their feedback gains using bicycle parameter values from Kooijman,[22] a totally different bicycle with a much lower mass. With this LPV controller, Andreo et al. showed that their bicycle was able to stabilise at low speeds (from 1.7 down to 1 m/s) and to balance despite an external impulsive roll torque disturbances (for speeds from 2.1 to 1.7 m/s). Saguchi et al. demonstrated stable behaviour for vertical roll angle target and for 10° roll angle target (steady cornering) at around 2.5 m/s. They also compared experimental results with simulations for straight ahead running with a lateral impulse on the rear frame and found very good agreement.

Three projects have successfully implemented steer angle (position) control: a motorcycle rider robot by Miyagishi et al. [124] and two bicycle robots by Tanaka and Murakami [77,131] and Yamaguchi.[125] However, while the motorcycle rider robot by Miyagishi et al. was position controlled, the motorcycle itself was torque controlled. The constructed rider robot was designed for carrying out objective handling quality tests, and therefore the rider robot was connected via springs and dampers to both sides (left and right) of the handlebar simulating rider passive properties and making the motorcycle itself torque controlled.

Initially, Miyagishi et al. [124] split the steering control algorithm into two parts which were constructed in parallel: a PD controller on the roll angle and rate for upright stability was successful. The implemented weighted proportional lateral position controller for tracking conflicted with the stability control. Thus, they went over to proportionally tracking a target roll angle which is then used as the set point for the posture control. Changing the stabilising and tracking algorithm to a serially implemented system.

Tanaka and Murakami [77,131] based the control of their robotic bicycle on the dynamics of a theoretical point mass model of a bicycle (Getz like [92]) with no steering dynamics. They too implemented separate controllers for stabilising and tracking in series. The stabilising controller consisted of a PD controller (again roll angle and rate). Two path tracking controllers were implemented: first, a lateral velocity controller was tried, based on proportional control

with respect to the (set) lateral velocity, which they found to destabilise the posture control as a result of unmodelled dynamics in the system. Second, they implemented a more robust proportional controller using the desired rate of change of path curvature per path length as the control variable. This tracking controller in combination with the stabilising controller was found to be stable.

Yamaguchi [125] recently applied 'steer into the lean' control to stabilise a (scaled) bicycle by a biped robot that can pedal and steer. The biped robot uses PID control of the roll rate signal from a gyroscope in the robot and uses servos to actuate the joints in the legs and arms. The bicycle is stabilised by the robot but the general heading is remotely controlled by a human. No information is available in the open literature about this bicycling biped robot as yet and thus it is unclear if it is really using steer angle control.

Finally, Lenkeit [126] was the first (and only) to develop a motorcycle robot using steer angle control at low speed and steer torque control at high speed. He had concluded from reviewing Weir's work [102,132] that to control the motorcycle with only a steer torque actuator (DC motor) that the steer motor had to be configured as a position servo below 30 km/h and a torque servo at higher speeds for which he only required the roll angle to be fed back. The general direction of the motorcycle was controlled by a human via a radio link (tracking not implemented), and switching from position to torque control was done manually at 30 km/h. He reportedly found that good control was achieved below 20 km/h and above 35 km/h, whilst between 20 and 35 km/h he describes a lightly damped oscillation being present, but no results were published.

3.3.2.2 Moving mass control.   Theoretical results indicate that stabilisation and tracking using only lean torque, by an inverted pendulum or laterally moving mass, is far more difficult than through steer control as far larger gains in the feedback are required. Only two projects have attempted to stabilise a bicycle using lean torque control. The first was van Zytveld [70] in the mid-1970s, who applied lean torque control to an inverted pendulum placed on a bicycle powered by a gasoline engine. The project failed to stabilise the bicycle due to the neglected geared inertia of the used electric motor.

The second, larger project which is still ongoing, by Yamakita et al.,[127–130,133] has taken advantage of the modern more controllable electric bicycle as the platform on which they have applied their pendulum control. They are particularly interested in extreme low-speed stabilisation and the possibility to stably adjust the vehicles vertical orientation (which is not possible with a gyroscope) and to track a desired vehicle orientation path in time (posture tracking). Therefore, they model the bicycle as a double-inverted pendulum (roll and lean angle, no steering) to carry out stabilisation control at stationary and very low speed (< 2 m/s). While the balance of a bicycle by an inverted pendulum model is interesting, it only works at low speed. The faster the vehicle moves, the less it looks like a double pendulum due to its ability to steer .

Yamakita et al. independently implemented two separate controllers: a non-linear controller for the stabilisation; and a linearised input–output controller for posture tracking. They note that as the two controllers were developed independently and use the same dynamical system they will cause some oscillations and offset to the balance control. Therefore, a shift on the lean angle and rate set point for the balance algorithm should be applied. Interestingly though, they do not need to perform this offset in either their simulations or experiments as the bike performs well enough without it. They carried out simulations to show that the bicycle is indeed stable and can track an orientation. The experimental machine confirmed this, but they had to use a modified control algorithm by adding an H_∞ controller (in the feedback loop) for robustness as the theoretical controller did not work on the experimental machine. The robustness of the controller was demonstrated by using the same controller in stationary and

low-speed trials.[127,130] But as yet no successful stabilisation and tracking control has been implemented at a wide range of speeds by moving mass control. In later studies, Yamakita et al. theoretically implemented trajectory tracking [129] using steer torque control and recently they adapted the moving mass controller and apparatus such that it can also be configured to act as a gyroscope.[133]

3.3.2.3 Control with a gyroscope.   Single-track vehicles with some form of gyroscope control applied have been successfully implemented by a number of authors.[86,134,135] Either the gyroscopic precession or the adjustment of the gyro speed supplies the required torque to keep the bicycle upright. The use of fast spinning gyros is mainly credited by the authors to the extreme level of continuous stability that this method can produce when compared with steer control and moving mass control for stationary and slow moving vehicle. The speed range that the authors generally investigate and apply control to is, therefore, also generally stationary and low speed (< 1 m/s). Different types of controllers for controlling the required gyroscopic precession have been implemented; based on root locus,[135] and H_2/H_∞ control.[86] Active stabilisation by the adjustment of the gyro speed has been implemented by Murata,[134] who made a robot riding a miniaturised bicycle using a gyroscope inside the robot's torso. The bicycle itself is not controlled, but the robot measures its orientation and calculates its centre of gravity and accelerates the gyroscope such that the centre of gravity comes over the wheel contact line. The resulting bicycle motion is very unnatural as can be expected from such a stabilised system as the rider has to remain in an upright position at all times to prevent the gyro from reaching its 'top' speed, and therefore no longer being able to provide the required stabilising torque. Although both precession and spin rate actively controlled gyroscopes have been shown to work, neither though seems to be an ideal candidate for automatic control though due to the required power.

Strictly speaking, the passive implementation of a gyroscope to slowdown the dynamics of the vehicle is not 'control', however, it can be used in combination with other forms of control such as steer control. This is the core of the Gyrobike [136] product, where a fast-rotating gyroscope inside the bicycle front wheel is used to reduce the level of instability of children's bicycles enabling the child to learn to cycle without the bicycle falling over as quickly as would happen without the gyroscope. By adjusting the gyroscope's rotational speed, the level of stability is adjusted. The added value of such a stability enhancement tool is questionable though. The gyroscope changes the dynamics of the bicycle significantly so the user still has to get used to a normal bicycle without a gyroscopic front wheel.

3.3.2.4 Multiple control methods.   Two sets of multiple control methods have been applied successfully. The first is the use of both steer and lean for the stabilisation and tracking by Nagai [74] and Iuchi et al.[123] Nagai and Iuchi both applied lean torque (position) control through the use of an inverted pendulum. The second method is the use of steer control in combination with a gyroscope for stabilisation and tracking which was implemented by the Berkeley University Blue Team on their Ghostrider Robot autonomous motorcycle. This motorcycle was designed to participate in the defense advanced research projects agency Grand Challenge 2005, a race across the desert for autonomous vehicles,[75,76] and was far more advanced than the vehicle by Nagai and Iuchi as it had global positioning system navigation and stereo vision implemented for its autonomous tracking capabilities.

Unlike the Ghostrider Robot, neither Nagai or Iuchi carried out experiments on the 'open road': Nagai placed the bicycle on a treadmill; Iuchi on rollers, thus the extent to which 'tracking' was taking place is debatable. However, both did carry out stabilisation experiments, whilst Iuchi's tracking task was simply to keep the bicycle on the rollers, and Nagai performed LC manoeuvres.

To investigate the required control for stabilising and tracking, both Nagai and Iuchi developed simplified linearised equations of motion for a bicycle with a leaned upper body. Nagai used a point mass bicycle (massless front frame and wheels), whilst Iuchi used a very simple double-pendulum model. For the control, Nagai used the lateral deviation from a preview point (as a function of steering angle) and the roll angle as control variables, Iuchi used the roll angle and roll rate of the bicycle as the reference inputs.

Nagai found good agreement with his models except for the situation in which only leaning for tracking control was used. Based on his LC experiments, Nagai concluded that the moving mass reduces the time required to carry out a LC manoeuvre, however, it also increases the size of the steer and roll angle response. Iuchi had to implement completely different control gains on the experimental bicycle compared to the model, in order to stabilise the bicycle, and even then he was not able to keep the bicycle on the rollers for long periods of time. This led him to conclude that the used bicycle model does not consider the physics sufficiently.

Ghostrider Robot's control was, similar to Nagai, based on a simple point mass bicycle model [92] and had to be tested extensively. Due to all the uncertainties between the model and the actual motorcycle, they initially used a fuzzy logic controller based on intuition and 125 parameters to stabilise the motorcycle, but this only worked at low speeds (3–6 mph). They then changed to a PID controller for which they found the parameter sensitivity to various terrains. This worked better but as it counted heavily on the integrating factor the directional response was slow. To further improve the system, they converted the PID controller to a run-time reinforcement learning (neural network) controller based on a stochastic model of the vehicle dynamics. Despite the installed gyroscope, they had trouble controlling vehicle whilst in the air (jumps).

From the above, it appears that it is essential to use a model that describes the motion of the vehicle sufficiently. Even with a relatively simple model (Nagai), good results can be achieved with a very simple controller for both stabilisation and tracking. On the other hand, when the model is probably not sufficient (Iuchi and the Ghostrider Robot), far more advanced control models are required and still the results can be mediocre.

3.4. *Rider modelling discussion and conclusions*

Models have found that steering is the dominant control method involved in the stabilising task for both bicycles and motorcycles and performed observation experiments confirm this. Classical control single output (steer only) motorcycle rider models for the stabilising task have been shown to compare well with actual rider data. Roll angle to steer torque was found to be the most effective transfer function for stabilising a motorcycle. But position classic control models have also been developed for a rider on a bicycle simulator, that compare well with experimental data. However, no bicycle rider models have been experimentally validated as yet. As both position and force control have been shown to model rider control well, it could, therefore, be interesting to investigate if steering impedance control is even better.

Observation experiments have found that riders appear not to use their upper bodies for stabilisation control when they have their hands on the handlebars. LQR optimal control and intuitive control models have also shown that it is highly unlikely that upper body lean will be used for stabilisation at low speed as the gains required are too large. Modelling a rider as rigidly attached to the bicycle is only able to perform steer actions (based on roll angle information), therefore, appears to be a good option for a rider model for stabilising control.

Classical control multiple output models (steer and lean) been developed for performing tracking and stabilising tasks that compare well to actual rider data. The addition of the

tracking task does not significantly alter the roll to steer gains for classical control models capable of stabilising by steering. Stabilisation and tracking have successfully been implemented in parallel in machines that use steering and moving mass control. However, for machines with only steering control (single output) tracking and stabilisation have been successfully implemented in series. An open question is if tracking and stabilisation can be implemented in a single controller.

Research into the optimal steering control for tracking with a motorcycle shows that good response can be achieved with very little controller effort by applying the control in the vehicle's dominant eigenmode (weave or wobble). Experimental observations performed with a bicycle and rider with hands on the handlebar have found that most steering motion occurs in the pedaling frequency. This could also be due to a similar mechanism as the pedaling motion causes a large cyclic perturbation that has a similar effect as the intrinsic weave or wobble vehicle modes.

The required preview distance for tracking capabilities depends on the vehicle being used. For bicycles 2.5 s is enough, but for motorcycles, the preview distance is speed dependent. Optimal control models have also shown that for a bicycle, the tracking performance improves with increased preview distance, but that there is a limit after which the extra preview distance does not add to performance anymore as its weighting factor is almost zero. These theoretical results, however, have not been experimentally validated. The preview time required to safely control a vehicle can be of interest to traffic and road planners as it can have an effect on the design of intersections and tight or blind corners.

Rider upper body motions, in general, have been shown to only have a small influence on the overall motion. Furthermore, it has been shown that upper body motions are most likely to be performed to control the orientation of the rider's head for comfort reasons and not the direct control of the vehicle. However, optimal control models have shown that upper body motion can contribute to manoeuvre performance, particularly for manoeuvres such as a LC. Motorcycle rider observations have shown that riders perform different controls depending on their level of experience. Novice riders apply more roll–steer torque than experienced riders to initiate a turn, whilst experienced riders can apply large upper body motions as part of their strategy to perform a manoeuvre more effectively. Therefore, for the development of a universal rider model, rider's experience will have to be parameterised. Interestingly though, the more difficult the manoeuvre task that is set, the more alike the experienced and inexperienced rider control actions become. Observations also indicate that for the successful completion of a manoeuvre, the timing of the applied control is more important than the amplitude of the applied control. This fares well with the idea that the applied control can be of an intermittent nature as found for stabilising control.

4. Handling qualities and manoeuvrability

The handling qualities of a vehicle are related to its stability and control characteristics. A vehicle's manoeuvrability is related to its ability to perform a specific (set of) manoeuvre(s).

The aircraft industry obviously had most to profit from research on handling qualities. Each airplane has to be controlled precisely in order to be able to land safely, fighter aircraft has to be highly manoeuvrable to avoid being shot down, yet still have to be controllable for the pilot, and many early aircraft suffered from pilot-induced oscillations during flight. Thus, it is not surprising that this is also where most of the insight into handling qualities initially was developed. Cooper and Harper [3] were the first to precisely define what they mean by handling qualities of aircraft, namely: 'Those qualities or characteristics of an aircraft that govern the

ADEQUACY FOR SELECTED TASK OR REQUIRED OPERATION*

AIRCRAFT CHARACTERISTICS ● DEMANDS ON THE PILOT IN SELECTED TASK OR REQUIRED OPERATION* — Pilot Rating

Aircraft Characteristics	Demands on the Pilot	Pilot Rating
Excellent Highly desireable	Pilot compensation not a factor for desired performance	1
Good Negligable deficiencies	Pilot compensation not a factor for desired performance	2
Fair - Some mildly unpleasant deficiencies	Minimal pilot compensation required for desired performance	3
Minor but annoying deficiencies	Desired performance requires moderate pilot compensation	4
Moderately objectionable deficiencies	Adequate performance requires considerable pilot compensation	5
Very objectionable but tolerable deficiencies	Adequate performance extensive pilot compensation	6
Major deficiencies	Adequate performance not attainable with maximum tolerable pilot compensation. Controllability not in question	7
Major deficiencies	Considerable pilot compensation is required for control	8
Major deficiencies	Intense pilot compensation is required to retain control	9
Major deficiencies	Control will be lost during some portion of required operation	10

Is it Satisfactory without improvement? — Yes / No → Deficiencies warrant improvement

Is adequate performance attainable with a tolerable pilot workload? — Yes / No → Deficiencies require improvement

Is it controllabe? — Yes / No → Improvement mandatory

Pilot decisions

*Definition of required operation involves designation of flight phase and/or subphases with accompanying conditions.
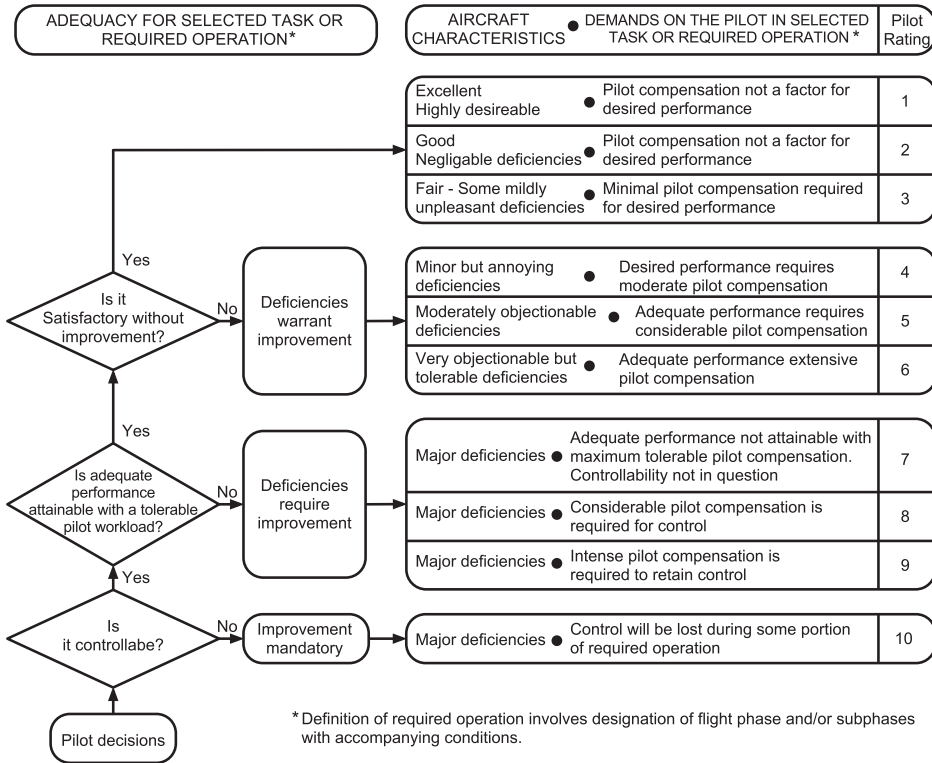
Figure 6. The Cooper–Harper handling qualities rating scale, from [3].

ease and precision with which a pilot is able to perform the tasks required in support of an aircraft role'. Where they defined 'task' as 'the actual work assigned to a pilot to be performed in completion of or as representative of a designated flight segment' and 'role' as 'the function or purpose that defines the primary use of an aircraft'.

Cooper and Harper state that both physical and mental workload need to be taken into account when rating a handling quality. They argue that a pilot can perform specific manoeuvres just as well in very differently behaving aircraft and that the measurable physical workload can be identical but that the mental workload can be very different. Therefore, they developed a 10-scale pilot rating system (shown in Figure 6) for determining aircraft handling qualities which became the norm for the industry and beyond. This rating system takes the mental workload into account.

Correlations have been found indicating that handling qualities can be linked directly to control effort. McRuer and Jex,[137] and Hess [138] found that the pilots' perception of the task difficulty, and therefore of vehicle handling qualities are highly correlated to the 'power' of the pilots output-rate feedback signal. Therefore, they only look at the physical workload and use it to define the handling qualities. This changes handling qualities to a control feedback problem. They found that the complete closed-loop system wants to act as a first-order system (20 dB/decade drop off in a Bode plot) around the cross-over frequency and where the desired band width is achieved by the pilot's control effort.

The most significant difference between aircraft and bicycles and motorcycles, with regard to designing for handling qualities and control strategies, is their ratio of pilot/rider to vehicle mass. The mass of a motorcycle rider is usually around 50% of the total mass, while for bicycle the rider can be as much as 90% of the total mass. On the other hand, for a fighter

aircraft, the pilot mass is typically less than 1% of the maximum take-off weight. The position, orientation and exact mass of a rider on a bicycle or motorcycle have a far greater influence on the open-loop dynamics of the system than they do in an aircraft. Furthermore, any motions executed by a aircraft pilot that do not disturb the control stick or rudder pedals will have little to no effect on the aircrafts trajectory, whilst for a bicycle or motorcycle, the body motions that do not disturb the handlebar directly can still cause a trajectory change of the bike as a result of the lean to steer coupling and relatively large mass of the rider. Examples of how the control strategy influences the trajectory of the vehicle, enabling different riders to complete the same manoeuvre in different ways were shown by Cossalter [139] for a U-turn manoeuvre and Rice [45] for a steady turn and for a LC manoeuvre. Rice found large differences in control strategy performed by novice and skilled riders. Both Rice and Cossalter concluded that a rider on a motorcycle can successfully carry out a specific manoeuvre in many different ways. Cossalter, however, also stresses that the rider's subjective interpretation determines the handling qualities of a vehicle. He points out that this subjective rating depends on the rider's driving style and sensitivity, and on the motorcycles; response to lateral acceleration and yaw rate, sideslip, sensitivity to external disturbances, response to control actions under different circumstances, and the feedback between rider and motorcycle.

Bicycle and motorcycle designers can, however, only develop the machine part of the complete machine–rider system. They design their machine generally for the following roles:

- accident avoidance manoeuvres: safety aspects
- normal riding: the safe use in and amongst traffic whilst obeying the traffic rules
- racing: the completion of a lap around a circuit in the shortest possible time

The first role is more a 'segment' of the 'designated flight plan' for which the designers would want to achieve maximum performance on in all circumstances and this has (logically) received most of the attention. The last two roles generally lead to very different looking vehicles, largely as a result of ergonomic and aerodynamic aspects. The next sections discuss these three roles in order.

4.1. *Handling qualities for accident avoidance: safety*

Most safety-related handling quality work has been done experimentally where the complete system: bicycle and rider, were tested simultaneously. No standard tests were used, making direct quantitative comparisons between experiments impossible. The various tests are shown in Table 4. The 'tasks' that have to be performed for safety are often categorised under the general terms 'manoeuvrability' and 'stability'. None of the studies actually defined what they exactly mean by these general terms, however, the general gist can be extracted from the experiments they carried out.

4.1.1. *Bicycles*

A number of studies were carried out to investigate the effect of different parameters on the 'stability' and 'manoeuvrability' of bicycles. Rice et al. [144] first investigated the lateral stability and control of two types of bicycle that were popular in the late 1960s and early 1970s and then investigated the effect of nine parameter changes on the same instrumented bicycle (load on the rear, the rider and the front, increasing the mass moment of inertia of the front wheel and under-inflating the tyres) for four experiments (straight line, obstacle avoidance and a narrow and wide slalom). They concluded that the standard bicycle is the best and that

Table 4. Bicycle manoeuvrability and stability experiments and how the performance is rated, from [5].

Term	Authors	Experiment	Performance measure
'Manoeuvrability'	Mortimer et al. [140]	Slalom at 5, 8, 10 and 12 mph and max speed	Crossing boundary and cones, max speed
	Arnberg and Tyden [141]	Block slalom, block pairs, 1 handed curve, 'relay' riding and steady-state circle	Time + interview
	Godthelp and Buist,[142] Godthelp and Wouters [143]	Complex slalom	Time
'Performance'	Rice and Roland [144]	Slalom	Minimum time
'Control'	Roland and Lynch [18]	Slalom	Max speed
	Mortimer et al. [140]	Circle, figure-eight, LC 10 mph, 90° corner	Time Minimum radius
	Arnberg and Tyden [141]	Stationary balance, ride between two narrow gates: (a) constant speed and (b) accelerate from rest	Time + interview
High-speed 'stability and manoeuvrability'	Godthelp and Buist,[142] Godthelp and Wouters [143]	Straight + bend with either left, right or both hands on handlebar	Time
Medium/high-speed 'stability'	Mortimer et al. [140]	Straight straight between two lines	Boundary crossings
	Arnberg and Tyden [141]	Looking backwards over shoulder for a number	Boundary crossing, recalling number
	Godthelp and Buist,[142] Godthelp and Wouters [143]	Straight straight between two lines	Relative time between lines
Low-speed 'stability'	Rice and Roland [144]	Hands free straight ahead	Minimum speed
	Roland and Lynch [18]	Straight line hands on	Minimum speed
	Mortimer et al. [140]	Straight between two lines	Boundary crossings
	Arnberg and Tyden [141]	Straight between two lines	Time + interview
	Godthelp and Buist,[142] Godthelp and Wouters [143]	Straight between two lines	Relative time between lines

load in the rear placed low is good for 'manoeuvrability', whilst load on the rear placed high is bad for 'manoeuvrability'.

The effect of different style handlebars (high rise, standard and racing) on the 'manoeuvrability' of the bicycle was experimentally investigated by Mortimer et al.[140] Riders carried out seven experiments and rated each bicycle and each task on a five-point scale. They concluded that

> since the high rise handlebar configuration allowed good manoeuvring performance it should be considered an acceptable design. Standard handlebars offer a good compromise between the characteristics of the racing and high rise types, and provided stable, low-speed tracking which is important for safe riding on streets in the mix of other traffic.

Godthelp, Buist and Wouters [142,143] developed a bicycle, which they could change geometric parameters such as the wheelbase, trail and moments of inertia of front frame and wheels, and carried out four experiments in each configuration (Table 4). They also carried out these experiments with four different style bicycles (and four different style mopeds) and conclude that all bicycles have the same high-speed stability. For low-speed stability and manoeuvrability, they conclude that the rider position is dominant and once again the racing bicycle and the high-rise handlebar bicycle were the worst.

The effect of different riding positions and bicycle styles on a child's ability to control a bicycle safely in traffic was investigated by Arnberg and Tyden.[141] They used the time to complete an experiment as a performance measure in 10 tests to measure the 'stability' and 'manoeuvrability' of six different bicycles when controlled by children for three styles (normal, collapsible and rodeo) of bicycle with two types of handlebar (normal and high rise). Similar to Mortimer et al.,[140] they concluded that bicycles with extreme handlebars have a poorer manoeuvrability performance than those with standard handlebars and that the race handlebars make the bicycle less manoeuvrable whilst high-rise handlebars are ok. Also, similar to Roland and Lynch,[18] they concluded that the rodeo style bicycles (mass high and to the rear) has the worst manoeuvrability performance out of the three tested models.

Similar safety experiments with young children were carried out by Wierda, Roos and Wolf [145,146] to investigate 'manoeuvrability'. However, they did not measure the time the rider used to complete the experiments; instead, they only recorded the errors made as they view 'safety' completely from the traffic point of view: to safely ride on the roads, the rider should be able to carry out the specified manoeuvre in a specific section of the road as any deviations could result in contact with another road user. They conclude, interestingly enough, that there are no major differences in 'manoeuvrability' between the different bicycle styles for children.

4.1.2. *Motorcycles*

The 'manoeuvrability' of a motorcycle and automobile was experimentally compared by Watanabe and Yoshida [147] by carrying out the same evasive manoeuvre with motorcycles and cars. They found that the motorcycles required significantly longer distances than cars, even though the cars are much wider than a motorcycle. They also found that less skilled riders require 15–20% more distance to avoid the obstacle than skilled riders. Furthermore, speed has less influence on the performance of skilled riders than unskilled riders who seemed unable to produce large steer torques at higher speeds. They conclude that for motorcycles riding at speeds above 30 km/h, they would consider manoeuvring around an object instead of attempting to stop before it as the better evasive manoeuvre due to the distance-to-speed relationship that a manoeuvre has whilst stopping has a distance-to-speed squared relationship.

Recently, Biral et al. [148,149] investigated safety support systems for motorcycles. They implemented a rider model on a real vehicle which acts as a co-rider that assists the human rider and detects potential dangers while curving and at intersections in advance.

4.1.3. *Safety discussion*

All the experimental ratings listed above are comparative, using relative scales and vehicle/rider combinations. Therefore, within a single study vehicles and vehicle properties can be compared, but this is not possible between studies as they are not universal. Also none followed the Cooper–Harper methodology whereby the physical and mental workload had to be measured. Most did not even interview the rider to get an indication of the mental workload level.

However, from the above, it does appear that most authors refer to 'stability' within the framework of 'safety' as the ability of the bicycle and rider system to remain upright and within a narrow straight path. Stability is measured in terms of deviations from that path or by the minimum speed that the manoeuvre can be carried out at. The term 'manoeuvrability' usually refers to the system's ability to change direction, such as in a slalom or LC. A more 'manoeuvrable' vehicle can carry out the same manoeuvre at a higher speed (slalom) or in a

shorter time/distance (LC). The manoeuvrability of vehicles appears to largely depend on two factors: the mass distribution of the system, in particular, the riders location and orientation (moment of inertia) and the style of the handlebar and thus the riders ergonomics.

4.2. *Handling qualities for normal riding*

Normal riding refers to the bulk of a vehicle's usage on open roads: not under extreme circumstances or at the performance limits.

Bicycle research on this role is surprisingly scarce. However, many have hypothesised that a self-stable bicycle is preferred over an unstable one as the unstable bicycle requires active rider control to be stabilised.[150] Herfkens [151] carried out bicycle model parameter investigations in the late 1940s. He concluded that to increase the low-speed stability of a typical Dutch bicycle, the head angle should be increased, the trail decreased, the mass of the front frame decreased and the mass of the front wheel increased.

Jones [67] in a quest to discover what makes a bicycle stable developed a number of what he called 'un-rideable bicycles'. He reasoned that a bicycle moving slowly is unstable and almost un-rideable, i.e. the rider cannot keep the bicycle upright, but a bicycle moving at high speed is stable and also easily rideable, and that the stability is, therefore, connected to a measure for how rideable the bicycle is. To discover more about the stability of a moving bicycle, he made examples that should be unstable and therefore un-rideable. He, however, found that the destabilising effect of a counter-rotating gyroscope had very little effect on the rider's ability to stabilise the bicycle, while the inherent stability of the bicycle was affected dramatically. On the other hand, he reasoned and experimentally found that by adjusting the bicycle's trail, he could make a bicycle that was both unstable and un-rideable or uncontrollable for the rider. He attributed this to the trail which has to remain positive and gravitational forces to be overcome to return to the upright straight ahead orientation. Jones' theories on stability were shown to be incomplete by Meijaard et al.[2]

The book 'Lords of the chainring' by Patterson [152] has been used by students in a bicycle design class to develop out of the ordinary bicycles with good handling. The book gives design guidelines based on aircraft handling quality analogies. It also discusses that the steering stiffness is an essential design parameter for bicycles. Design guidelines are given including equations such as for the roll control authority which links the roll rate to the hand movement. These equations are only subject to geometrical parameters of the bicycle and not the masses and inertias, making the validity doubtful, however, good results are claimed to have been achieved with this method by Patterson and Leone.[153]

More has been done for motorcycles. Tony Foale [154] has a whole book on Motorcycle handling and chassis design. He defines handling as: 'By this we mean the ease, style and feel with which the motorcycle does our bidding' (p.1-1). Foale continues by saying that this depends on the overall geometry, chassis stiffness, weight and its distribution, tyre type and size and that also the rider responses have a major influence on the handling characteristics. In a penultimate chapter, he talks about 'feel', by which he means the proprioceptive feedback to the rider as a sensor of how far he can get with braking, cornering, etc., and points out that this is an important factor in good rider control. In the last chapter, he reports on some nice experiments on a BMW R75/5 motorcycle with various head angles and trails. His conclusion is that there is nothing magical about currently used values (27° and 9 cm) almost anything (positive!) is rideable, even given some moderate forward speed with hands-off.

The other research on motorcycles has been largely focussed on developing test manoeuvres and then correlating experimental results with simulation results and using simulations for predicting handling.

Table 5. Manoeuvres used for rating motorcycle handling, the indexes used to rate them and the corresponding values for good handling according to Cossalter and Saduackas.[48,139]

Test	Handling index	Good handling achieved when
Steady turning	Roll factor $= \tau/\varphi$ Acceleration factor $= \dfrac{\tau}{V^2/R_c} \cong \dfrac{\tau}{g \cdot \tan \varphi}$ Yaw factor $= \dfrac{\dot{\psi}}{\delta}$	Low values, small negative steer torque
U-turn	Koch index $= \dfrac{\tau_{\text{peak}}}{V \cdot \dot{\varphi}_{\text{peak}}}$	Low values
Slalom	Roll transfer function $= \dfrac{\tau}{\varphi}$	Small phase
LC	LC Roll Index $= \dfrac{\tau_{p-p}}{\dot{\varphi}_{p-p} \cdot V_{\text{avg}}}$	Low values
Obstacle avoidance	Time lag between τ and $\dot{\varphi}$	Small lag

Note: τ is steer torque, φ roll angle, ψ yaw angle, δ steer angle and v forward speed.

To rate different manoeuvring aspects such as steady-state and transient behaviour in separate manoeuvres, five tests have been defined (steady turning, U-turn, slalom, LC, and obstacle avoidance test).[19,139] For each test, handling indexes have been developed, as described in Table 5. The LC manoeuvre has received most attention from researchers.

Koch [155] defined the 'Koch index' to classify the ease with which a turn is entered (transient response) by relating the peak in steer torque to the first opposing peak in roll rate. Later, Cossalter and Saduackas [48] defined the 'lane change roll index' (LC index) to classify the transient LC manoeuvre by relating the peak-to-peak rider input steering torque to the peak-to-peak roll rate of the motorcycle. Both indexes are normalised by the forward speed. These handling indexes have been found to correspond well with what is perceived as 'good handling'.

The LC index was shown to be an objective function for comparing motorcycles,[48] and in the same study, it was shown with an analytic approximation of the LC index that motorcycle 'manoeuvrability' is dominated by front wheel inertia properties. Earlier, 'performance maps' for a LC, which are the loci (*xy* plot) of the steer torque and roll angle (the quotient of which is the roll factor), were investigated by Rice.[45] With these plots, he was able to distinguish between successful and unsuccessful manoeuvres and different riders.

LC manoeuvres on motorcycles were simulated and carried out experimentally by Rice and Kunkel [19] and Zellner and Weir.[110] The latter also developed a steady-state turn manoeuvre and used the roll and yaw factors to compare experiments with simulations with mixed success.

The rider's perceived steering effort in a LC manoeuvre has been shown to correlate with the resultant steer torque and the rate of change of rider control torque (T_2 and T_{rc} see Table 6) for a LC experiment by Kuroiwa et al.[156] Interestingly, these measures include the rider applied roll–steer torque and was found for a wide range of rider experience.

The influence of the dominant rider mass on the lateral dynamics of mopeds compared to motorcycle dynamics was analytically investigated by Zellner and Weir.[157] They concluded that the moped is more sensitive than the motorcycle for steer torque control yet the required rider lean input is (surprisingly) the same for the motorcycle and much lighter moped.

4.3. *Handling qualities for racing*

In racing, the main goal is to complete a specified course in the shortest possible time. Here, rider comfort is only deemed of importance if it is a limiting factor for increasing speed and the decreasing lap time. Handling qualities for the racing 'role' are, therefore, linked to performance factors.

Table 6. Characteristic measures used by Kuroiwa et al. [156] to investigate steering effort in a LC manoeuvre.

Characteristic value	Nomenclature
Peak steering torque in stationary test	T_{s1}
Peak roll–steer torque in stationary test	T_{r1}
Static torque ratio	$\epsilon_1 = (T_{r1}/T_{s1})$
Peak steering torque in actual running test	T_{s2}
Peak roll–steer torque in actual running test	T_{r2}
Dynamic torque ratio	$\epsilon_2 = (T_{r2}/T_{s2})$
Resultant dynamic steering torque	$T_2 = \sqrt{(T_{s2})^2 + (T_{r2})^2}$
Rider control torque	$T_{rc} = (\epsilon_2 - \epsilon_1)T_{s2}$

Note: They perform two experiments, the first stationary with the motorcycle held upright where the rider applies a $\pm 5°$ sinusoidal steering input. The second experiment is a LC test.

Oddly enough, and despite there being a massive bicycle racing industry, handling quality research within the racing 'role' has only been performed on motorcycles. The only exception is the recent work by Cangley et al. [158] where they model the bicycle (including aerodynamic drag), track and rider, to determine the optimal bicycle for a specific time trial track. On the other hand, a plethora of biomechanics, aerodynamics, physiology, frame and component stiffness and mass, and suspension dynamics research has been carried out over the years with respect to increasing cycling race performance levels.[159,160] These investigations only focus on optimising either the riders physical output level, or the material they were using and were never aimed at optimising speeds for specific corners or the required rider control. Generally, but certainly for road bicycles this is the case, very little has changed over the last 110 years with respect to bicycle geometry other than a sloping upper-tube and the result of adapting material and fabrication techniques. If this is because the design is already near to optimal with respect to handling for racing through the evolution process, or because there is no need for good handling qualities, we do not know. The rest of this section, on handling qualities for racing, will only focus on motorcycles.

The most active group in the motorcycle racing handling qualities area has been the group of Cossalter at Padua University, Italy. For roughly the last 20 years, they have been investigating motorcycle dynamics and control. Cossalter [139] has clearly defined what he means when talking about 'directional stability', 'manoeuvrability' and 'handling' with respect to racing. They are:

- *Directional stability*: The ease with which a motorcycle naturally tends to maintain its equilibrium and follow a rectilinear path. It depends on the intrinsic vehicle characteristics; inertial properties of the motorcycle, forward speed, geometric properties of the steering head (which collectively determine the aligning effect of the trail), gyroscopic effects and tyre properties.
- *Manoeuvrability*: An intrinsic vehicle performance measure relating its ability to do manoeuvres to the time required to do the manoeuvre. One can, therefore, quantify a vehicle's 'manoeuvrability' by finding the best performance (shortest time, shortest distance, etc.) that a the vehicle can do on a specific manoeuvre and relate that to the performance of other vehicles on the same manoeuvre.
- *Handling*: Is the ability of the vehicle to do complex manoeuvres taking into consideration the driver's limits. It does, however, not include the rider's mental workload. It is evaluated by comparing the control effort required for the different vehicles to perform their specific optimal manoeuvre, where less effort relates to a better handling vehicle.

To find an objective measure for manoeuvrability of motorcycles independently of the rider the theoretical 'optimal manoeuvre method' was developed by Cossalter et al.[40,82] This optimal method was discussed earlier in Section 3.2.2. The required optimal control for a specific manoeuvre is vehicle dependent; therefore, vehicles can be rated by comparing the optimal performance on a specific manoeuvre. Within the context of the 'optimal manoeuvre method' when the rider limitations (physical and physiological limits such as the maximum torque the rider is able to apply or maximum steering rate they are able to reach) are included as limitations in the optimal 'manoeuvre', then the achieved performance quantifies the 'handling'.

Another approach to understanding manoeuvrability and handling is the concept of the instantaneous screw axis or Mozzi axis about which the vehicle rotates. Cossalter et al. [139, 161,162] link the instantaneous screw axis with the concept of handling by realising that large movements of the screw axis trace comply with demanding tasks. Using this concept they are able to distinguish different kinds of behaviour in transient manoeuvres and identify the different phases of a manoeuvre such as entering and exiting a curve.

Certain parts of a manoeuvre are more 'complex' than others and this has been linked to the change in kinetic energy associated with the change in orientation of the whole system. To investigate racing rider skill and effort as well as the motorcycles performance, an energy analysis of transient manoeuvres was carried out on data collected under race conditions and analysed by Cossalter et al.[162] They found that in a U-turn, there are three locations where there are large changes in kinetic energy and these locations coincide with the counter steer phase (initiating the turn), the point at which maximum rotational energy occurs (after which the throttle is applied again) and at the exit. They, therefore, predict that the time rate of the rotational kinetic energy can be used to extract phases of the manoeuvre and the dynamic limit.

Many researchers have investigated handling and manoeuvrability for racing using multi-body packages in combination with control. Berritta and Mitolo [163] used performance indexes as a measure to investigate how design parameters affected the performance of a U-turn manoeuvre. Capitani et al. [117] modelled a scooter in ADAMS/motorcycle multi-body dynamics software and compared with measurements made with a real scooter for a LC, J-turn, large and small radius 90^{circ} turn and a figure eight. The results did not compare well, which they suspected was a result of unmodelled rider motion, as the rider (inadvertently or unconsciously) used movement as part of the scooter manoeuvre control. Giner et al. [32] implemented rider motion as an inverted pendulum in a multi-body model of the motorcycle and rider based on motion capture data of real riders on a stationary motorcycle simulator. The pendulum control was based on the bikes' location in the corner and the equivalent motion capture measurements.

To research the level of detail necessary in a motorcycle model to predict handling, Frendo et al. [164] investigated the differences with three levels of model detail. They found that a non-linear tyre model greatly influences the results. Interestingly, the geometric parameter study on handling they carried out, found, trend wise, very little difference between the three different models, indicating that simpler models can be used to predict handling improvements relative to an initial setup.

4.4. *Handling qualities discussion and conclusions*

Bicycle handling quality research has only been interested in safety: accident avoidance. But no standard handling quality tests for bicycles have been developed. There is, therefore, no way to quantitatively compare the results of different bicycle handling experiments that have been carried out by the different authors.

In the bicycle research performed, most authors do not accurately define what they mean by the terms: handling quality, stability or manoeuvrability, but from the experiments it can be deduced that most mean the same. However, authors interested in normal riding generally measure stability as the ability to remain upright, whilst authors working on bicycle safety have an additional requirement that the bicycle continues in the same direction, which is defined as directional stability.

Motorcycle handling quality research has focussed mostly on determining quantifiable measures and repeatable testing procedures based around normal riding and racing situations. This has enabled authors to compare simulations and experiments often performed by different authors and often years apart. Furthermore, indexes have been developed for the different standard manoeuvres with which good and bad handling can be predicted.

For bicycle research, it is also essential to develop a standardised set of tests and handling indexes, in a similar manner to those that exist for motorcycles, such that bicycle handling can be compared and quantified both experimentally and in simulations. Another advantage of such a set of handling tests is that a set of handling qualities for normal riding can be determined, such that the designers, who now apply a time-consuming trial and error method to developing new bicycle concepts, can determine a priori what the handling qualities will be.

According to Cooper and Harper, handling qualities (for aircraft) are a factor of the complete rider(pilot)–vehicle system and depend upon both the physical and mental workload of the (pilot)rider. For aircraft, handling and manoeuvrability can be determined based on frequency and damping of the main dynamic modes. This method could also be applied to single-track vehicles where the (unstable) weave and wobble modes are most likely to be modes of interest.

In single-track vehicles, the rider has a large influence on the actual implementation of the manoeuvre, therefore, comparing different riders output can be problematic. Furthermore, large differences have also been noted between the rider control for novice and experienced motorcycle riders. The development of a set of design guidelines based on rider control effort for different rider levels of experience could help manufacturers design bikes that are better suited to a certain group such as novice, intermediate or expert riders.

5. Discussion and conclusions

Much effort has gone into the development of bicycle and motorcycle models. These models have been experimentally validated and can accurately model the dynamics of the vehicle. For instance, the Whipple bicycle model, which is a low-dimensional model, is still capable of showing basic bicycle dynamics like unstable and stable weave and capsize motions. On the other hand, only a small number of rider control models have been developed and of which the majority have been developed to simulate the stabilising task control. None of the bicycle rider models have been experimentally validated. There is thus still much to explore on the rider control side of the modelling of bicycling. While steering has been identified as the main control output, it is still important to explore the contribution of upper body motions. For motorcycles, it has been shown that upper body motions only have a small contribution to the overall control of a vehicle, but they are most likely to be important for rider comfort and can be used for achieving better performance during transient manoeuvres. However, it is unclear if completely dismissing upper body motion on bicycles, where the rider to machine mass is much larger than on a motorcycle and cyclic motions due to pedaling take place, is valid.

There is no coupling at the moment between bicycle human rider control models and handling qualities. In fact, only a few handling quality indexes have been defined for single-track vehicles of which all but one have been developed for motorcycles. Furthermore, the only

handling quality measure for bicycling that has been defined so far has yet to be experimentally validated. While many authors have performed bicycle handling experiments they have all done this according to their own specifications making only qualitative comparisons between different experiments possible. The development of standard performance tests and handling quality measures for bicycling, therefore, is an exciting area still waiting to be explored.

Notes

1. This and many other interesting facts about Drais are discussed in Hans-Erhard Lessing's book.[165]
2. Most authors talk about the three orthogonal rotations of the rear frame of a single-track vehicle as yaw; the rotation about the vertical axis, roll; the rotation about the longitudinal axis, and pitch; the rotation about the lateral axis. When a rider is taken into account, the rider is often modelled as two bodies, being; a lower body is rigidly attached to the vehicle rear frame and an upper body which can rotate about the longitudinal axis of the rear frame. The angle that the upper body makes with respect to the rear frame is called the lean angle or rider lean angle. Sometimes the rider upper body is also able to rotate about the lateral axis of the rear frame, this is called rider pitch. When the rider is rigidly connected as a single body to the vehicle such as is the case for the benchmark bicycle,[2] there is no (rider) lean or rider pitch. However, to confuse matters, the authors then talk about lean of the rear frame instead of roll. There, the lean is the absolute roll angle that the rider and vehicle make with respect to the vertical axis.

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