TAM 674
Applied Multibody Dynamics

Spring Term 2003, Mon & Wed 10:10-11:00, 202 Thurston Hall, 3 credits.

Homework assignment 11

A bungee jumper is modeled by a solid block with a length of $l = 1.87$ m, a width of $w = 0.3$ m, and a 'thickness' of $d = 0.2$ m. The total mass of the jumper is $m = 95$ kg, which is assumed to be uniformly distributed over the body. The bungee is a linear elastic cord with a relative material damping of 10 % (in the case of a simple linear mass-spring-damper system for the bungee and jumper). The compliance and the unstretched length of the bungee are such that the jumper has a maximal acceleration of $3g$ at the bottom dead position. The jumper starts from a height of 80 m above level ground, whereas we assume that the bottom dead point is at 1/4 of this height. The bungee is attached at the bottom surface of the jumper in an asymmetric way such that it is 25 % out of the centre in both the width and the thickness direction. The air drag is modeled by three forces D_i applied at the centre of mass of the body according to

$$D_i = -\frac{1}{2}\rho A_i c_d \frac{v_i}{|\mathbf{v}|}|\mathbf{v}|^2, \quad i = x..z. \tag{1}$$

With the specific mass of the air $\rho = 1.25$ kg/m^3, the frontal areas A_i m^2, the drag coefficient $c_d = 1.2$ as in a flat plate, and the velocities v_i m/s of the center of mass of the jumper (no summation over i). The initial conditions are such that the jumper falls forward from rest as if pivoting around the edge of the platform until he loses contact. This is where the real simulation starts and we call this moment in time $t = 0$. The gravity field strength is $g = 9.81$ N/kg.

a. Determine the stiffness k N/m and the damping c Ns/m for the bungee from a simple linear mass-spring-damper model.

b. Determine the initial conditions, that is: position, orientation, velocity and angular velocity, for the jumper at $t = 0$.

c. Determine the motion of the jumper by numerical integration of the equations of motion over a time period of at least 17 seconds. Give a clear representation of this motion in a number of graphs of your own choose. A minimal set should include the time history of all state variables and all state derivatives. Use Euler parameters for the description of the orientation of the body. Express the constrained equations of motion in the DAE form and use as state variables the position of the center of mass of the jumper expressed in the global coordinate system, the four Euler parameters, the velocity of the center of mass expressed in the global coordinate system and the angular velocity of the jumper expressed in the local coordinate system of the jumper as in

$$\mathbf{x} = (x, y, z, \lambda_0, \lambda_1, \lambda_2, \lambda_3, v_x, v_y, v_z, \omega'_x, \omega'_y, \omega'_z) \tag{2}$$

Use the coordinate projection method locally on the four Euler parameters to 'stabilize' the constraint. Give an indication of the accuracy of your results.

d. Make a graph of the ratio of the rotational Kinetic Energy of the jumper over the total Kinetic Energy of the jumper during the motion.

e. Which part of the motion is not realistic and how could you improve your model?