

TAM 674

Applied Multibody Dynamics

Spring Term 2003, Mon & Wed 10:10-11:00, 202 Thurston Hall, 3 credits.

Homework assignment 9

- a. Determine the rotation matrix \mathbf{R} and its inverse for the (3-1-3) Euler angles ϕ , θ , and ψ . The recipe for these Euler angles is to first rotate an angle ϕ about the z -axis, then rotate an angle θ about the rotated x -axis, and finally rotate an angle ψ about the rotated z -axis. The angles ϕ , θ , and ψ are known as the *precession*, *nutation*, and *spin* angles, respectively.

Determine for these Euler angles the expressions for the angular velocities, in both the space fixed and the body fixed coordinate system, in terms of the Euler angles and its time derivatives. Also determine the inverse relations, i.e. the time derivatives of the Euler angles in terms of the Euler angles and the angular velocities, in both the space fixed and the body fixed coordinate system. For which Euler angles are these last two transformations singular? Please give a physical interpretation of this singularity.
- b. Redo the above analysis for the sequence of rotations commonly used in vehicle system dynamics the so-called *yaw*, *pitch*, and *roll* angles (3-2-1). The recipe for these angles is to first rotate an angle ψ (yaw) about the z -axis, then rotate an angle θ (pitch) about the rotated y -axis, and finally rotate an angle ϕ (roll) about the rotated x -axis. In aircraft dynamics the angles ψ , θ , and ϕ are also known as the *heading*, *attitude*, and *bank* angles, respectively.
- c. Consider two successive finite rotation described by the Euler parameters p and q , successive meaning first with p and then with q . Proof that the composed rotation is described by the Euler parameters r which follows from the quaternion product, denoted by ‘ \circ ’, as $q \circ p = r$.
- d. Consider two successive finite rotations described by the Euler parameters p and q . Show that the linear interpolated values r , as in $r = (1 - \xi) * p + \xi * q$, $0 \leq \xi \leq 1$, do *not* represent a rotation for all ξ . Can you think of another way of interpolating where the result always represents a rotation?
- e. Derive the expressions for the angular velocity, in both the space fixed and the body fixed coordinate system, in terms of Euler parameters and its time derivatives. Also determine the inverse relations, i.e. the time derivatives of the Euler parameters in terms of the Euler parameters and the angular velocities, in both the space fixed and the body fixed coordinate system. For which Euler parameters are these last two transformations singular? Please discuss.