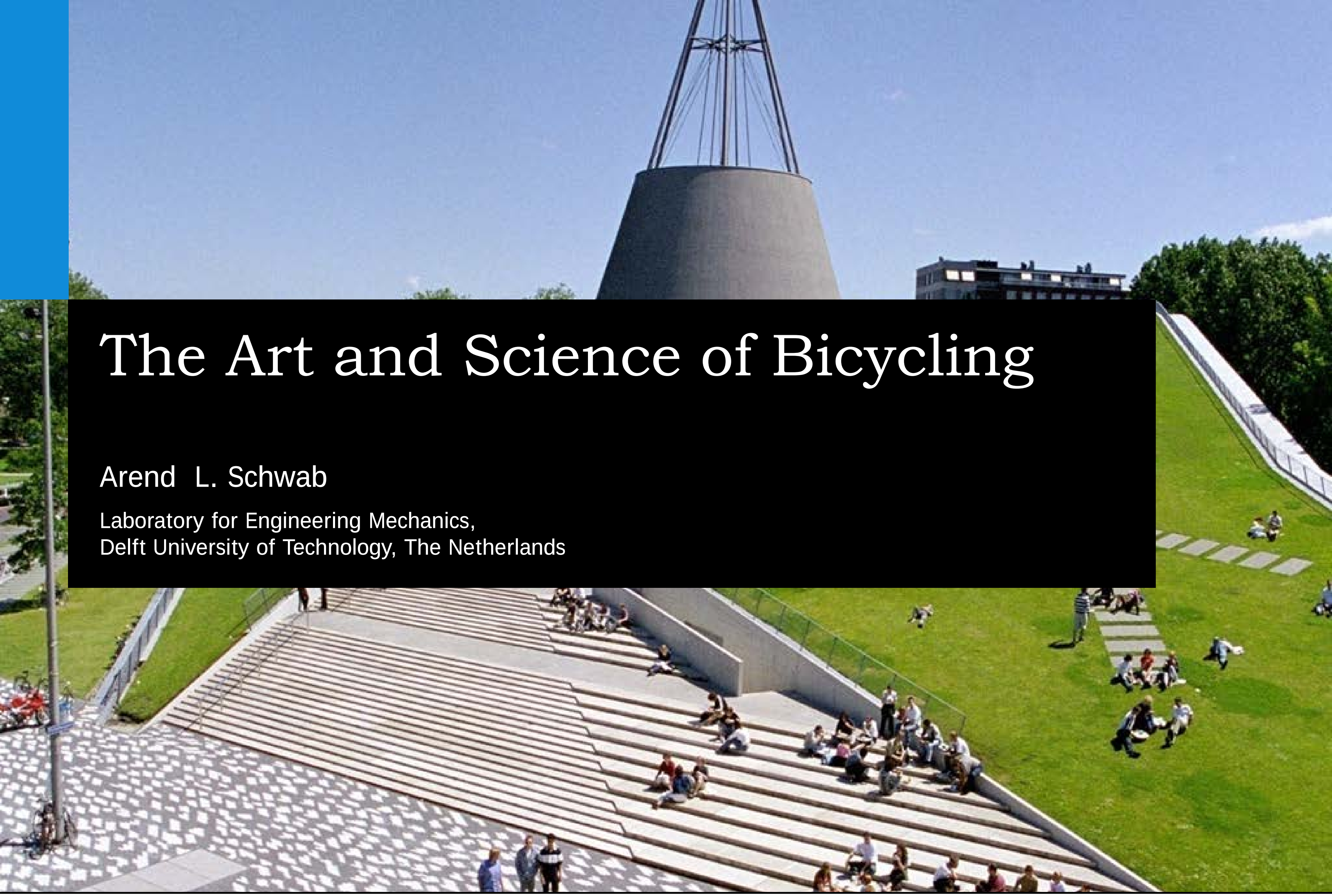

The Art and Science of Bicycling

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Acknowledgement

TUDelft:

Jaap Meijaard

Jodi Kooiman

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J. P. Meijaard, Jim M. Papadopoulos, Andy Ruina, A. L. Schwab (2007) Linearized dynamics equations for the balance and steer of a bicycle: a benchmark and review. *Proc. R. Soc. A.* **463**, 1955-1982.

J. D. G. Kooijman, J. P. Meijaard, Jim M. Papadopoulos, Andy Ruina, and A. L. Schwab (2011) A bicycle can be self-stable without gyroscopic or caster effects, *Science* 15 April: **332**(6027), 339-342.

www.bicycle.tudelft.nl

1.

Bicycle Dynamics, some observations

Movie

Jour de Fête van Jacques Tati, 1949

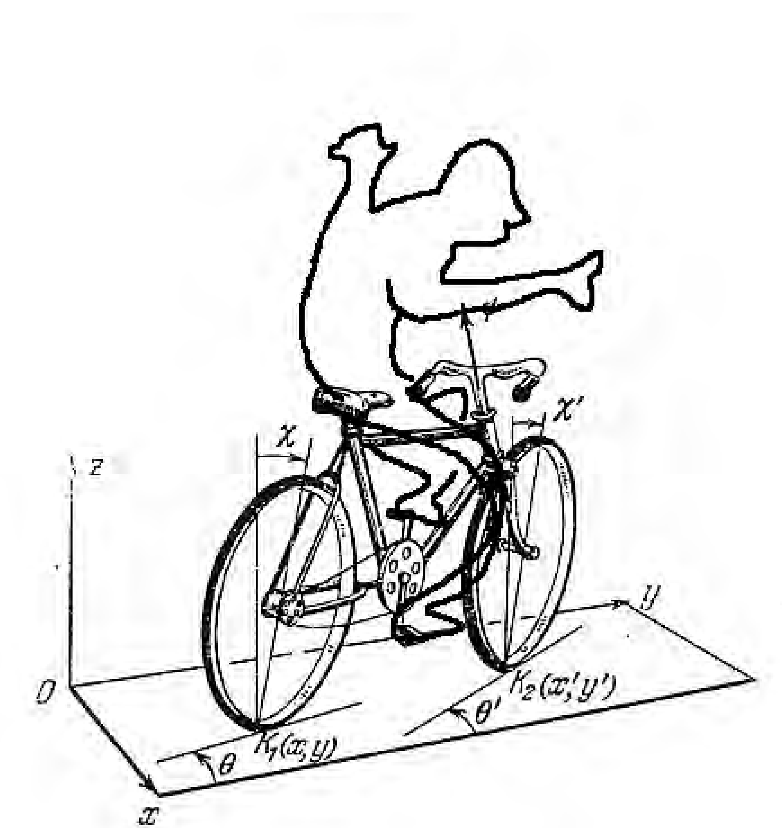
Experiment

Yellow Bike in the Car Park, Cornell University, Ithaca, NY.

Experiment

Yellow Bike in the Car Park, Cornell University, Ithaca, NY.

The Whipple/Carvallo Model (1899)



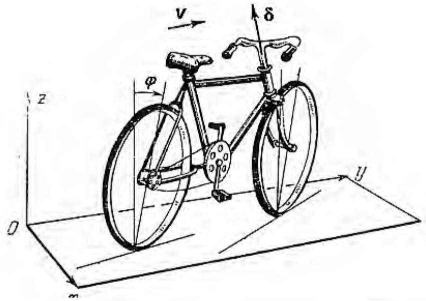
Modelling Assumptions:

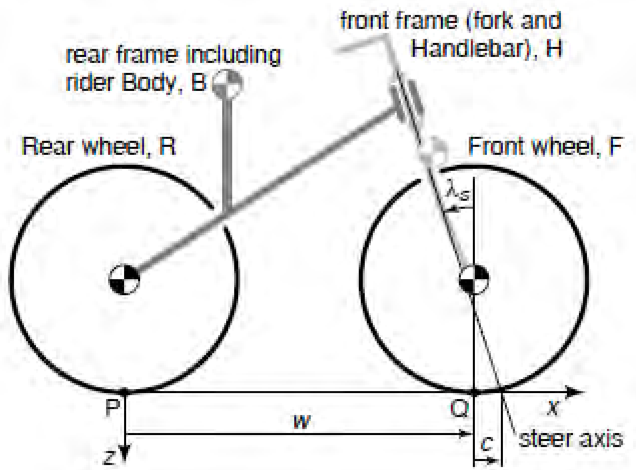
- rigid bodies
- fixed rigid rider
- hands-free
- symmetric about vertical plane
- knife-edge wheels
- point contact, no side slip
- flat level road
- no friction or propulsion

➡ 3 velocity degrees of freedom

Note: Energy Conservative

The Whipple/Carvallo Model (1899)





parameter	symbol	value for benchmark
wheel base	w	1.02 m
trail	c	0.08 m
steer axis tilt ($\pi/2-$head angle)	λ	$\pi/10$ rad (90°−72°)
gravity	g	9.81 N kg^{-1}
forward speed	v	various m s^{-1} (table 2)
R*ear wheel R*		
radius	$r_{\rm R}$	0.3 m
mass	$m_{\rm R}$	2 kg
mass moments of inertia	$(I_{\rm Rxx}, I_{\rm Ryy})$	(0.0603, 0.12) kg m^2
rear **B***ody and frame assembly B*		
position centre of mass	$(x_{\rm B}, z_{\rm B})$	(0.3, −0.9) m
mass	$m_{\rm B}$	85 kg
mass moments of inertia	$\begin{bmatrix} I_{\rm Bxx} & 0 & I_{\rm Bxz} \\ 0 & I_{\rm Byy} & 0 \\ I_{\rm Bxz} & 0 & I_{\rm Bzz} \end{bmatrix}$	$\begin{bmatrix} 9.2 & 0 & 2.4 \\ 0 & 11 & 0 \\ 2.4 & 0 & 2.8 \end{bmatrix}$ kg m^2
front **H***andlebar and fork assembly H*		
position centre of mass	$(x_{\rm H}, z_{\rm H})$	(0.9, −0.7) m
mass	$m_{\rm H}$	4 kg
mass moments of inertia	$\begin{bmatrix} I_{\rm Hxx} & 0 & I_{\rm Hxz} \\ 0 & I_{\rm Hyy} & 0 \\ I_{\rm Hxz} & 0 & I_{\rm Hzz} \end{bmatrix}$	$\begin{bmatrix} 0.05892 & 0 & -0.00756 \\ 0 & 0.06 & 0 \\ -0.00756 & 0 & 0.00708 \end{bmatrix}$ kg m^2
F*ront wheel F*		
radius	$r_{\rm F}$	0.35 m
mass	$m_{\rm F}$	3 kg
mass moments of inertia	$(I_{\rm Fxx}, I_{\rm Fyy})$	(0.1405, 0.28) kg m^2

3 velocity degrees of freedom:

- lean rate
- steer rate
- forward speed *v*

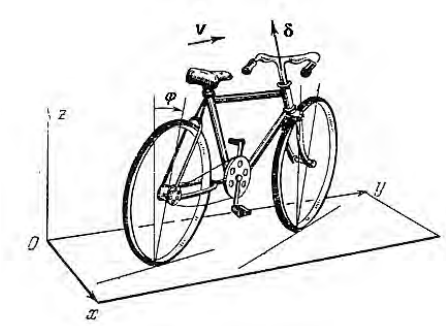
25 bicycle parameters!

Linearized Eqn's of Motion

For the straight ahead upright motion with lean angle φ, steering angle δ and forward speed v:

$$[\mathbf{M}]\begin{bmatrix}\ddot{\varphi}\\ \ddot{\delta}\end{bmatrix} + [\mathbf{C}_1 v]\begin{bmatrix}\dot{\varphi}\\ \dot{\delta}\end{bmatrix} + [\mathbf{K}_0 + \mathbf{K}_2 v^2]\begin{bmatrix}\varphi\\ \delta\end{bmatrix} = \mathbf{0}$$

$$\dot{v} = 0$$



Standard bicycle + rider :

$$\mathbf{M} = \begin{bmatrix}130 & -3\\ -3 & 0.3\end{bmatrix},\ \mathbf{C}_1 = \begin{bmatrix}0 & -40\\ 0.6 & 1.8\end{bmatrix},\ \mathbf{K}_0 = \begin{bmatrix}-1003 & 27\\ 27 & -8.8\end{bmatrix},\ \mathbf{K}_2 = \begin{bmatrix}0 & -96\\ 0 & 2.7\end{bmatrix}$$

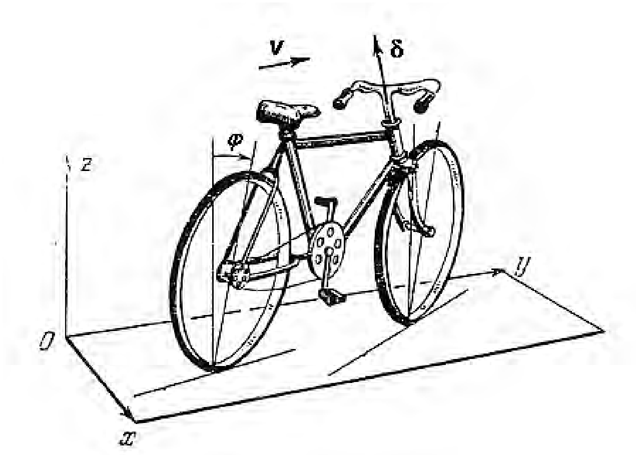
Assume motions: $\varphi = \varphi_0 e^{\lambda t}, \delta = \delta_0 e^{\lambda t}$

Characteristic equation : $\det(\lambda^2[\mathbf{M}] + \lambda[\mathbf{C}_1 v] + [\mathbf{K}_0 + \mathbf{K}_2 v^2]) = 0$

leads to a fourth order characteristic polynomial in eigenvalues λ:

$$\lambda^4 + a_3(v)\lambda^3 + a_2(v)\lambda^2 + a_1(v)\lambda + a_0(v) = 0$$

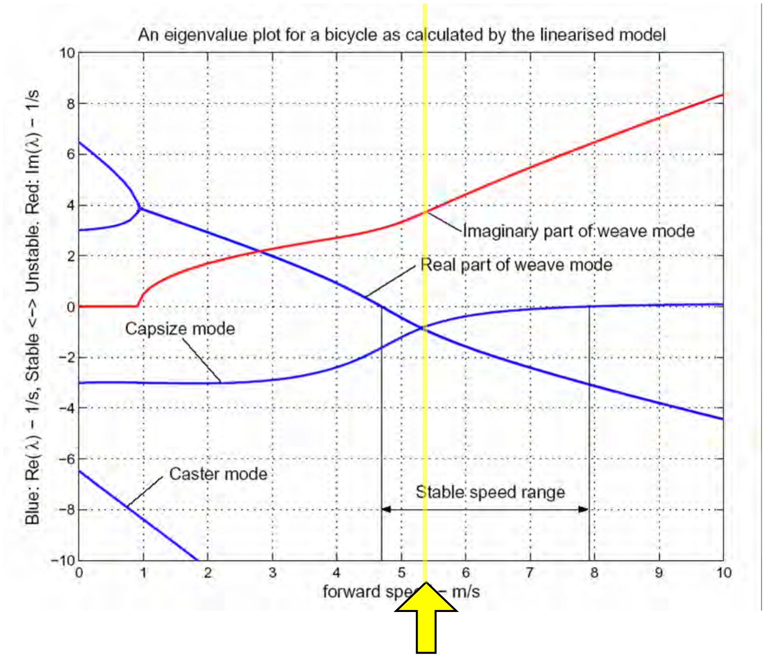
The Whipple/Carvallo Model (1899)



3 velocity degrees of freedom:

- lean rate $\dot{\varphi}$
- steer rate $\dot{\delta}$
- forward speed v

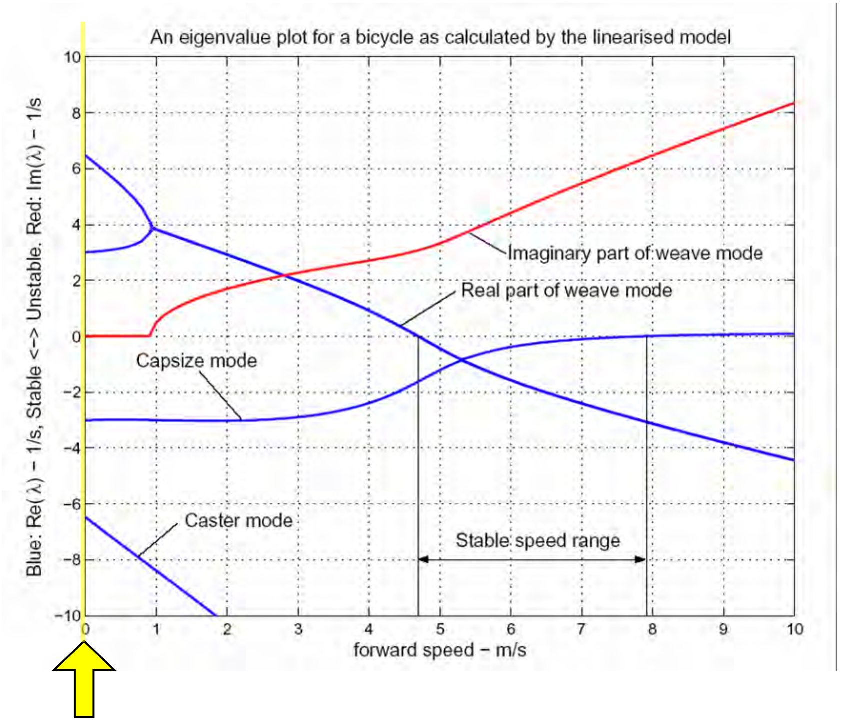
$$\det(\lambda^2[\mathbf{M}] + \lambda[\mathbf{C}_1 v] + [\mathbf{K}_0 + \mathbf{K}_2 v^2]) = 0$$

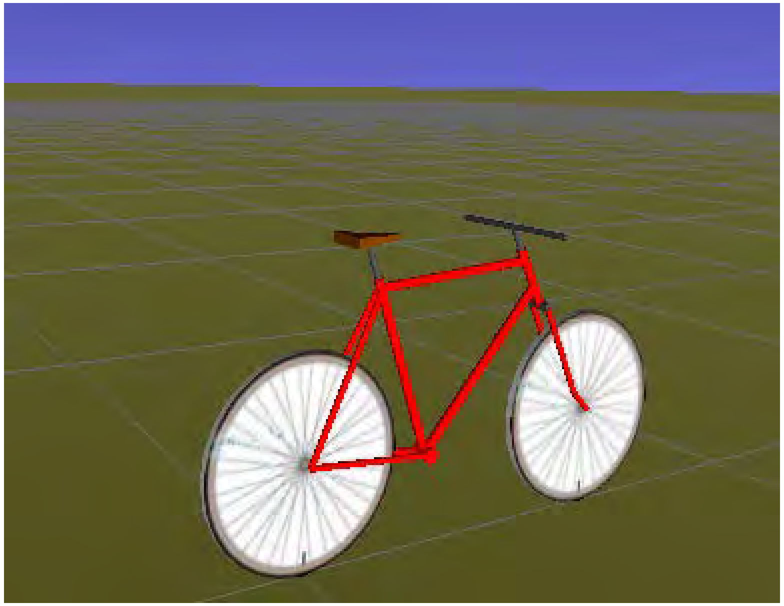


selfstable: 4.6 < v < 7.9 m/s

$$\varphi = \varphi_0 e^{\lambda t}, \delta = \delta_0 e^{\lambda t}$$

Root Loci



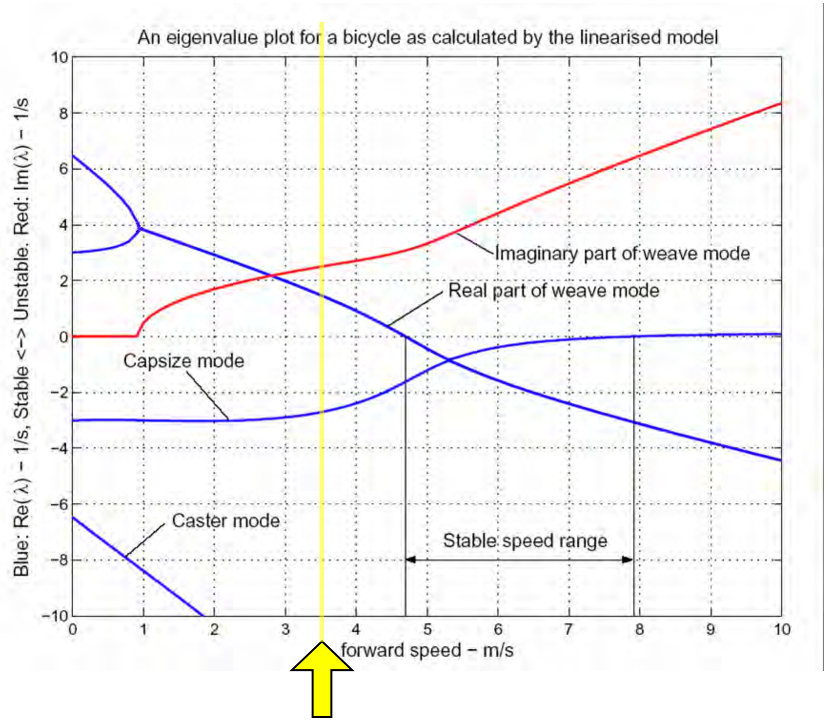


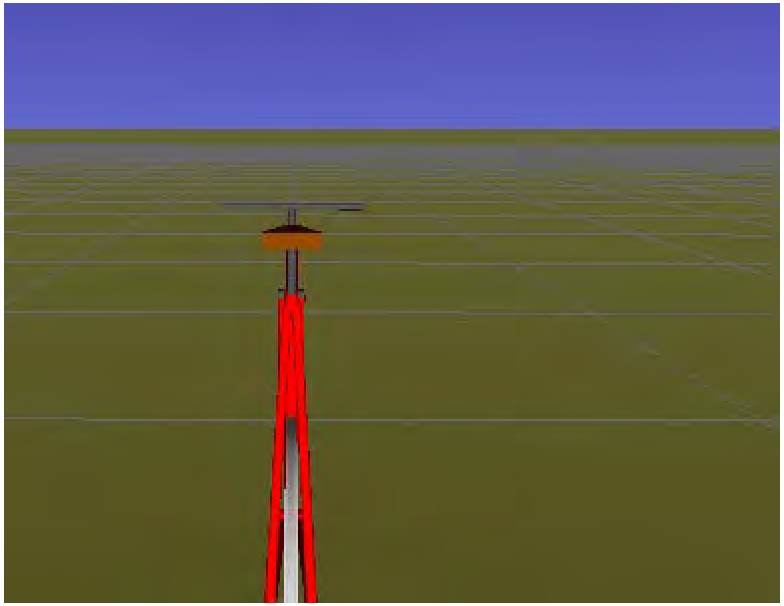
forward speed v=0 m/s, unstable

Parameter: forward speed *v*

Stable forward speed range
4.5 < v < 8.0 m/s

Root Loci



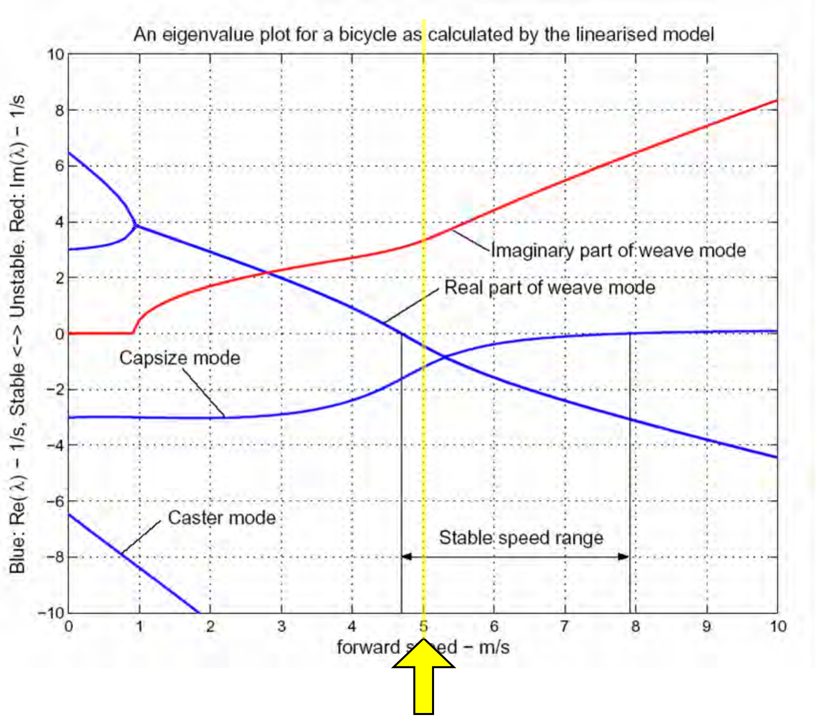


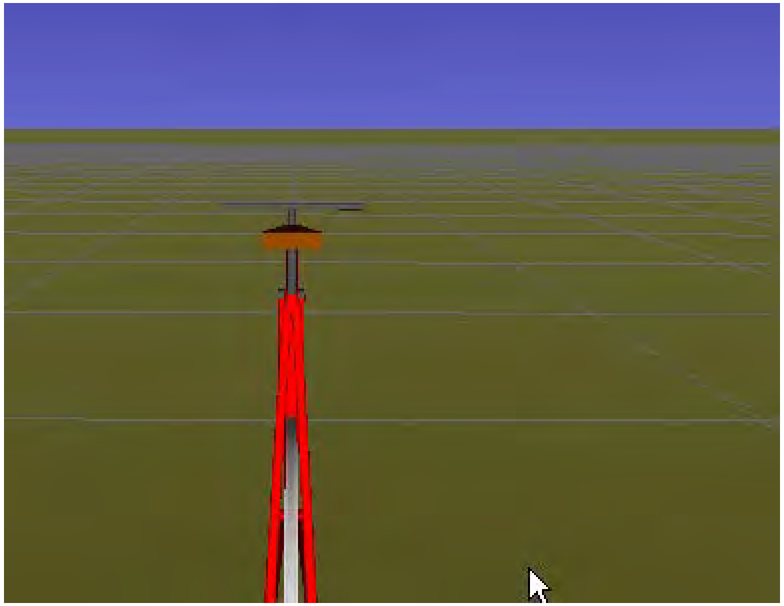
forward speed v=3.5 m/s, unstable

Parameter: forward speed v

Stable forward speed range
4.5 < v < 8.0 m/s

Root Loci



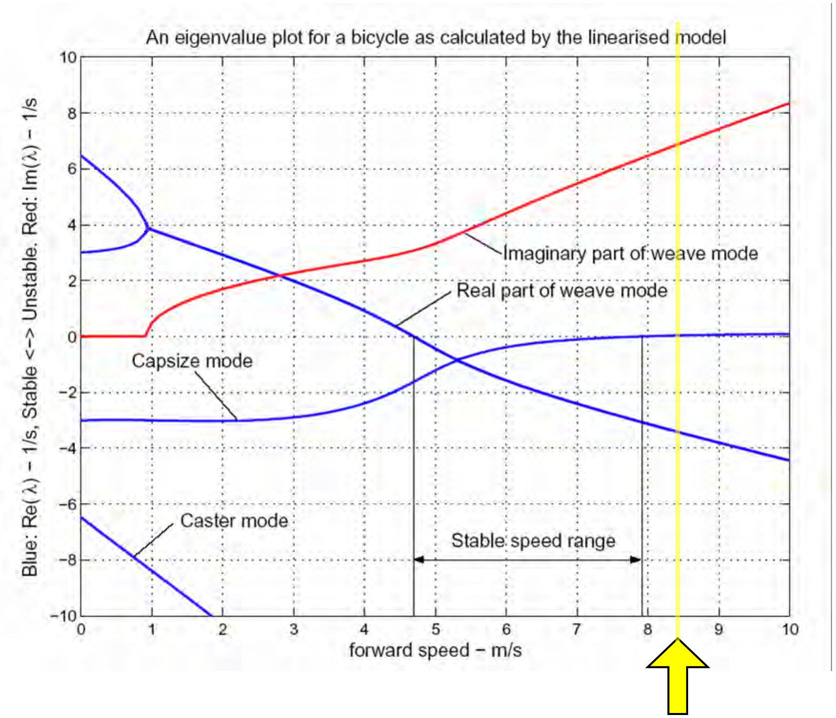


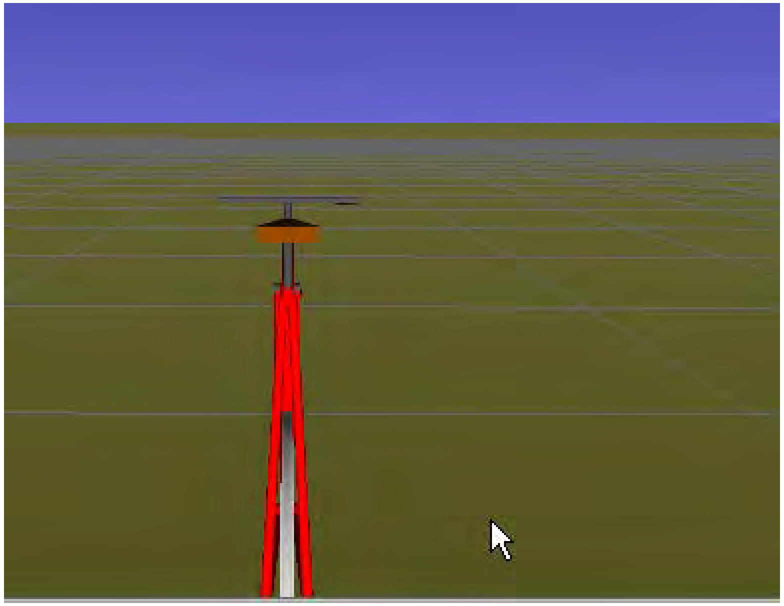
Parameter: forward speed *v*

Stable forward speed range
4.5 < v < 8.0 m/s

forward speed v=5.0 m/s, stable

Root Loci





forward speed v=8.5 m/s, unstable

Parameter: forward speed v

Stable forward speed range
4.5 < v < 8.0 m/s

2.

Experimental validation

Experimental Validation

Instrumented Bicycle, uncontrolled

2 rate gyros:

-lean rate $\dot{\varphi}$

-yaw rate $\dot{\psi}$

1 speedometer:

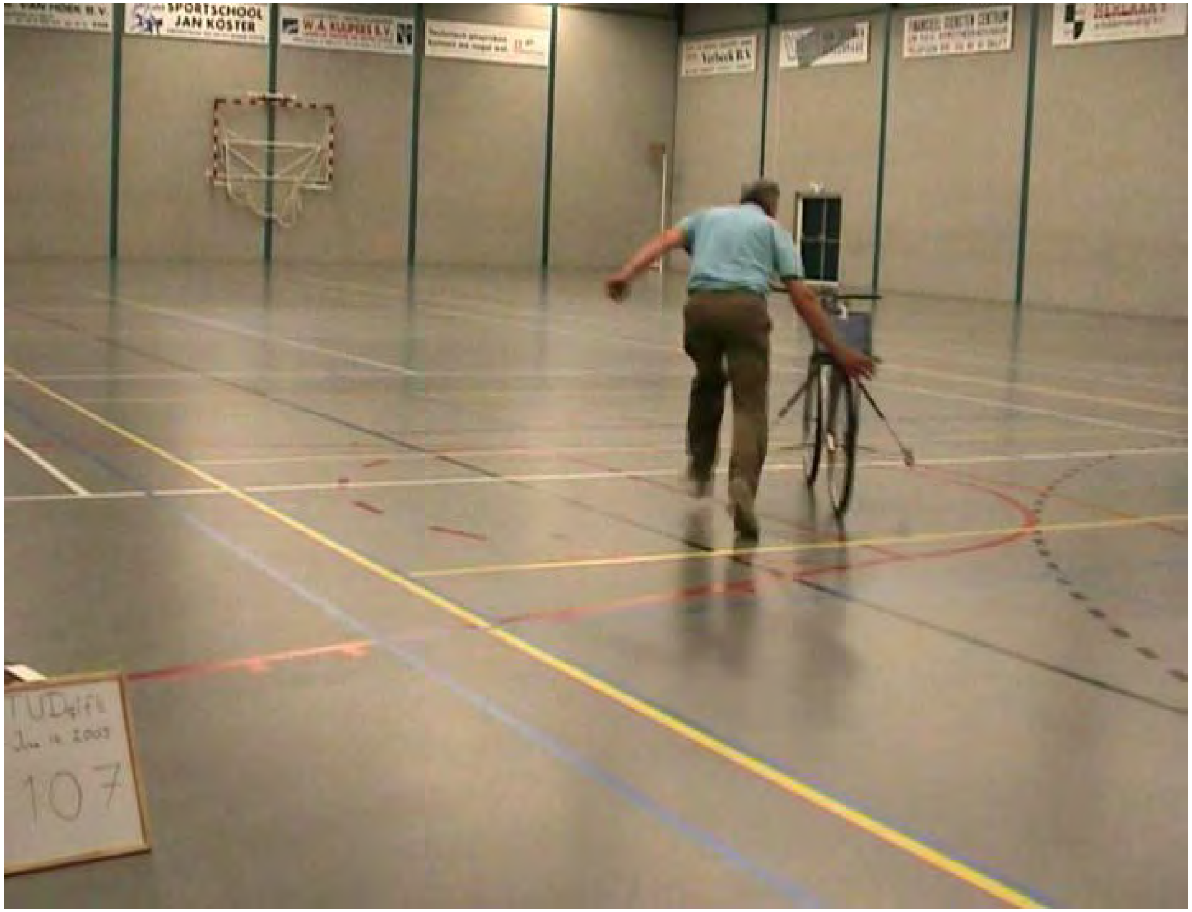
-forward speed v

1 potentiometer

-steering angle δ

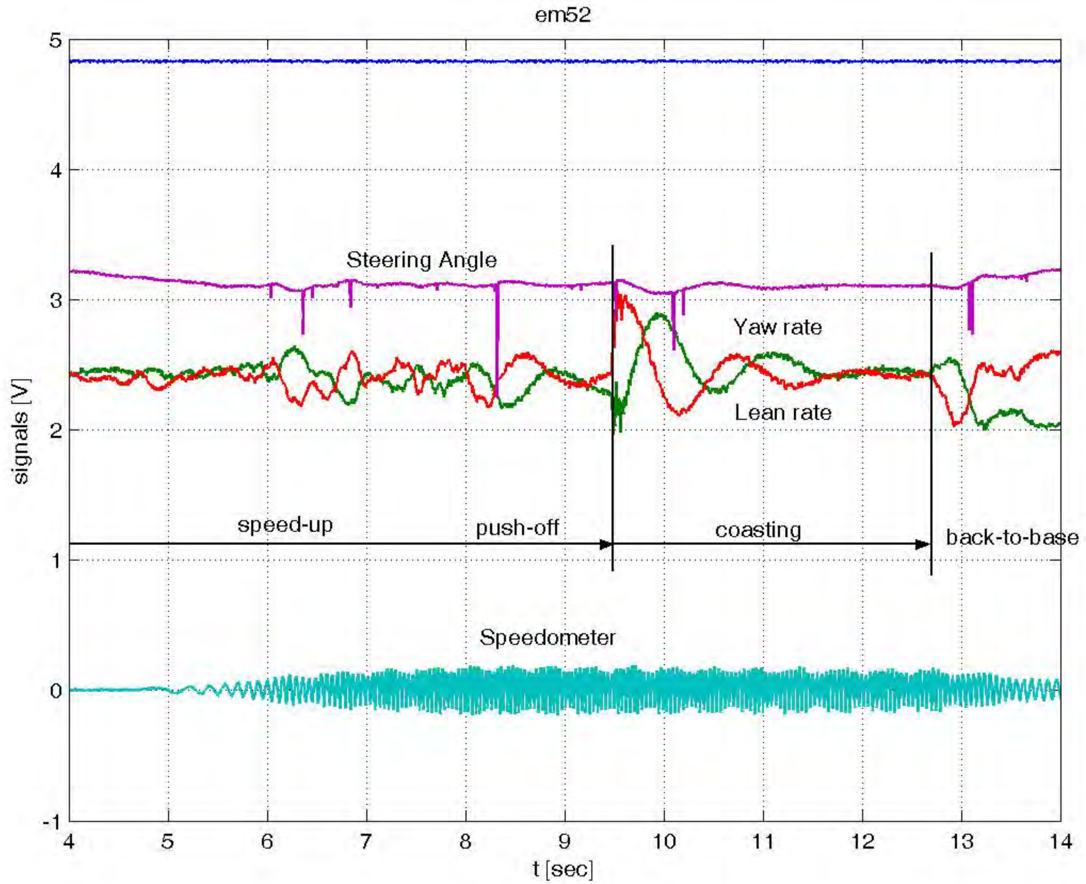
Laptop Computer running LabVIEW

An Experiment



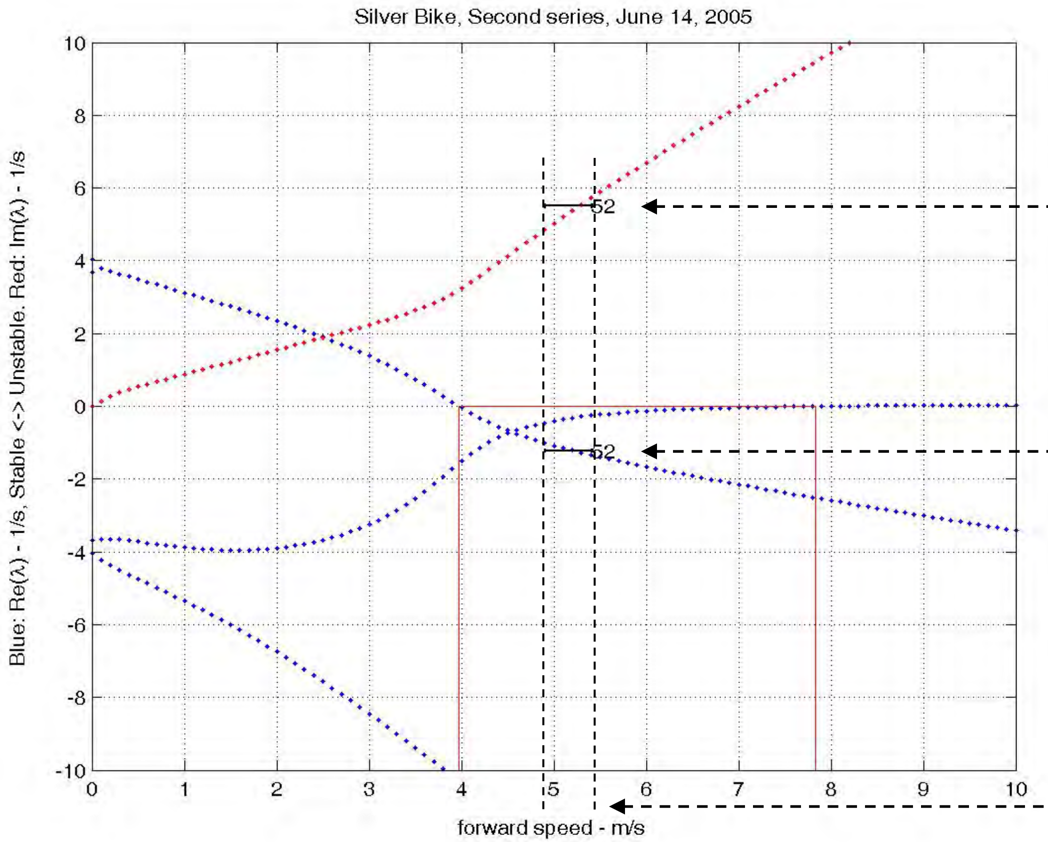
An Experiment

Measured Data



Fit function for the lean rate: $\dot{\varphi} = c_1 + e^{\lambda_1 t}[c_2 \cos(\lambda_2 t) + c_3 \sin(\lambda_2 t)]$

Compare with Linearized Results



λ_2 = 5.52 [rad/s]

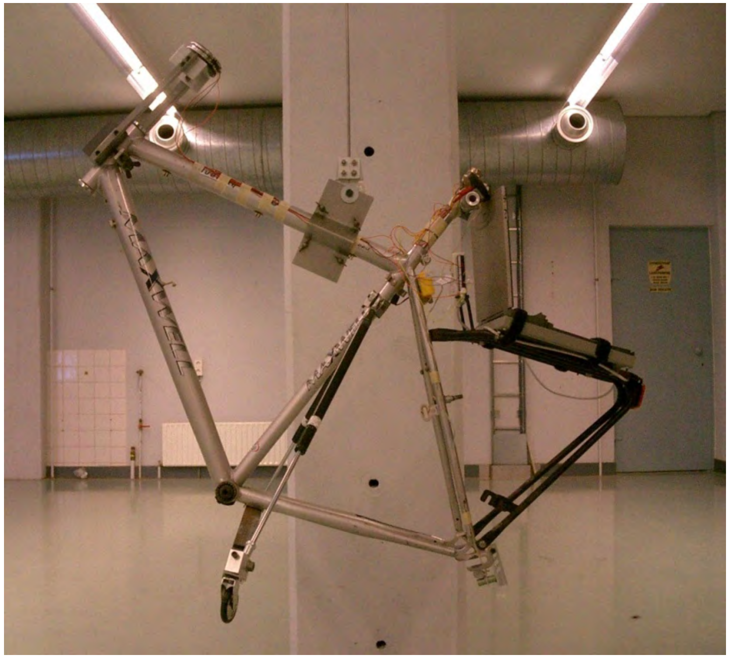
λ_1 = -1.22 [rad/s]

forward speed:

4.9 < v <5.4 [m/s]

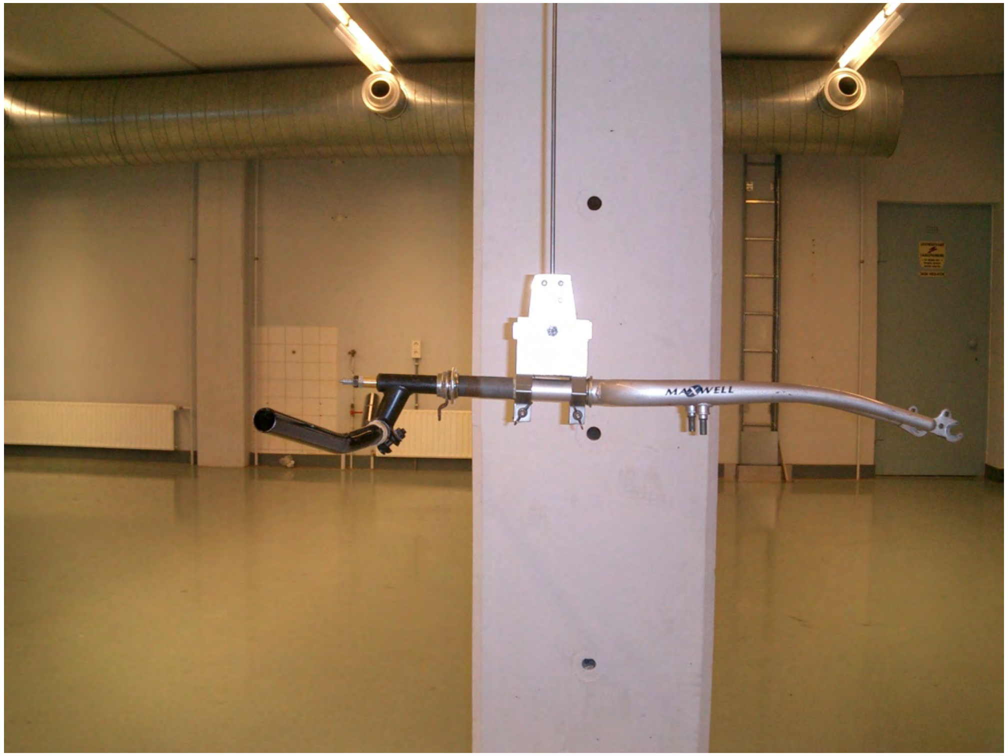
Measure Bicycle Parameters

Mass Moments of Inertia



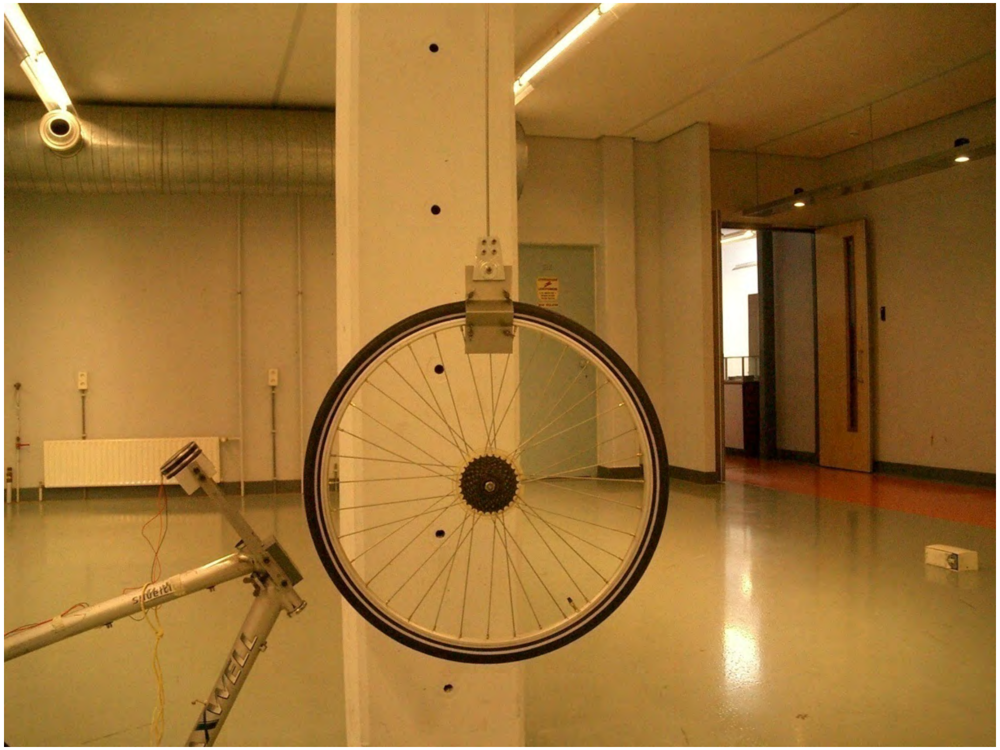
Measure Bicycle Parameters

Mass Moments of Inertia

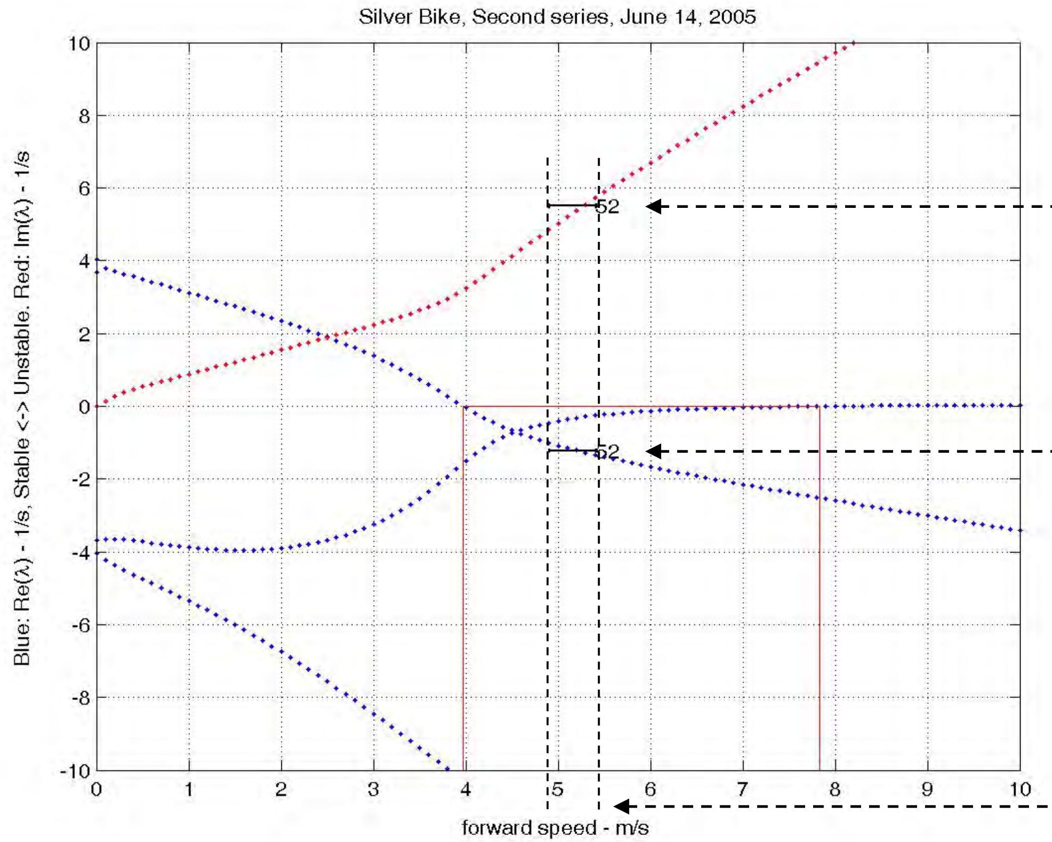


Measure Bicycle Parameters

Mass Moments of Inertia



Compare with Linearized Results



λ_2 = 5.52 [rad/s]

λ_1 = -1.22 [rad/s]

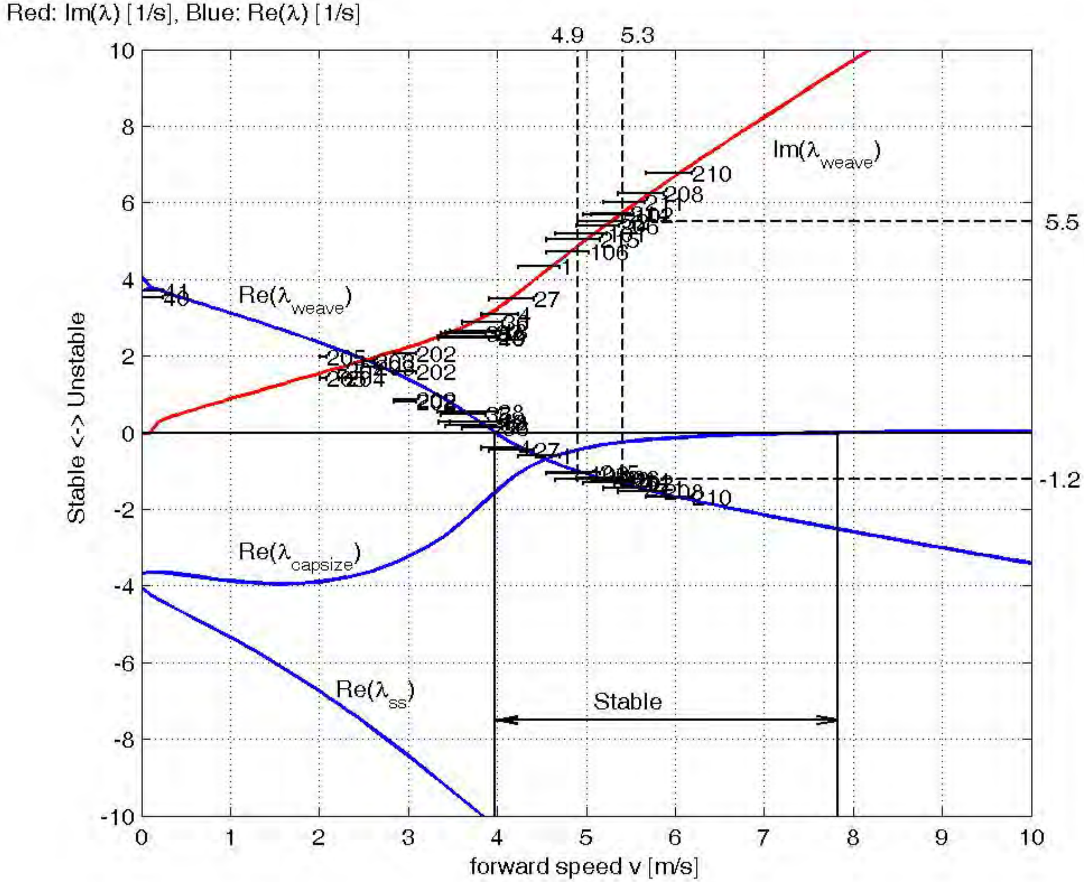
forward speed:

4.9 < v <5.4 [m/s]

Fit function for the lean rate $\dot{\varphi} = c_1 e^{\lambda_1 t}[c_2 \cos(\lambda_2 t) + c_3 \sin(\lambda_2 t)]$

Below critical weave speed

Compare in a broad speed range



Conclusion:
Experimental data in good agreement with linearized analysis on 3 dof model.

3.

Bicycle Selfstability

Selfstable: automagic control?

How do we balance a bicycle?

We balance an inverted pendulum by accelerating the support in the direction of the fall.

Selfstable: automagic control?

We balance an inverted pendulum by accelerating the support in the direction of the fall.

Balance the bicycle by steer into the fall!

(lateral acceleration contact point: $a \approx v^2/w\ \delta$)

Anecdote

LEGO Mindstorms NXT Bicycle, built by Joep Mutsaerts, MSc TUDelft

Automagic control? Steer-into-the-fall !

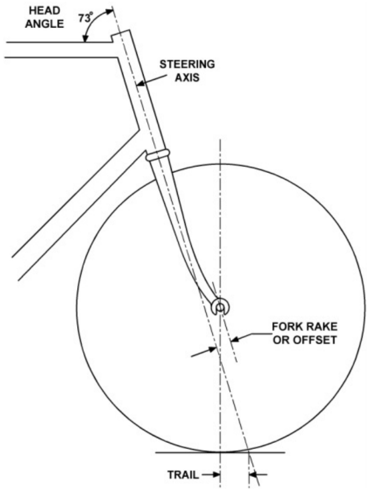
Control Law: SteerMotorVoltage=8*LeanRate

LEGO Mindstorms NXT Bicycle, built by Joep Mutsaerts, MSc TUDelft

A selfstable bicycle

Yellow Bike in the Car Park (slow motion), Cornell University, Ithaca, NY.

A bicycle is selfstable because ....



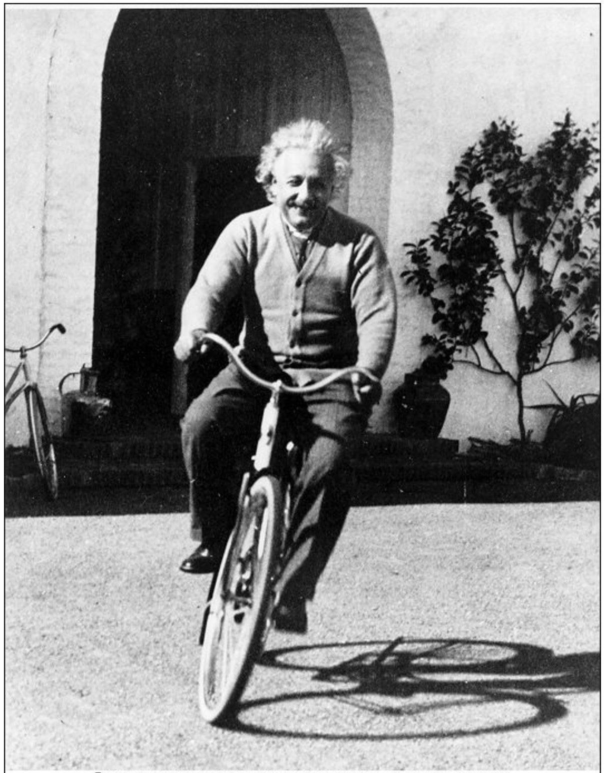
Gyroscopic effect of the front wheel?

Trail on the front wheel?

4.

Control and Handling

How do we control the mostly unstable bicycle?



by steer and balance

Control

To turn RIGHT you have to steer ...

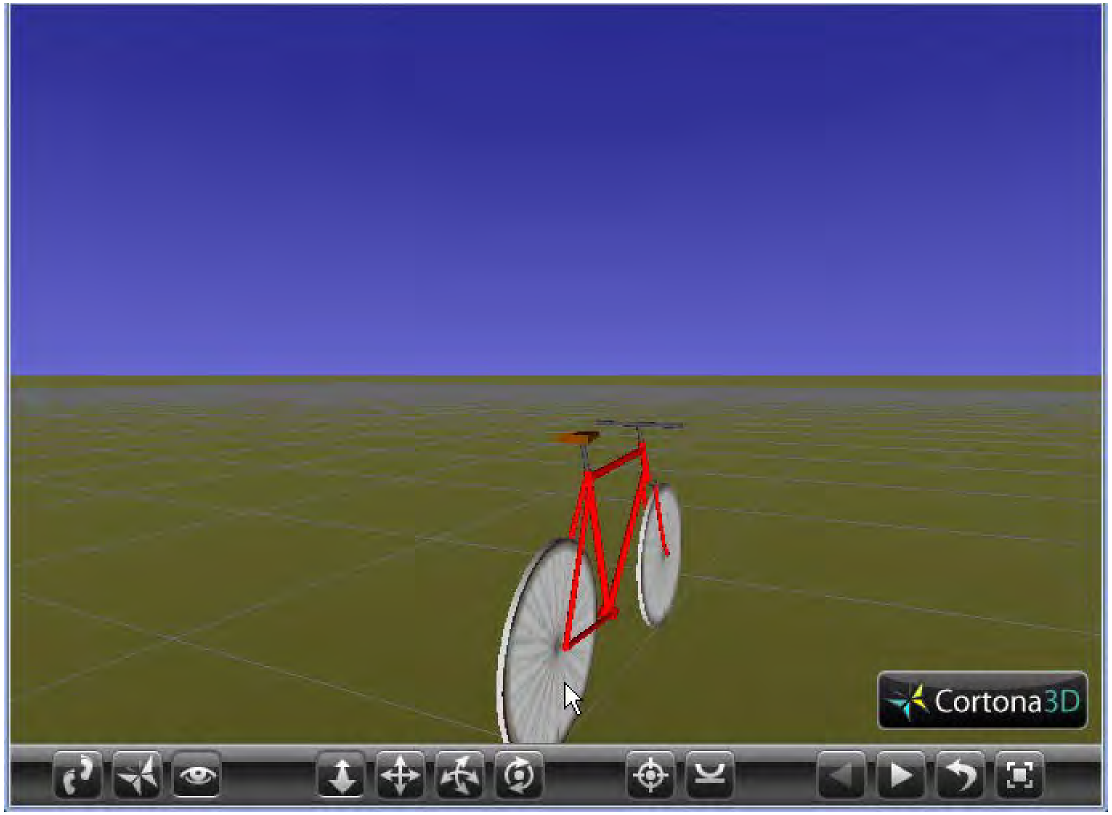
briefly to the LEFT, and then let go of the handle bars.

Control

To turn RIGHT you have to steer ...

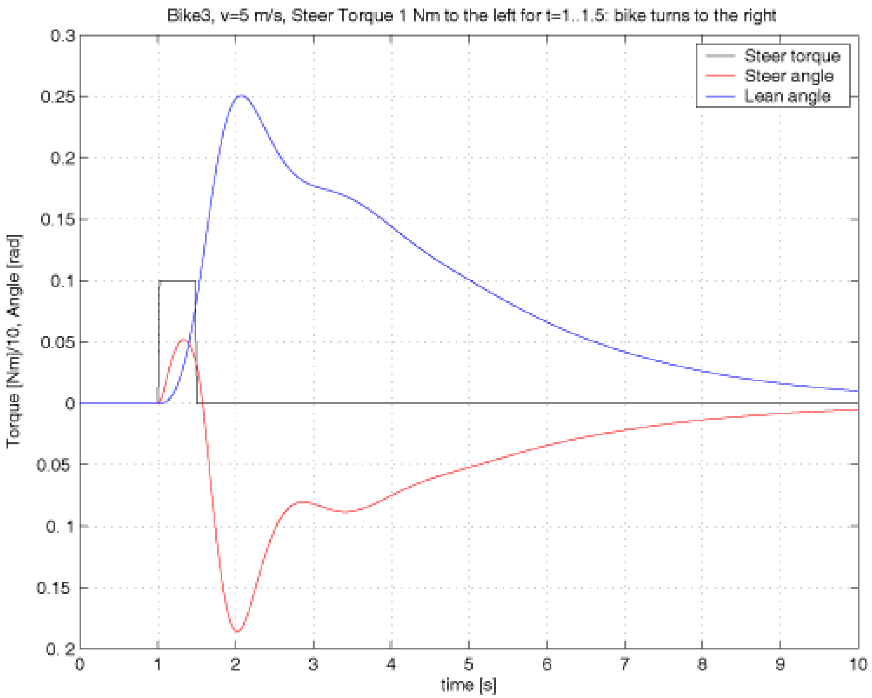
briefly to the LEFT, and then let go of the handle bars.

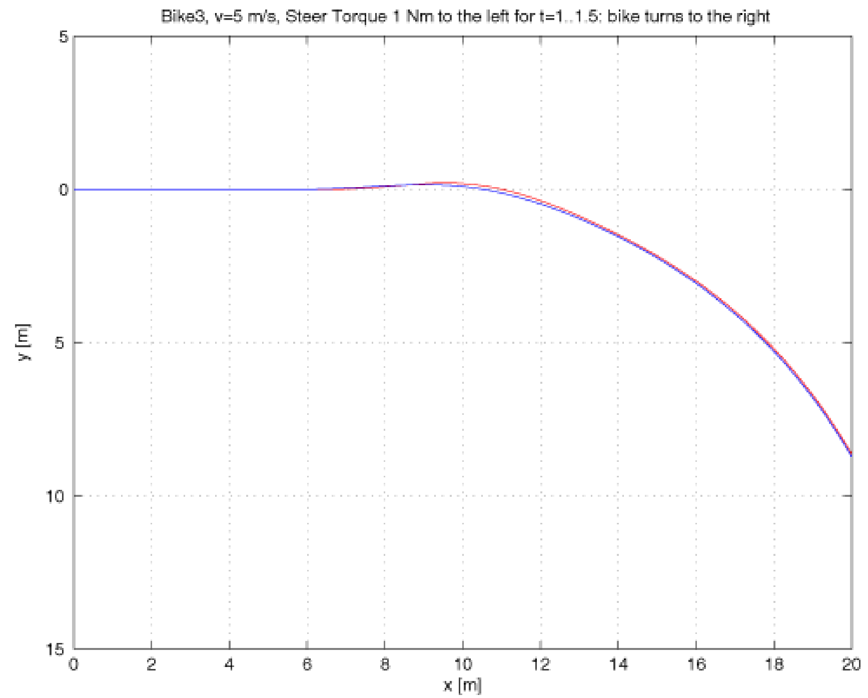
Slow motion:



Steering a Bike

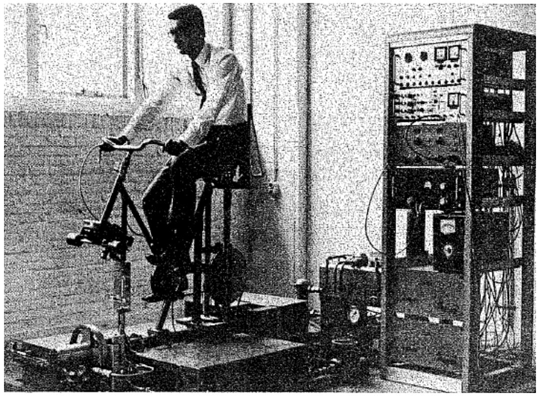
Standard bike with rider at a stable forward speed of 5 m/s, after 1 second we apply a steer torque of 1 Nm for ½ a second and then we let go of the handle bars.





How do we steer and balance?

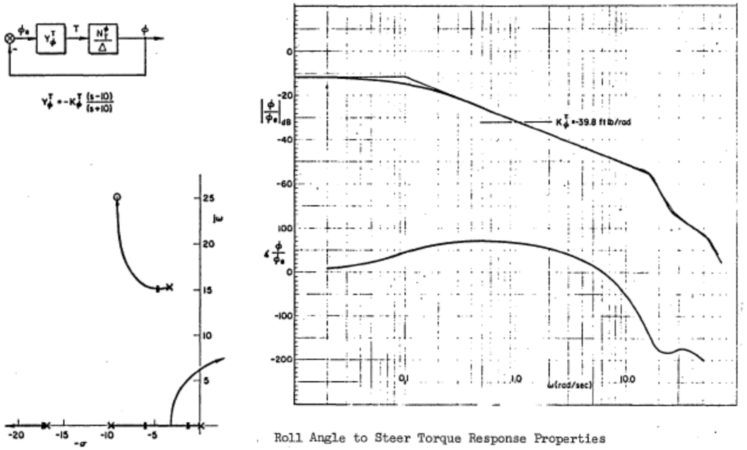
- A. van Lunteren and H. G. Stassen. On the variance of the bicycle rider's behavior. In *Proceedings of the 6th Annual Conference on Manual Control*, April 1970.



$$\hat{H}_1(s) = +1.07\left[1+0.15s\right]\, e^{-0.16s} \quad \text{(handle bar)}$$

$$\hat{H}_2(s) = -0.13\left[1+1.6s\right]\, e^{-0.09s} \quad \text{(upper body)}$$

- David Herbert Weir. *Motorcycle Handling Dynamics and Rider Control and the Effect of Design Configuration on Response and Performance.* PhD thesis, University of California, LA, 1972.



Roll Angle to Steer Torque Response Properties

A Ride into Town

Measure rider control on an instrumented bicycle

3 rate gyros: Lean, Yaw and Steer

1 steer angle potentiometer

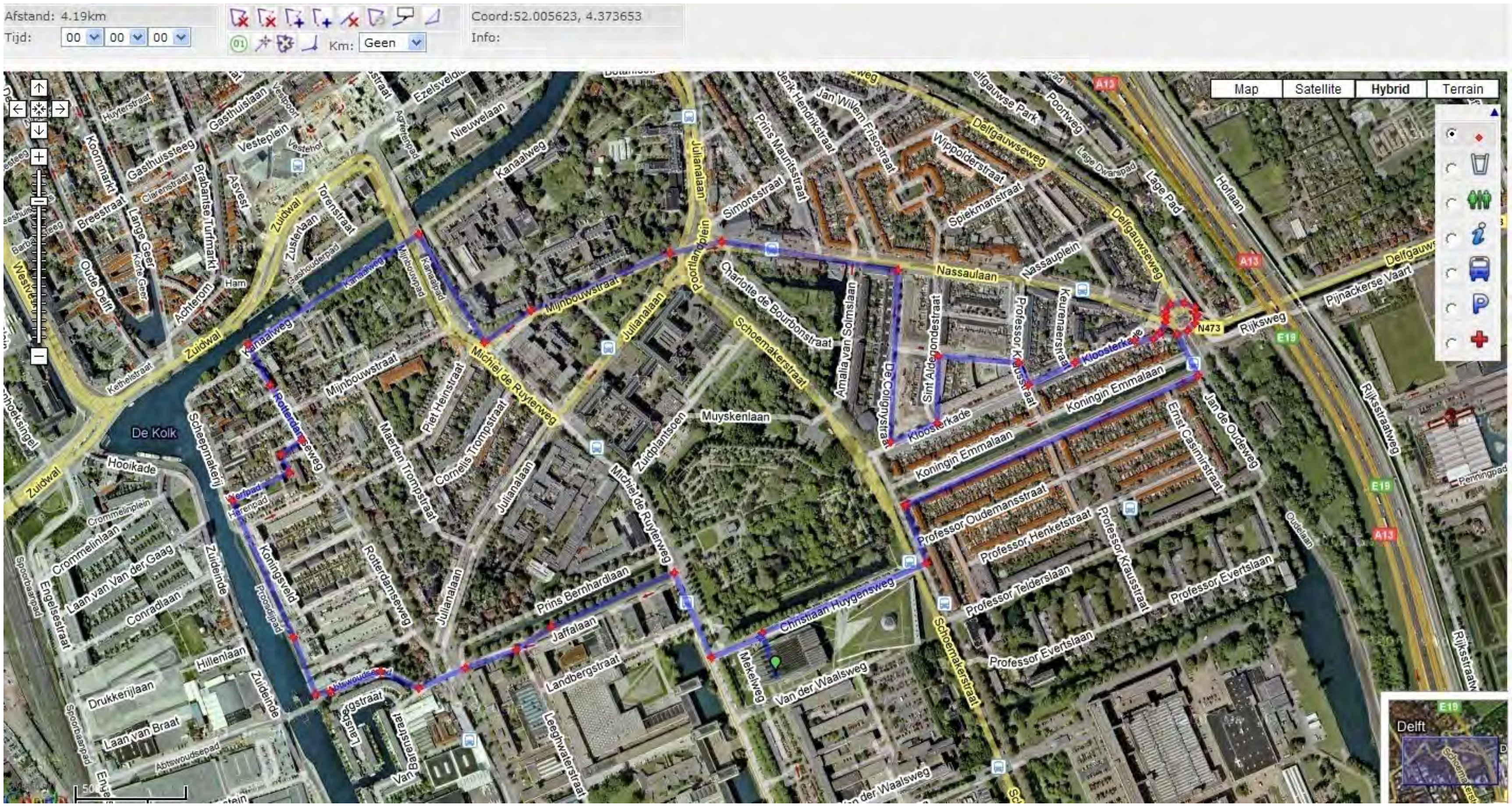
2 forward speed

1 pedal cadence pickup

1 video camera

Compac Rio data collection

A ride into Town



A Ride into Town

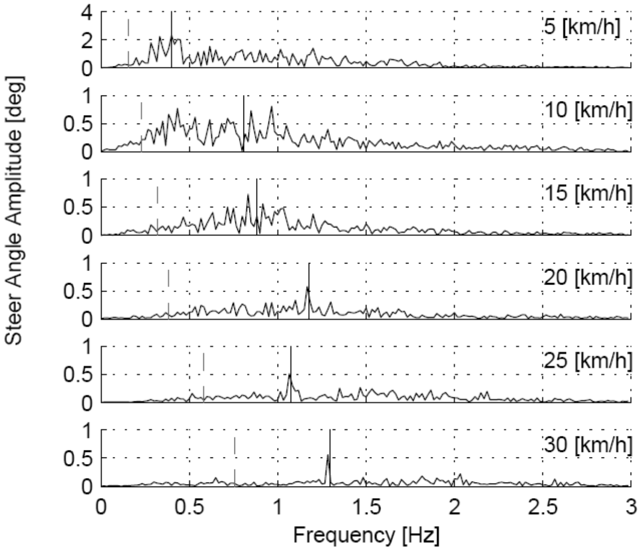
Treadmill experiments

Vrije Universiteit Amsterdam , 3 x 5 m treadmill, vmax=35 km/h

Treadmill experiments

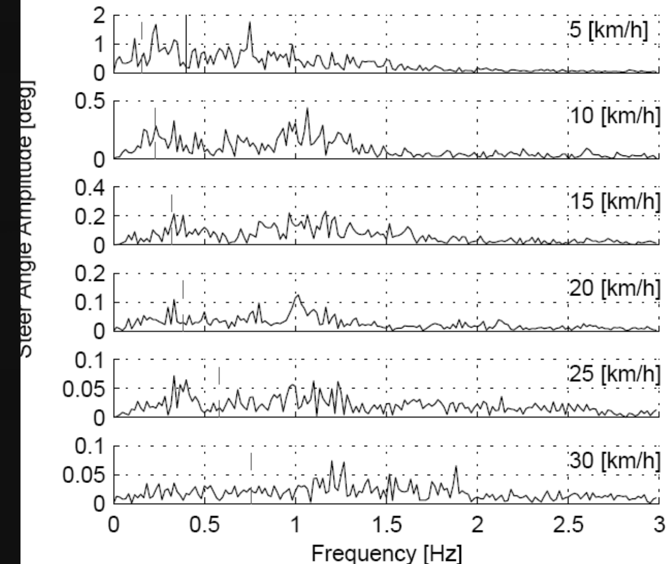
Rider Control Observations

Treadmill Experiments Camera Bicycle – Normal Cycling, Pedaling



Rider Control Observations

Treadmill Experiments Camera Bicycle – Towing



Rider Control

Full Human Motion Capture, Optotrack active marker system

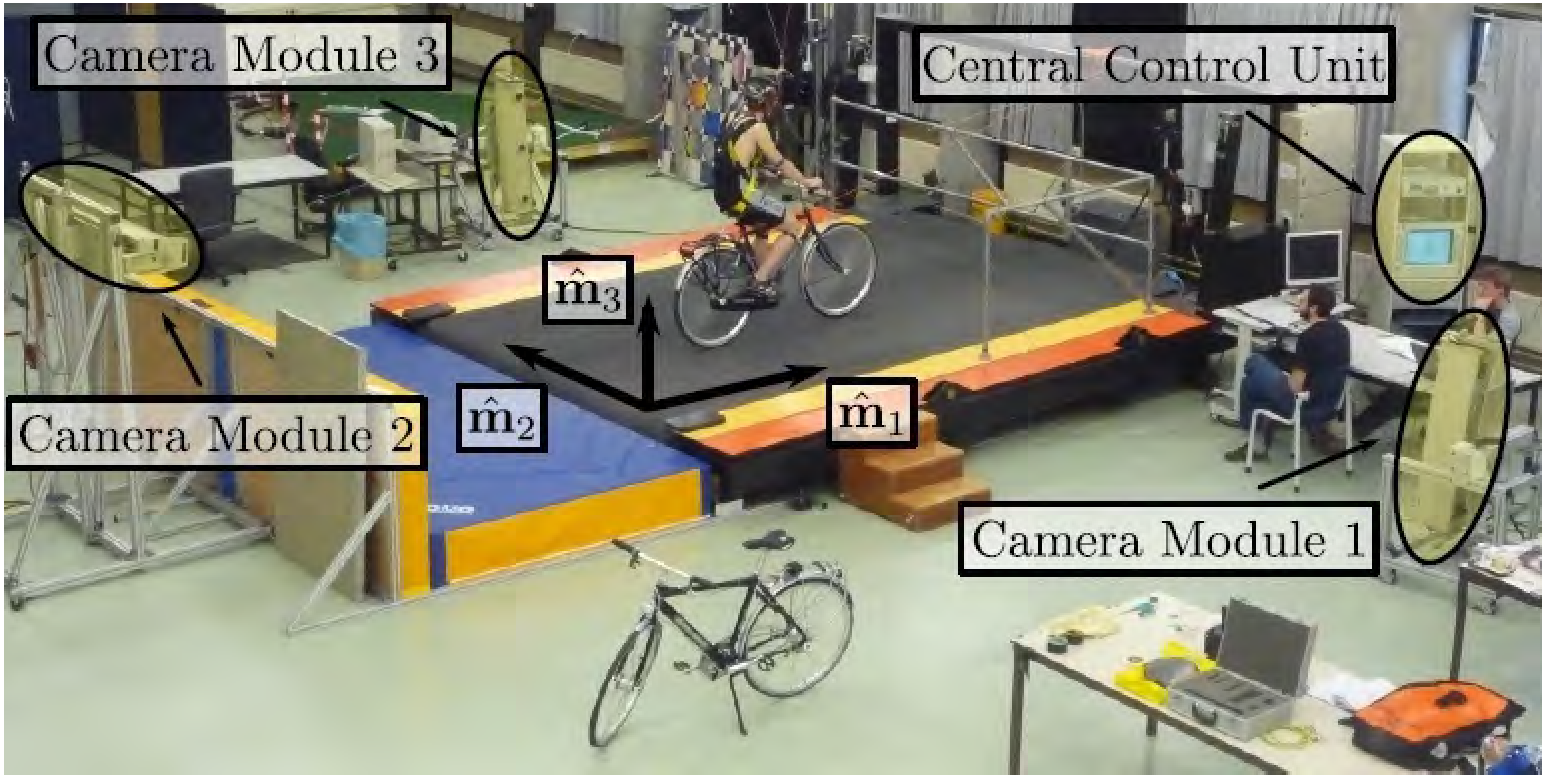
- 31 markers xyz-coor.
- Sample freq 100 Hz
- Sample time 1 min

One run is 600,000 numbers.

Data reduction by Principal Component Analysis (PCA).

Rider Control

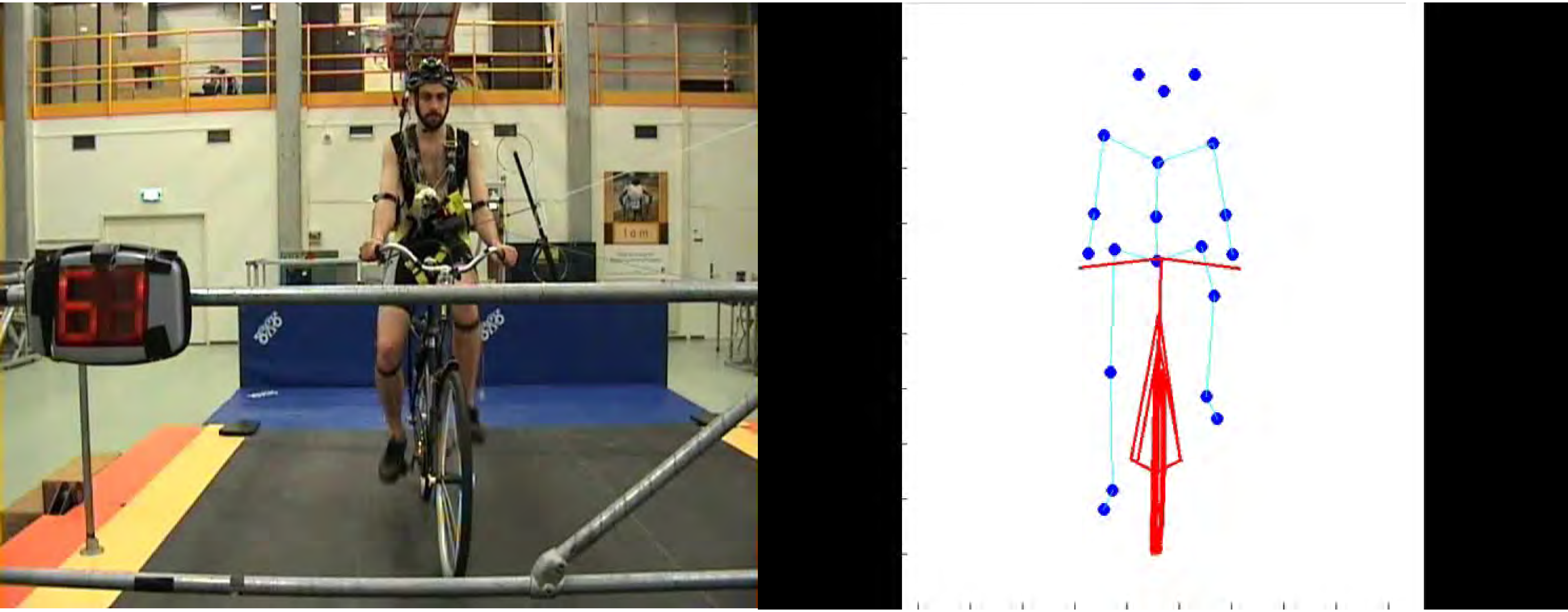
Full Human Motion Capture, Optotrack active marker system



3 x 5 m treadmill at the Vrije Universiteit Amsterdam

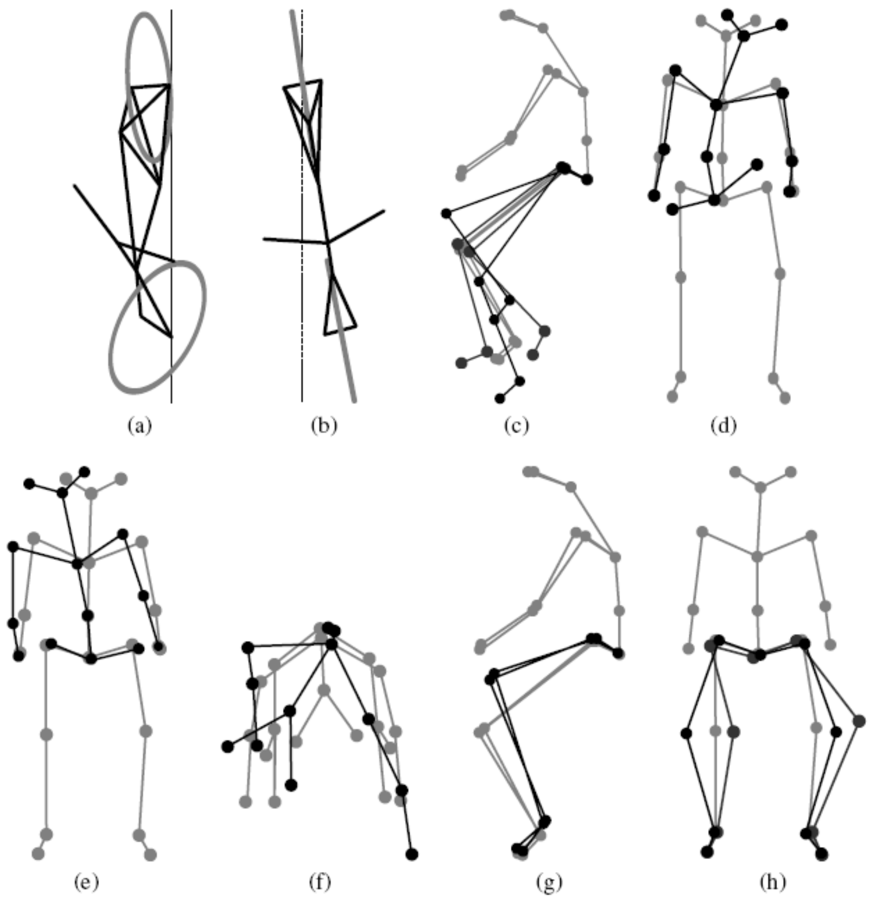
Rider Control

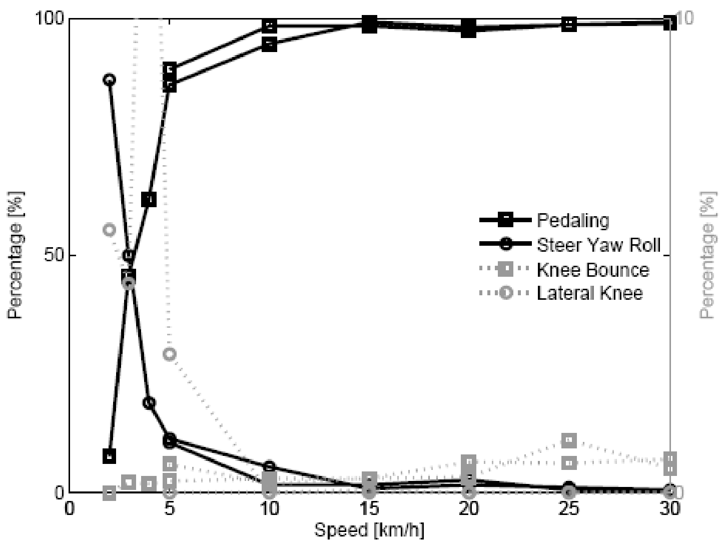
Treadmill Experiments Full Human Motion Capture - Normal Cycling, Pedaling



Rider Control

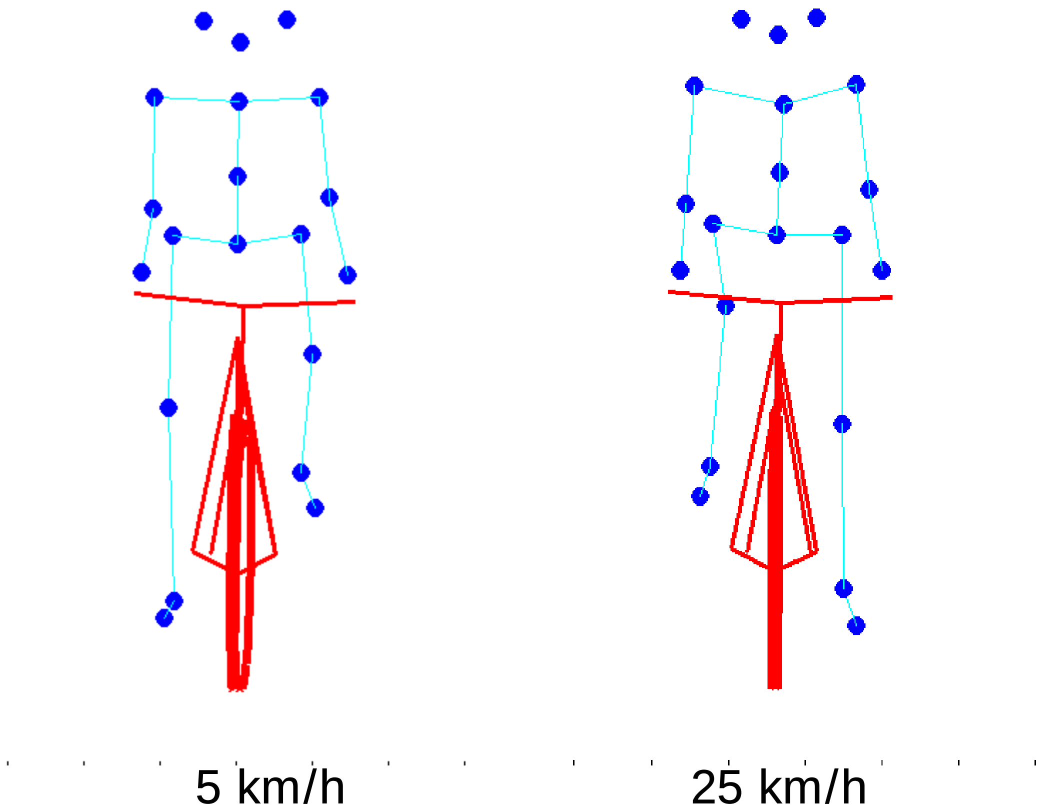
Treadmill Experiments, Full Human Motion Capture – PCA Motion Groups





Rider Control

Treadmill Experiments, Full Human Motion Capture - Compare



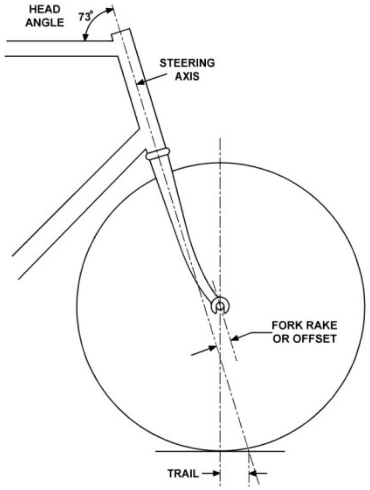
Rider Control: Conclusions

- During normal bicycling the dominant upper body motions: lean, bend, twist and bounce, are all linked to the pedaling motion.
- We hypothesize that lateral control is mainly done by steering since we observed only upper body motion in the pedaling frequency.
- If upper body motions are used for control then this control is in the pedaling frequency.
- When pedaling at low speed we observe lateral knee motions which are probably also used for control.

5.

Selfstability, revisited

A bicycle is selfstable because ....

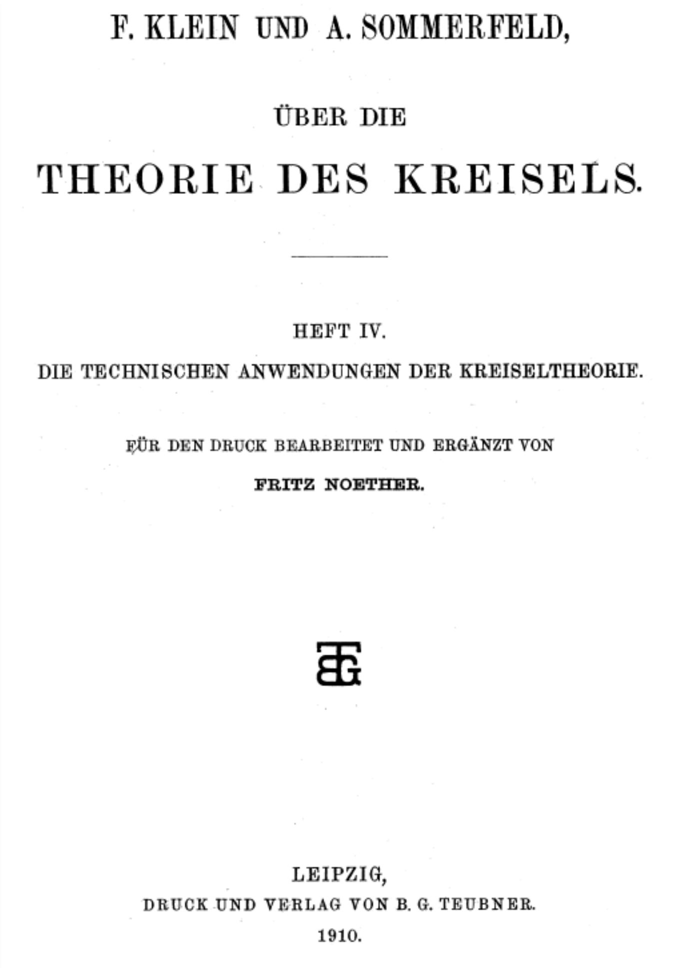


Gyroscopic effect of the front wheel?

Trail on the front wheel?

Gyroscopic Effect?

Klein & Sommerfeld 1910


F. KLEIN UND A. SOMMERFELD,

ÜBER DIE

THEORIE DES KREISELS.

HEFT IV.

DIE TECHNISCHEN ANWENDUNGEN DER KREISELTHEORIE.

FÜR DEN DRUCK BEARBEITET UND ERGÄNZT VON

FRITZ NOETHER.

LEIPZIG,

DRUCK UND VERLAG VON B. G. TEUBNER.

1910.

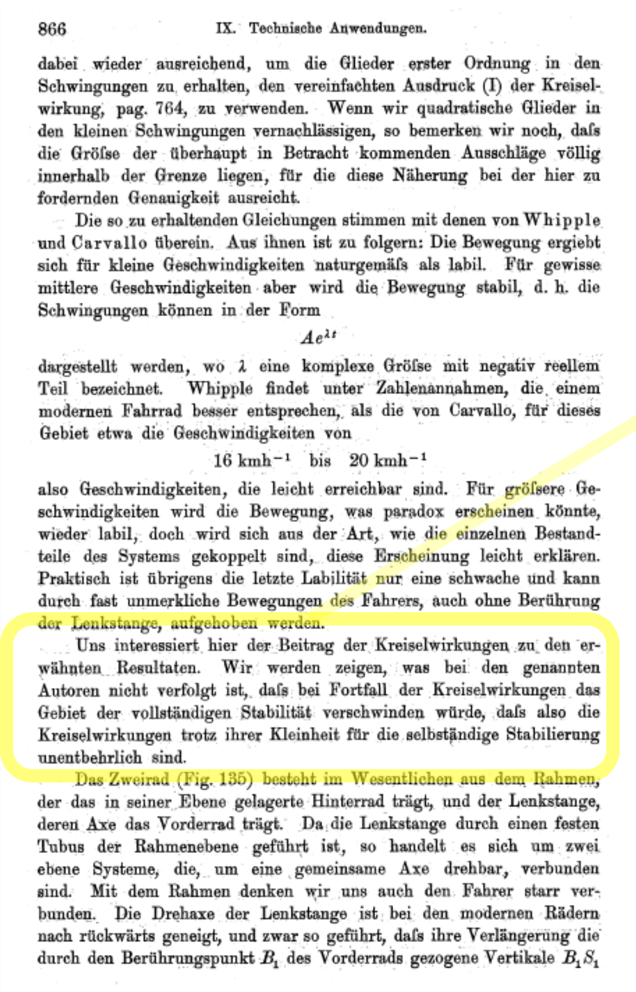
Felix Klein
(1849-1925)
from the Klein bottle

Arnold Sommerfeld
(1868-1951)
81 Nobel prize nominations

Fritz Noether
(1884-1941)
brother of Emmy

Gyroscopic Effect?

Klein & Sommerfeld 1910


866 IX. Technische Anwendungen.

dabei wieder ausreichend, um die Glieder erster Ordnung in den Schwingungen zu erhalten, den vereinfachten Ausdruck (I) der Kreiselwirkung, pag. 764, zu verwenden. Wenn wir quadratische Glieder in den kleinen Schwingungen vernachlässigen, so bemerken wir noch, dafs die Gröfse der überhaupt in Betracht kommenden Ausschläge völlig innerhalb der Grenze liegen, für die diese Näherung bei der hier zu fordernden Genauigkeit ausreicht.

Die so zu erhaltenden Gleichungen stimmen mit denen von Whipple und Carvallo überein. Aus ihnen ist zu folgern: Die Bewegung ergiebt sich für kleine Geschwindigkeiten naturgemäfs als labil. Für gewisse mittlere Geschwindigkeiten aber wird die Bewegung stabil, d. h. die Schwingungen können in der Form

$$Ae^{\lambda t}$$

dargestellt werden, wo λ eine komplexe Gröfse mit negativ reellem Teil bezeichnet. Whipple findet unter Zahlenannahmen, die einem modernen Fahrrad besser entsprechen, als die von Carvallo, für dieses Gebiet etwa die Geschwindigkeiten von

$$16\ \mathrm{kmh}^{-1} \quad \text{bis} \quad 20\ \mathrm{kmh}^{-1}$$

also Geschwindigkeiten, die leicht erreichbar sind. Für gröfsere Geschwindigkeiten wird die Bewegung, was paradox erscheinen könnte, wieder labil, doch wird sich aus der Art, wie die einzelnen Bestandteile des Systems gekoppelt sind, diese Erscheinung leicht erklären. Praktisch ist übrigens die letzte Labilität nur eine schwache und kann durch fast unmerkliche Bewegungen des Fahrers, auch ohne Berührung der Lenkstange, aufgehoben werden.

Uns interessiert hier der Beitrag der Kreiselwirkungen zu den erwähnten Resultaten. Wir werden zeigen, was bei den genannten Autoren nicht verfolgt ist, dafs bei Fortfall der Kreiselwirkungen das Gebiet der vollständigen Stabilität verschwinden würde, dafs also die Kreiselwirkungen trotz ihrer Kleinheit für die selbständige Stabilierung unentbehrlich sind.

Das Zweirad (Fig. 135) besteht im Wesentlichen aus dem Rahmen, der das in seiner Ebene gelagerte Hinterrad trägt, und der Lenkstange, deren Axe das Vorderrad trägt. Da die Lenkstange durch einen festen Tubus der Rahmenebene geführt ist, so handelt es sich um zwei ebene Systeme, die, um eine gemeinsame Axe drehbar, verbunden sind. Mit dem Rahmen denken wir uns auch den Fahrer starr verbunden. Die Drehaxe der Lenkstange ist bei den modernen Rädern nach rückwärts geneigt, und zwar so geführt, dafs ihre Verlängerung die durch den Berührungspunkt B_1 des Vorderrads gezogene Vertikale B_1S_1

Here, we are interested in the contribution of the gyroscopic effects to the results mentioned above [that bicycles can be self-stable]. We shall show, what has not been pursued by [Whipple and Carvallo], that by leaving out the gyroscopic effects the region of full stability would disappear; therefore that the gyroscopic effects, despite their smallness, are indispensable for the self-stability.

Gyroscopic Effect?

Klein & Sommerfeld 1910

880 IX. Technische Anwendungen.

Wir ergänzen diese Resultate durch den Nachweis, daſs die vollständige Stabilierung ohne Kreiselwirkungen nicht möglich wäre. Zu dem Zweck berechnen wir den Koeffizienten $(\delta_1 u + \delta_2 u^3)$ von λ aus der Determinante Δ. Wenn wir zur Abkürzung das gesamte Schweremoment $M_1 h_1 + M_2 h_2$ mit Mh bezeichnen, wird dieser:

$$
\begin{aligned}
& g(-M_1 h_1 c_1 + M_2 h_2 c_2) \sin\sigma \cdot N \\
& - g Mh [c_2 \cos\sigma A_v + c_1 (\cos B_v + \sin\sigma B_{hv})] \frac{u}{l} \\
& - g Mh (c_1^2 + c_2^2) \sin\sigma \frac{N}{l} \\
& - g Mh [c_2^2 M_1 s_1 + c_1^2 M_2 s_2] \frac{u}{l} \\
& + g(c_2 M_1 s_1 + c_1 M_2 s_2)\left[(c_1 + c_2)\frac{N}{l} + (M_1 h_1 c_2 + M_2 h_2 c_1)\frac{u}{l} + B_{hv}\frac{u}{l}\right] \\
& + (c_2 + c_1) \cos\sigma \cdot \frac{N}{l} [2Nu + Mhu^2 - g M_2 c_1 r]
\end{aligned}
$$

und reduziert sich noch zu:

$$
\begin{aligned}
& - g Mh \cos\sigma (c_2 A_v + c_1 B_v) \frac{u}{l} \\
& + g B_{hv}\left(- M_1 h_1 \sin\sigma + M_2 r \frac{c_1}{l} \cos\sigma\right) u \\
& - g M_1 h_1 M_2 h_2 l \sin\sigma \cdot u - g M_1 M_2 h_1 c_1 r \cos\sigma \cdot u \\
& + \frac{c_2 + c_1}{l} \cos\sigma N [2Nu + Mhu^2].
\end{aligned} \tag{13}
$$

In diesem Ausdruck enthält das letzte Glied, da N mit u proportional ist, den Faktor u^3, die anderen nur den Faktor u. Von diesen überwiegen die negativen Glieder weit über das positive, da das letztere die beiden kleinen Faktoren c_1 und r enthält; daher wird für kleine Fahrtgeschwindigkeit u der ganze Koeffizient negativ. Er bliebe immer negativ, und damit die aufrechte Bewegung labil, wenn die Kreiselwirkungen unberücksichtigt blieben, also $N = 0$ angenommen würde (d. h. da die Umlaufsgeschwindigkeit proportional zu u ist, wenn das Trägheitsmoment der Räder um ihre Rotationsaxe vernachlässigt würde). Durch das letzte, von den Kreiselwirkungen herrührende Glied, das den Faktor u^3 enthält, wird der Koeffizient bei genügend groſser Geschwindigkeit positiv. (Welches die Gröſsenordnung der hier als klein und genügend groſs unterschiedenen Geschwindigkeitsintervalle ist, können wir aus den oben angegebenen Whipple'schen Zahlen ersehen. Die Grenze zwischen beiden bildet der Wert $u_1 = 12$ km/h.)

Die von Whipple gefundene Stabilität des Fahrrads für die Geschwindigkeiten von 16—20 km/h ist daher nur durch die Kreiselwirkungen der rotierenden Räder ermöglicht.

The stability of the bicycle found by Whipple for the speeds from 16–20 km/h is therefore only made possible through the gyroscopic effects of the wheels.

Gyroscopic Effect?

Klein & Sommerfeld 1910

Wir ergänzen diese Resultate durch den Nachweis, daſs die vollständige Stabilierung ohne Kreiselwirkungen nicht möglich wäre. Zu dem Zweck berechnen wir den Koeffizienten $(\delta_1 u + \delta_2 u^3)$ von λ aus der Determinante Δ. Wenn wir zur Abkürzung das gesamte Schweremoment $M_1 h_1 + M_2 h_2$ mit Mh bezeichnen, wird dieser:

$$
\begin{aligned}
&g(-M_1 h_1 c_1 + M_2 h_2 c_2) \sin\sigma \cdot N \\
&- g\,Mh\,[c_2 \cos\sigma\, A_v + c_1(\cos B_v + \sin\sigma\, B_{hv})]\,\frac{u}{l} \\
&- g\,Mh\,(c_1{}^2 + c_2{}^2) \sin\sigma\,\frac{N}{l} \\
&- g\,Mh\,[c_2{}^2 M_1 s_1 + c_1{}^2 M_2 s_2]\,\frac{u}{l} \\
&+ g(c_2 M_1 s_1 + c_1 M_2 s_2)\Big[(c_1 + c_2)\frac{N}{l} + (M_1 h_1 c_2 + M_2 h_2 c_1)\frac{u}{l} + B_{hv}\frac{u}{l}\Big] \\
&+ (c_2 + c_1) \cos\sigma \cdot \frac{N}{l}\,[2Nu + Mhu^2 - g M_2 c_1 r]
\end{aligned}
$$

und reduziert sich noch zu:

Two sign errors:

$$
\begin{aligned}
&- g\,Mh \cos\sigma\,(c_2 A_v + c_1 B_v)\,\frac{u}{l} \\
&+ g B_{hv}\Big(- M_1 h_1 \sin\sigma + M_2 r\,\frac{c_1}{l} \cos\sigma\Big) u \\
&- g M_1 h_1 M_2 h_2 l \sin\sigma \cdot u - g M_1 M_2 h_1 c_1 r \cos\sigma \cdot u \\
&+ \frac{c_2 + c_1}{l} \cos\sigma\, N [2Nu + Mhu^2].
\end{aligned}
\tag{13}
$$

In diesem Ausdruck enthält das letzte Glied, da N mit u proportional ist, den Faktor u^3, die anderen nur den Faktor u. Von diesen überwiegen die negativen Glieder weit über das positive, da das letztere die beiden kleinen Faktoren c_1 und r enthält; daher wird für kleine Fahrtgeschwindigkeit u der ganze Koeffizient negativ. Er bliebe immer negativ, und damit die aufrechte Bewegung labil, wenn die Kreiselwirkungen unberücksichtigt blieben, also $N = 0$ angenommen würde (d. h. da die Umlaufsgeschwindigkeit proportional zu u ist, wenn das Trägheitsmoment der Räder um ihre Rotationsaxe vernachlässigt würde). Durch das letzte, von den Kreiselwirkungen herrührende Glied, das den Faktor u^3 enthält, wird der Koeffizient bei genügend groſser Geschwindigkeit positiv. (Welches die Gröſsenordnung der hier als klein und genügend groſs unterschiedenen Geschwindigkeitsintervalle ist, können wir aus den oben angegebenen Whipple'schen Zahlen ersehen. Die Grenze zwischen beiden bildet der Wert $u_1 = 12$ km/h.)

Die von Whipple gefundene Stabilität des Fahrrads für die Geschwindigkeiten von 16—20 km/h ist daher nur durch die Kreiselwirkungen der rotierenden Räder ermöglicht.

$$
\begin{aligned}
&-gMh\cos\sigma(c_2 A_v + c_1 B_v)\frac{u}{l} \\
&+gB_{hv}\big(\underbrace{+}_{\uparrow}\, M_1 h_1 \sin\sigma + M_2 r\frac{c_1}{l}\cos\sigma\big)u \\
&-gM_1 h_1 M_2 h_2 l \sin\sigma \cdot u\, \underbrace{+}_{\uparrow}\, gM_1 M_2 h_1 c_1 r \cos\sigma \cdot u \\
&+\frac{c_2 + c_1}{l}\cos\sigma N[2Nu + Mhu^2].
\end{aligned}
$$

Two sign errors corrected.

Trail?

Jones 1970

THE STABILITY OF THE BICYCLE

Tired of quantum electrodynamics, Brillouin zones, Regge poles? Try this old, unsolved problem in dynamics—how does a bike work?

David E. H. Jones

ALMOST EVERYONE can ride a bicycle, yet apparently no one knows how they do it. I believe that the apparent simplicity and ease of the trick conceals much unrecognized subtlety, and I have spent some time and effort trying to discover the reasons for the bicycle's stability. Published theory on the topic is sketchy and presented mainly without experimental verification. In my investigations I hoped to identify the stabilizing features of normal bicycles by constructing abnormal ones lacking selected features (see figure 1). The failure of early unridable bicycles led me to a careful consideration of steering geometry, from which—with the aid of computer calculations—I designed and constructed an inherently unstable bicycle.

The nature of the problem

Most mechanics textbooks or treatises on bicycles either ignore the matter of their stability, or treat it as fairly trivial. The bicycle is assumed to be balanced by the action of its rider who, if he feels the vehicle falling, steers into the direction of fall and so traverses a curved trajectory of such a radius as to generate enough centrifugal force to correct the fall. This theory is well formalised mathematically by S. Timoshenko and D. H. Young,[1] who derive the equation of motion of an idealized bicycle, neglecting rotational moments, and demonstrate that a falling bicycle can be saved by proper steering of the front wheel. The theory explains, for example, that the ridability of a bicycle depends crucially on the freedom of the front forks to swivel (if they are locked, even dead ahead, the bicycle can not be ridden), that the faster a bicycle moves the easier it is to ride (because a smaller steering adjustment is needed to create the centrifugal correction) and that it can not be balanced when stationary.

Nevertheless this theory can not be true, or at least it can not be the whole truth. You experience a powerful sense, when riding a bicycle fast, that it is inherently stable and could not fall over even if you wanted it to. Also a bicycle pushed and released riderless will stay up on its own, traveling in a long curve and finally collapsing after about 20 seconds, compared to the 2 sec it would take if static. Clearly the machine has a large measure of self-stability.

The next level of sophistication in current bicycle-stability theory invokes the gyroscopic action of the front wheel. If the bike tilts, the front wheel precesses about the steering axis and steers it in a curve that, as before, counteracts the tilt. The appeal of this theory is that its action is perfectly exemplified by a rolling hoop, which indeed can run stably for just this reason. A bicycle is thus assumed to be merely a hoop with a trailer.

The lightness of the front wheel distresses some theorists, who feel that the precession forces are inadequate to stabilize a heavily laden bicycle.[2,3] K. I. T. Richardson[4] allows both theories and suggests that the rider himself twists the front wheel to generate precession, hence staying upright. A theory of the hoop and bicycle on gyroscopic principles is given by R. H. Pearsall[5] who includes many rotational moments and derives a complex fourth-order differential equation of motion. This is not rigorously solved but demonstrates on general grounds the possibility of self-righting in a gyroscopically stable bicycle.

A non-gyroscopic bicycle

It was with vague knowledge of these simple bicycle theories that I began my series of experiments on bicycle stability. It occurred to me that it would be fun to make an unridable bicycle, which by canceling the forces of stability would baffle the most experienced rider. I therefore modified a standard bicycle by mounting on the front fork a second wheel, clear of the ground, arranged so that I could spin

"UNRIDABLE" BICYCLES. David Jones is seen here with three of his experimental machines, two of which turned out to be ridable after all. At top of this page is URB I, with its extra counter-rotating front wheel that tests the gyroscopic theories of bicycle stability. At left is URB III, whose reversed front forks give it great stability when pushed and released riderless. URB IV (immediately above) has its front wheel mounted ahead of the usual position and comes nearest to being "unridable." —FIG. 1

David E. H. Jones took bachelor's and doctor's degrees in chemistry at Imperial College, London, and has since alternated between the industrial and academic life. Currently he is a spectroscopist with ICI in England.

Trail?

Jones 1970

to provide perfect centrifugal stability, and that was why all bicycles have more or less the same steering geometry. As for the strange behavior of URB III, awkward to ride but incredibly stable if riderless, perhaps BICYC would provide a clue.

But further calculations shattered my hopes. Even with the bicycle dead upright, the forkpoint fell as the wheel turned out of plane (thus neatly disproving the contention of reference 7 that a bicycle tends to run true because its center of gravity rises with any turn out of plane), and the minimal height occurred at an absurdly large steering angle, 60 deg. Even worse, as the bike tilted, this minimum occurred at angles nearer and nearer the straight-ahead position (figure 4) until at 40 deg of tilt the most stable position was only 10 deg out of plane (these values are all for a typical observed steering geometry). Clearly the tilting wheel never reaches its minimal-energy position, and the minimum can not be significant for determining the stability of the bicycle.

I looked instead at the slope of the height versus steering-angle curve at zero steering angle, because this slope is proportional to the twisting torque on the front wheel of a tilted bike. Then, if H is the height of the forkpoint, the torque varies as $-dH/d\alpha$ at small values of α, the steering angle.

The curves in figure 4 show clearly that $dH/d\alpha$ varies linearly with lean angle L for small angles of lean. The more the bike leans, the bigger is the twisting torque, as required. The constant of proportionality for this relationship is $d^2H/d\alpha dL$, and the sign convention I adopted implies that a bicycle is stable if this parameter is negative. That is, for stability the forkpoint falls as the wheel turns into the lean when the bike is tilted.

COMPUTERIZED BICYCLES. These data, from BICYC output, show that the minimal height of the forkpoint occurs nearer to the straight-ahead position for greater angles of lean. Note also that $dH/d\alpha$ varies linearly with lean angle L for small L. Curves, computed for typical steering geometry (20-deg fork angle, 0.2 radii front projection), are vertically staggered for clarity. **—FIG. 4**

I therefore computed $d^2H/d\alpha dL$ for a wide range of steering geometries, and drew lines of constant stability on a diagram connecting the two parameters of steering geometry—the angle of the front-fork steering axis and the projection of the wheel center ahead of this axis. I then plotted on my stability diagram all the bicycles I could find—ranging from many existing models to old high-wheeled "penny-farthings" to see if they supported the theory.

The results (figure 5) were immensely gratifying. All the bicycles I plotted have geometries that fall into the stable region. The older bikes are rather scattered but the modern ones are all near the onset of instability defined by the $d^2H/d\alpha dL = 0$ line. This is immediately understandable. A very stable control system responds sluggishly to perturbation, whereas one nearer to instability is more responsive; modern bicycle design has emphasized nimbleness and maneuverability. Best of all, URB III comes out much more stable than any commercial bike. This result explains both its wonderful self-righting properties and also why it is difficult to ride—it is too stable to be steered. An inert rider with no balancing reflexes and no preferred direction of travel would be happy on URB III, but its characteristics are too intense for easy control.

This mathematical exercise also made it plain that the center-of-gravity lowering torque is developed exactly as shown in figure 6, and is identical with that postulated in reference 6. But it does not vanish when the bicycle's lean is in equilibrium with centrifugal force, as therein supposed (BICYC calculated the height of the forkpoint in the plane of the bicycle—the "effective vertical"—to allow for this). It can only vanish when the contact point of the front wheel is intersected by the steering axis, which BICYC shows clearly is the condition for minimal height. There is thus an intimate connection between the "trail" of a bicycle, as defined in figure 6, and $d^2H/d\alpha dL$, in fact the $d^2H/d\alpha dL$ line in figure 4 coincides with the locus of zero trail.

Two further courses of action remained. First, I could make URB IV with a steering geometry well inside the unstable region, and second, I had to decide what force opposes the twisting torque on a bike's front wheel and prevents it reaching

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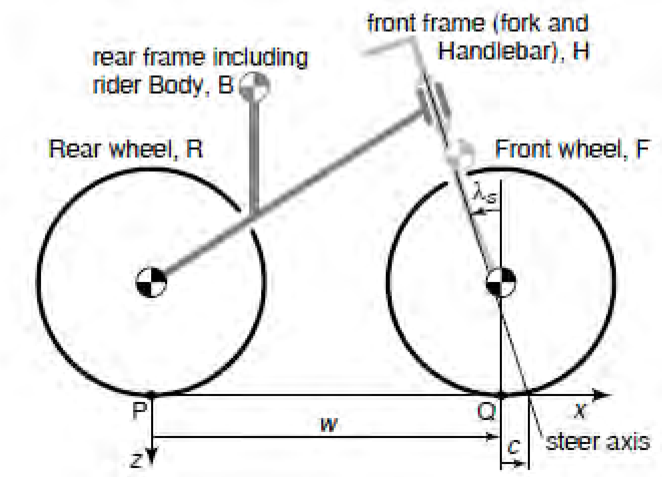
Jones calculated handlebar torque via the derivative of system gravitational (potential) energy H with respect to steer angle at fixed lean.

But careful analysis on the full dynamical system (linearized equations of motion) shows:

$$\text{Steer torque} = \underbrace{gK_{0\delta\phi}\phi_0}_{\text{Jones Static Couple}} + \underbrace{(-M_{\delta\phi}/M_{\phi\phi})gK_{0\phi\phi}\phi_0}_{\text{Dynamic Correction}}.$$

Gyroscopic Effect and/or Trail?

Whipple model



Linearized equations of motion:

$$\mathbf{M}\ddot{\mathbf{q}} + v\mathbf{C}_1\dot{\mathbf{q}} + [g\mathbf{K}_0 + v^2\mathbf{K}_2]\mathbf{q} = \mathbf{f},$$

$$\mathbf{M} = \begin{bmatrix} M_{\phi\phi} & M_{\phi\delta} \\ M_{\delta\phi} & M_{\delta\delta} \end{bmatrix}, \quad \mathbf{C}_1 = \begin{bmatrix} 0 & C_{1\phi\delta} \\ C_{1\delta\phi} & C_{1\delta\delta} \end{bmatrix},$$

$$\mathbf{K}_0 = \begin{bmatrix} K_{0\phi\phi} & K_{0\phi\delta} \\ K_{0\delta\phi} & K_{0\delta\delta} \end{bmatrix}, \quad \mathbf{K}_2 = \begin{bmatrix} 0 & K_{2\phi\delta} \\ 0 & K_{2\delta\delta} \end{bmatrix}.$$

Stability: $\mathbf{q} = \mathbf{q}_0 \exp(\lambda_i t), \qquad \det\left(\mathbf{M}\lambda^2 + v\mathbf{C}_1\lambda + g\mathbf{K}_0 + v^2\mathbf{K}_2\right) = 0,$

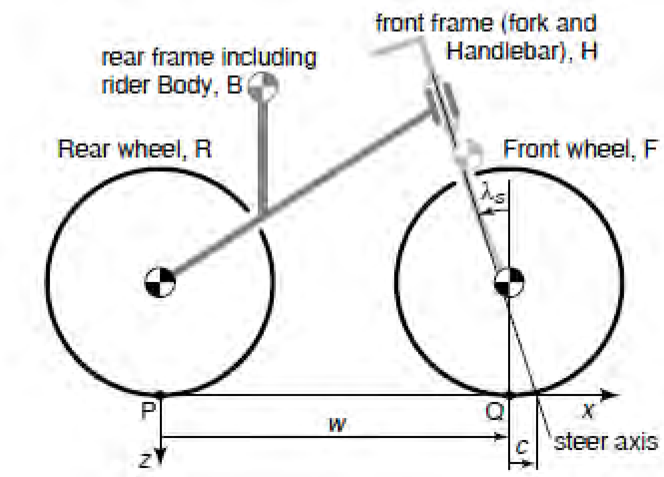
Characteristic eqn: $A\lambda^4 + B\lambda^3 + C\lambda^2 + D\lambda + E = 0.$

Routh stability criteria A,B,C,D,E,X>0:

$$\begin{aligned} A &= A_0 \\ B &= B_1 v \\ C &= C_0 + C_2 v^2 \\ D &= D_1 v + D_3 v^3 \\ E &= E_0 + E_2 v^2. \\ X &= BCD - ADD - EBB = X_2 v^2 + X_4 v^4 + X_6 v^6, \end{aligned}$$

Gyroscopic Effect and/or Trail?

Whipple model



parameter	symbol	value for benchmark
wheel base	w	1.02 m
trail	c	0.08 m
steer axis tilt ($\pi/2-$head angle)	λ	$\pi/10$ rad ($90°-72°$)
gravity	g	9.81 N kg^{-1}
forward speed	v	various m s^{-1} (table 2)
R*ear wheel R*		
radius	r_R	0.3 m
mass	m_R	2 kg
mass moments of inertia	(I_{Rxx}, I_{Ryy})	(0.0603, 0.12) kg m^2
rear **B***ody and frame assembly B*		
position centre of mass	(x_B, z_B)	(0.3, −0.9) m
mass	m_B	85 kg
mass moments of inertia	$\begin{bmatrix} I_{Bxx} & 0 & I_{Bxz} \\ 0 & I_{Byy} & 0 \\ I_{Bxz} & 0 & I_{Bzz} \end{bmatrix}$	$\begin{bmatrix} 9.2 & 0 & 2.4 \\ 0 & 11 & 0 \\ 2.4 & 0 & 2.8 \end{bmatrix}$ kg m^2
front **H***andlebar and fork assembly H*		
position centre of mass	(x_H, z_H)	(0.9, −0.7) m
mass	m_H	4 kg
mass moments of inertia	$\begin{bmatrix} I_{Hxx} & 0 & I_{Hxz} \\ 0 & I_{Hyy} & 0 \\ I_{Hxz} & 0 & I_{Hzz} \end{bmatrix}$	$\begin{bmatrix} 0.05892 & 0 & -0.00756 \\ 0 & 0.06 & 0 \\ -0.00756 & 0 & 0.00708 \end{bmatrix}$ kg m^2
F*ront wheel F*		
radius	r_F	0.35 m
mass	m_F	3 kg
mass moments of inertia	(I_{Fxx}, I_{Fyy})	(0.1405, 0.28) kg m^2

25 bicycle parameters!

$$A\lambda^4 + B\lambda^3 + C\lambda^2 + D\lambda + E = 0.$$

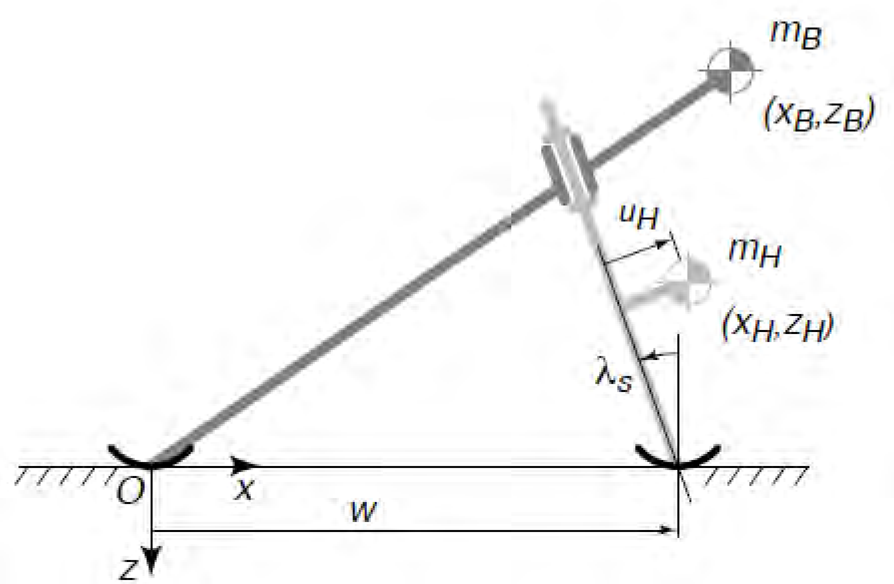
Routh stability criteria A,B,C,D,E,X>0:

$$\begin{aligned} A &= A_0 \\ B &= B_1 v \\ C &= C_0 + C_2 v^2 \\ D &= D_1 v + D_3 v^3 \\ E &= E_0 + E_2 v^2. \end{aligned}$$

$$X = BCD - ADD - EBB = X_2 v^2 + X_4 v^4 + X_6 v^6,$$

Two-mass-skate (TMS) bicycle

Only 8 bicycle parameters



Two point masses and wheels are replaced by skates

Parameter	Symbol	Value
Wheel base	w	1 m
Steer axis tilt	λ_s	5°
Rear frame assembly B mass	m_B	10 kg
Rear frame assembly B center of mass	(x_B, z_B)	(1.2, –0.4) m
Front fork and handlebar assembly H mass	m_H	1 kg
Front fork and handlebar assembly H center of mass	(x_H, z_H)	(1.02, –0.2) m

Only 8 non-zero bicycle parameters

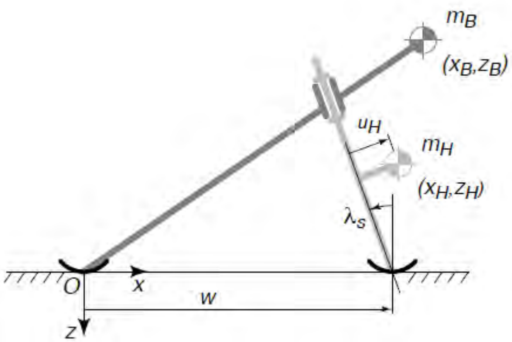
Two-mass-skate (TMS) bicycle

Routh criteria

A,B,C,D,E,X>0:

$$
\begin{aligned}
A &= A_0 \\
B &= B_1 v \\
C &= C_0 + C_2 v^2 \\
D &= D_1 v + D_3 v^3 \\
E &= E_0 + E_2 v^2 . \\
X &= BCD - ADD - EBB = X_2 v^2 + X_4 v^4 + X_6 v^6 ,
\end{aligned}
$$

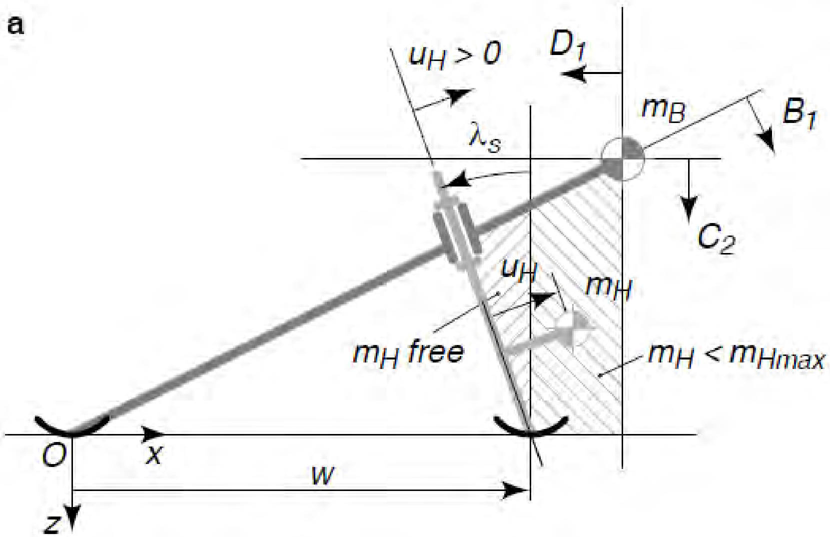
$$
\begin{aligned}
A_0 &= m_\mathrm{B}\, m_\mathrm{H}\, {u_\mathrm{H}}^2\, {z_\mathrm{B}}^2 \\
B_1 &= -m_\mathrm{B}\, m_\mathrm{H}\, u_\mathrm{H}\, z_\mathrm{B}\, (x_\mathrm{B}\, z_\mathrm{H} - x_\mathrm{H}\, z_\mathrm{B})\, / \bar{w} \\
C_0 &= -g\, m_\mathrm{H}\, u_\mathrm{H}\, (m_\mathrm{B}\, \sin\lambda_\mathrm{s}\, {z_\mathrm{B}}^2 - m_\mathrm{B}\, u_\mathrm{H}\, z_\mathrm{B} + m_\mathrm{H}\, \sin\lambda_\mathrm{s}\, {z_\mathrm{H}}^2 + m_\mathrm{H}\, u_\mathrm{H}\, z_\mathrm{H}) \\
C_2 &= m_\mathrm{B}\, m_\mathrm{H}\, u_\mathrm{H}\, z_\mathrm{B}\, (z_\mathrm{B} - z_\mathrm{H})\, / \bar{w} \\
D_1 &= -g\, m_\mathrm{B}\, m_\mathrm{H}\, u_\mathrm{H}\, z_\mathrm{B}\, (x_\mathrm{B} - x_\mathrm{H})\, / \bar{w} \\
D_3 &= 0 \\
E_0 &= -g^2\, m_\mathrm{H}\, u_\mathrm{H}\, (m_\mathrm{H}\, (x_\mathrm{H} - w)\, \cos\lambda_\mathrm{s} + m_\mathrm{B}\, z_\mathrm{B}\, \sin\lambda_\mathrm{s}) \\
E_2 &= 0 \\
X_2 &= -g^2\, ({m_\mathrm{B}}^2\, {m_\mathrm{H}}^3\, {u_\mathrm{H}}^3\, {z_\mathrm{B}}^2\, (z_\mathrm{B} - z_\mathrm{H})\, (m_\mathrm{B}\, \sin\lambda_\mathrm{s}\, {x_\mathrm{B}}^2\, z_\mathrm{B}\, z_\mathrm{H} + m_\mathrm{B}\, u_\mathrm{H}\, {x_\mathrm{B}}^2\, z_\mathrm{B} \\
&\quad - m_\mathrm{B}\, \sin\lambda_\mathrm{s}\, x_\mathrm{B}\, x_\mathrm{H}\, {z_\mathrm{B}}^2 - m_\mathrm{B}\, u_\mathrm{H}\, x_\mathrm{B}\, x_\mathrm{H}\, z_\mathrm{B} + m_\mathrm{H}\, \sin\lambda_\mathrm{s}\, x_\mathrm{B}\, x_\mathrm{H}\, {z_\mathrm{H}}^2 + m_\mathrm{H}\, u_\mathrm{H}\, x_\mathrm{B}\, x_\mathrm{H}\, z_\mathrm{H} \\
&\quad - m_\mathrm{H}\, \sin\lambda_\mathrm{s}\, {x_\mathrm{H}}^2\, z_\mathrm{B}\, z_\mathrm{H} - m_\mathrm{H}\, u_\mathrm{H}\, {x_\mathrm{H}}^2\, z_\mathrm{B}))/\bar{w}^2 \\
X_4 &= g\, {m_\mathrm{B}}^3\, {m_\mathrm{H}}^3\, {u_\mathrm{H}}^3\, {z_\mathrm{B}}^3\, (x_\mathrm{B}\, z_\mathrm{H} - x_\mathrm{H}\, z_\mathrm{B})\, (x_\mathrm{B} - x_\mathrm{H})\, (z_\mathrm{B} - z_\mathrm{H})\, / \bar{w}^3 \\
X_6 &= 0 \quad .
\end{aligned}
$$

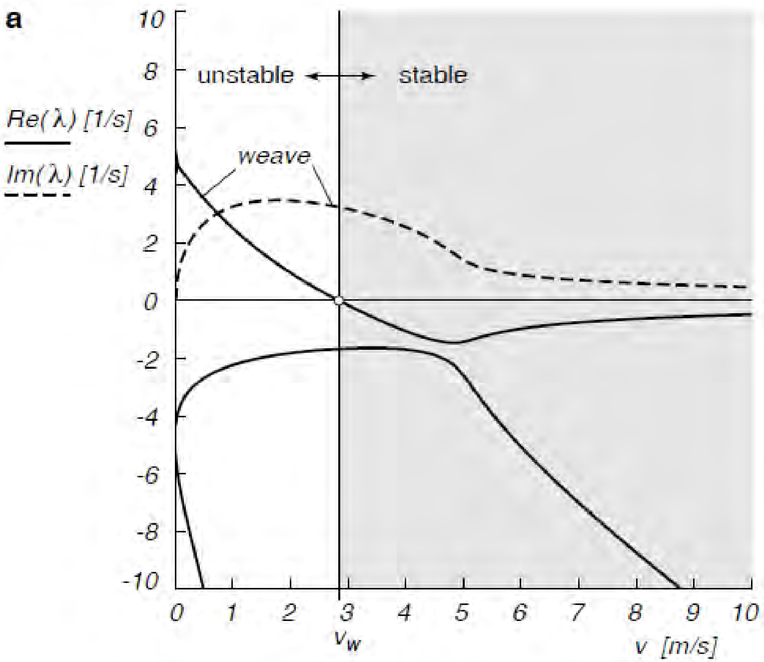


More manageable expressions ....

Two-mass-skate (TMS) bicycle

Stable when....





Eigenvalue plot

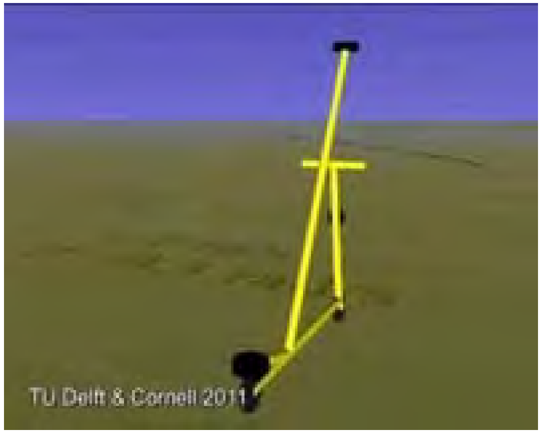
Stable when front mass m_H is within shaded area,

where $$m_{\mathrm{Hmax}} = m_{\mathrm{B}} \frac{-z_{\mathrm{B}}}{(x_{\mathrm{H}} - w)} \tan \lambda_{\mathrm{s}}$$

Stable without Gyroscopic or Trail effect!

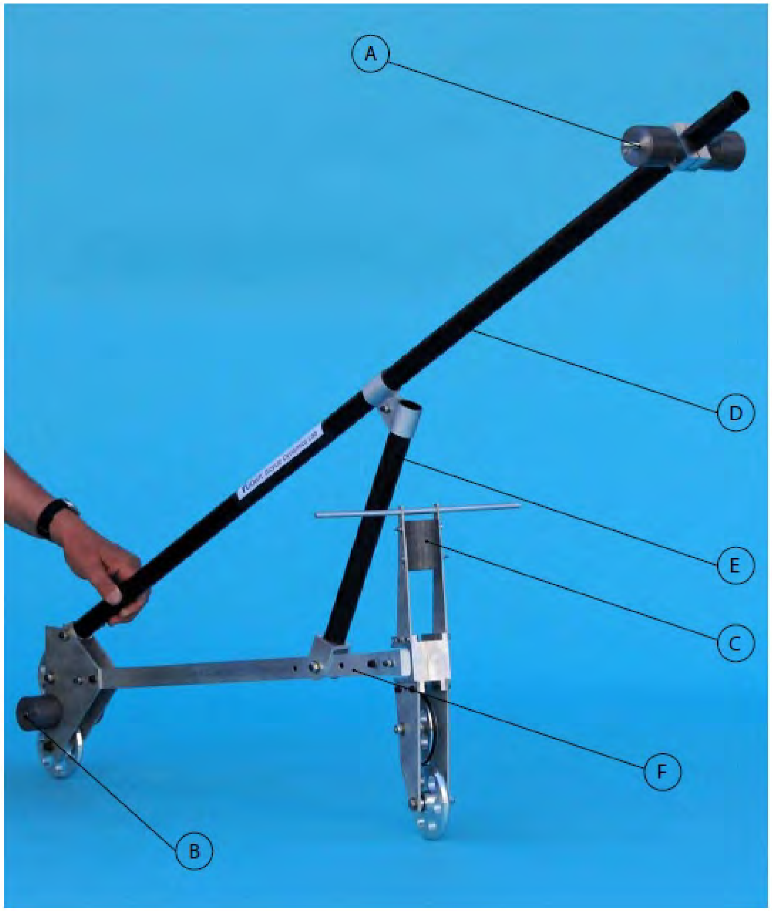
Two-mass-skate (TMS) bicycle

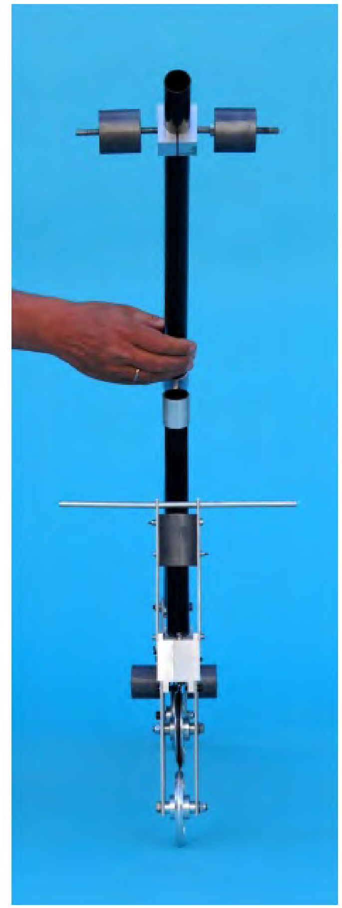
Full non-linear simulation



Two-mass-skate (TMS) bicycle

Experimental Bicycle

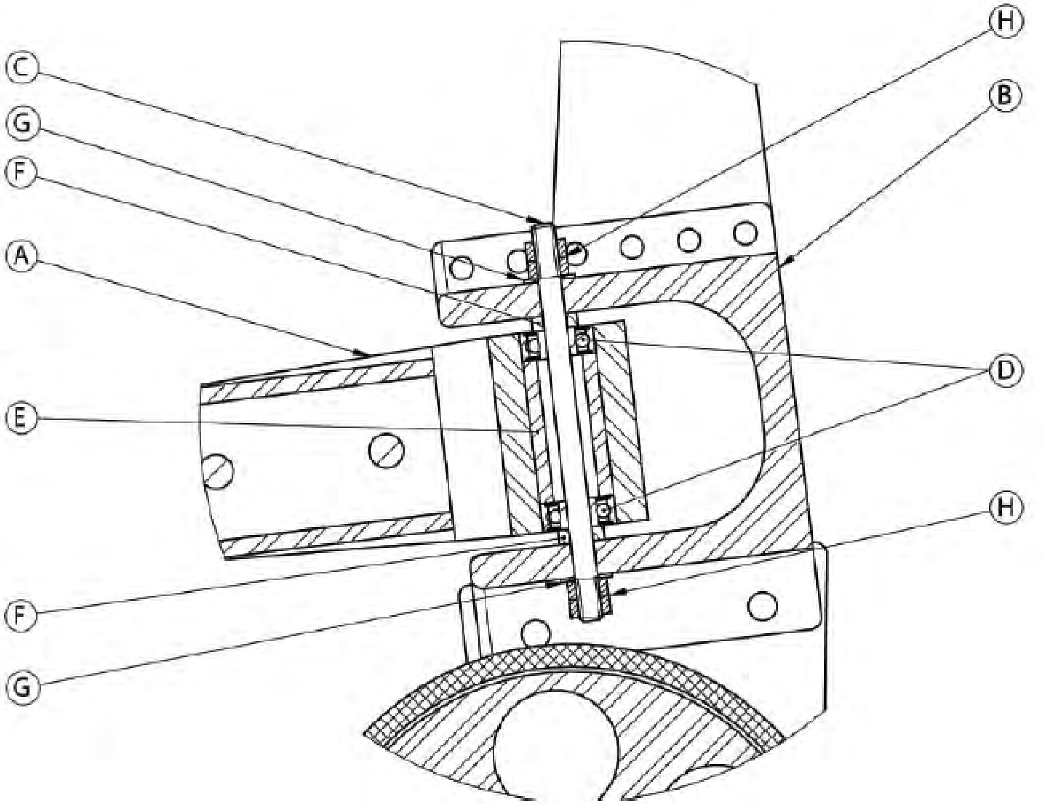




Wheelbase 0.75 m, point masses 2 kg, 99.5 % gyro free, trail -4 mm

Two-mass-skate (TMS) bicycle

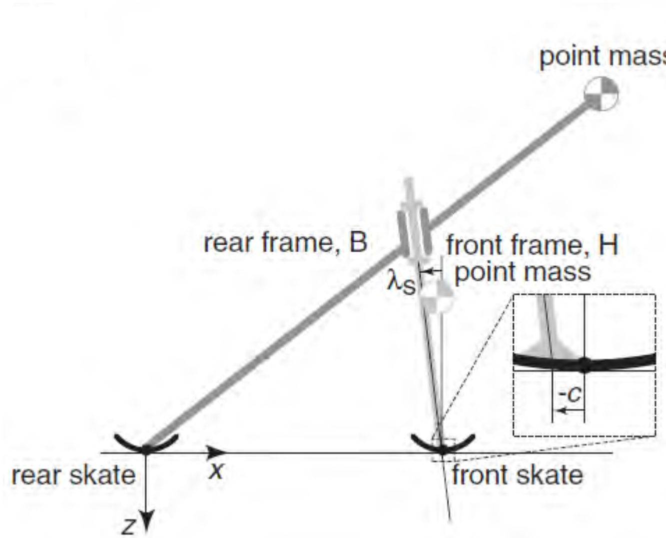
Experimental Bicycle



Front assembly, steering head design detail

Two-mass-skate (TMS) bicycle

Physical Model

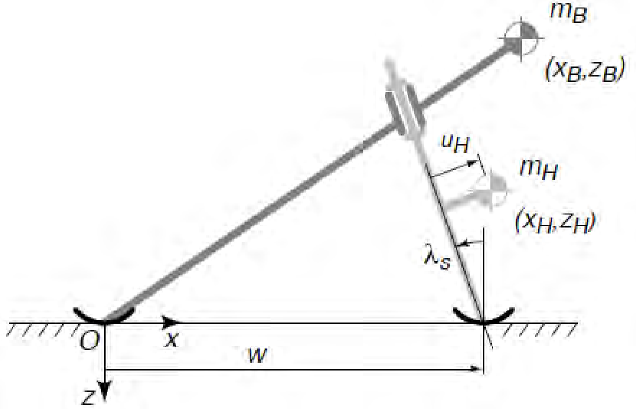


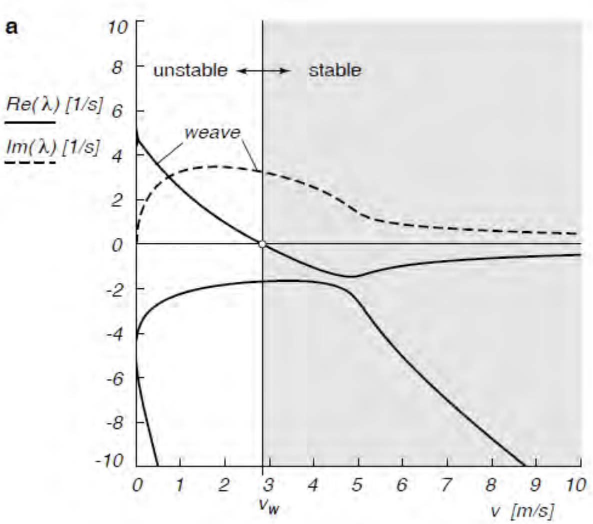
Parameter	Symbol	Value
Wheel base	w	0.750 m
Trail	c	-0.004 m
Steer axis tilt (90° − head angle)	λ_s	7° (90° − 83°)
Gravity constant	g	9.81 N/kg
Forward speed	v	*various* m/s
Rear wheel R		
Radius	r_R	0.050 m
Mass	m_R	0 kg
Effective spin inertia	I_{Ryy}	$1.8 \cdot 10^{-5}$ kgm^2
Rear Body and frame assembly B		
Position center of mass	(x_B, z_B)	(0.5044, −0.4279) m
Mass	m_B	6.425 kg
Mass moments of inertia	$\begin{bmatrix} I_{Bxx} & 0 & I_{Bxz} \\ 0 & I_{Byy} & 0 \\ I_{Bxz} & 0 & I_{Bzz} \end{bmatrix}$	$\begin{bmatrix} 0.875295 & 0 & 1.18665 \\ 0 & 2.59262 & 0 \\ 1.18665 & 0 & 1.73573 \end{bmatrix}$ kgm^2
Front Handlebar and fork assembly H		
Position center of mass	(x_H, z_H)	(0.7338, −0.3022) m
Mass	m_H	2.412 kg
Mass moments of inertia	$\begin{bmatrix} I_{Hxx} & 0 & I_{Hxz} \\ 0 & I_{Hyy} & 0 \\ I_{Hxz} & 0 & I_{Hzz} \end{bmatrix}$	$\begin{bmatrix} 0.038384 & 0 & -0.00055657 \\ 0 & 0.038071 & 0 \\ -0.00055657 & 0 & 0.00143206 \end{bmatrix}$ kgm^2
Front wheel F		
Radius	r_F	0.050 m
Mass	m_F	0 kg
Effective spin inertia	I_{Fyy}	$1.8 \cdot 10^{-5}$ kgm^2

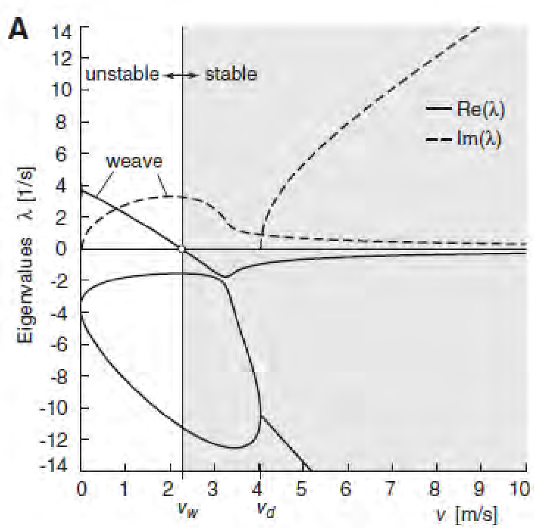
Wheelbase 0.75 m, point masses 2 kg, 99.5 % gyro free, trail -4 mm

Two-mass-skate (TMS) bicycle

From theory to practice





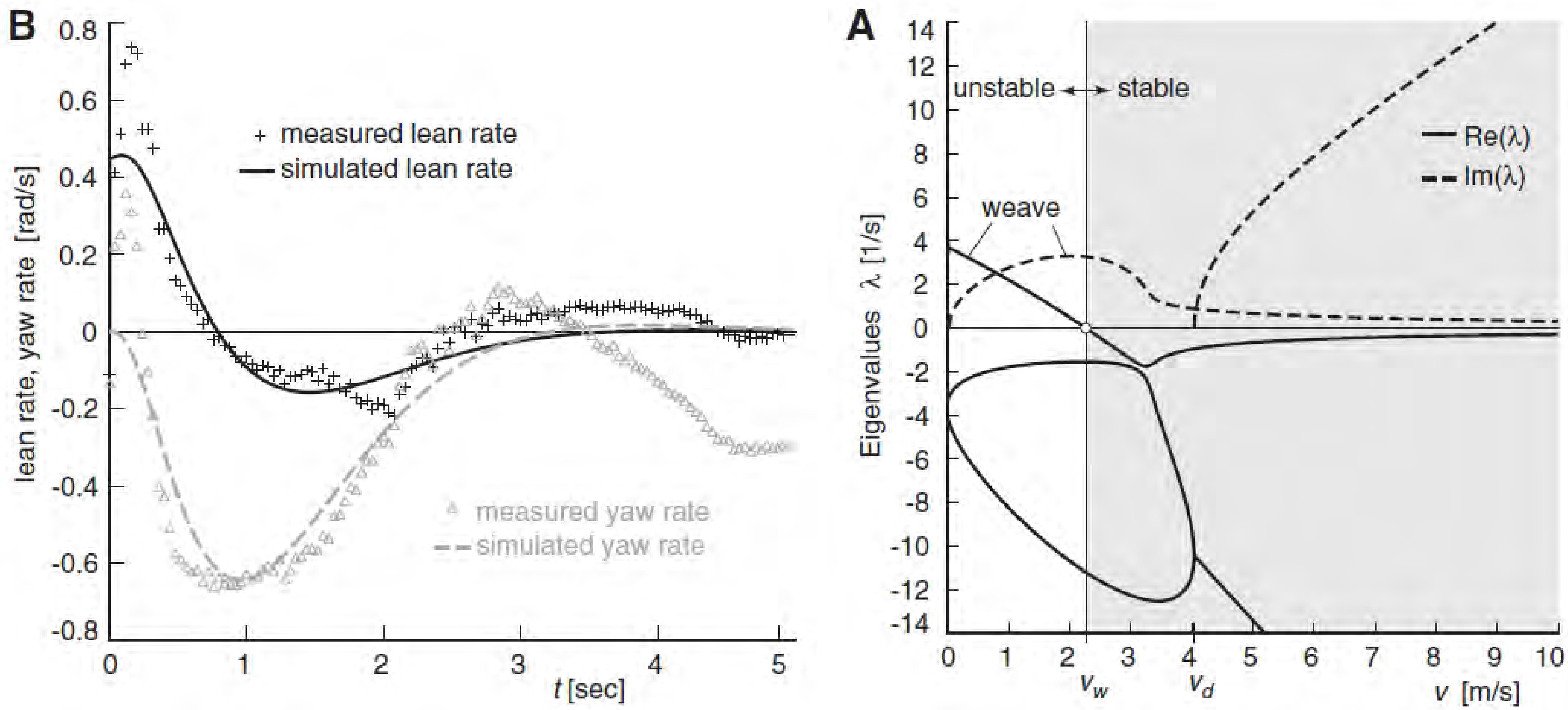


Two-mass-skate (TMS) bicycle

Basic Experiment

Two-mass-skate (TMS) bicycle

Compare measured motions with simulation



Transient motion after a disturbance for the experimental TMS bicycle. Measured and predicted lean and yaw (heading) rates of the rear frame are shown. The predicted motions show the theoretical (oscillatory) exponential decay.
Note at t=0: v=3.6 m/s-> t=3 sec: 2.4 m/s.

End of Story?

A necessary condition for self-stability

Linearized equations of motion: $\mathbf{M}\ddot{\mathbf{q}} + v\mathbf{C}_1\dot{\mathbf{q}} + [g\mathbf{K}_0 + v^2\mathbf{K}_2]\mathbf{q} = \mathbf{f},$

Stability: $\mathbf{q} = \mathbf{q}_0 \exp(\lambda_i t), \qquad \det\left(\mathbf{M}\lambda^2 + v\mathbf{C}_1\lambda + g\mathbf{K}_0 + v^2\mathbf{K}_2\right) = 0,$

Characteristic eqn: $A\lambda^4 + B\lambda^3 + C\lambda^2 + D\lambda + E = 0.$

Routh stability criteria A,B,C,D,E,X>0:

$$\begin{aligned}
A &= A_0 \\
B &= B_1 v \\
C &= C_0 + C_2 v^2 \\
D &= D_1 v + D_3 v^3 \\
&\boxed{E = E_0 + E_2 v^2 = \det\left(g\mathbf{K}_0 + v^2\mathbf{K}_2\right),} \\
X &= BCD - ADD - EBB = X_2 v^2 + X_4 v^4 + X_6 v^6,
\end{aligned}$$

Steer into the Fall

Steady turn steer torque: $[g\mathbf{K}_0 + v^2\mathbf{K}_2]\begin{bmatrix}\phi \\ \delta\end{bmatrix} = \begin{bmatrix}0 \\ T_\delta\end{bmatrix}. \implies T_\delta = \frac{\det(g\mathbf{K}_0 + v^2\mathbf{K}_2)}{g m_T z_T}\delta.$

Self-stability: $E > 0 \implies \mathrm{sgn}(T_\delta) = -\mathrm{sgn}(\delta).$

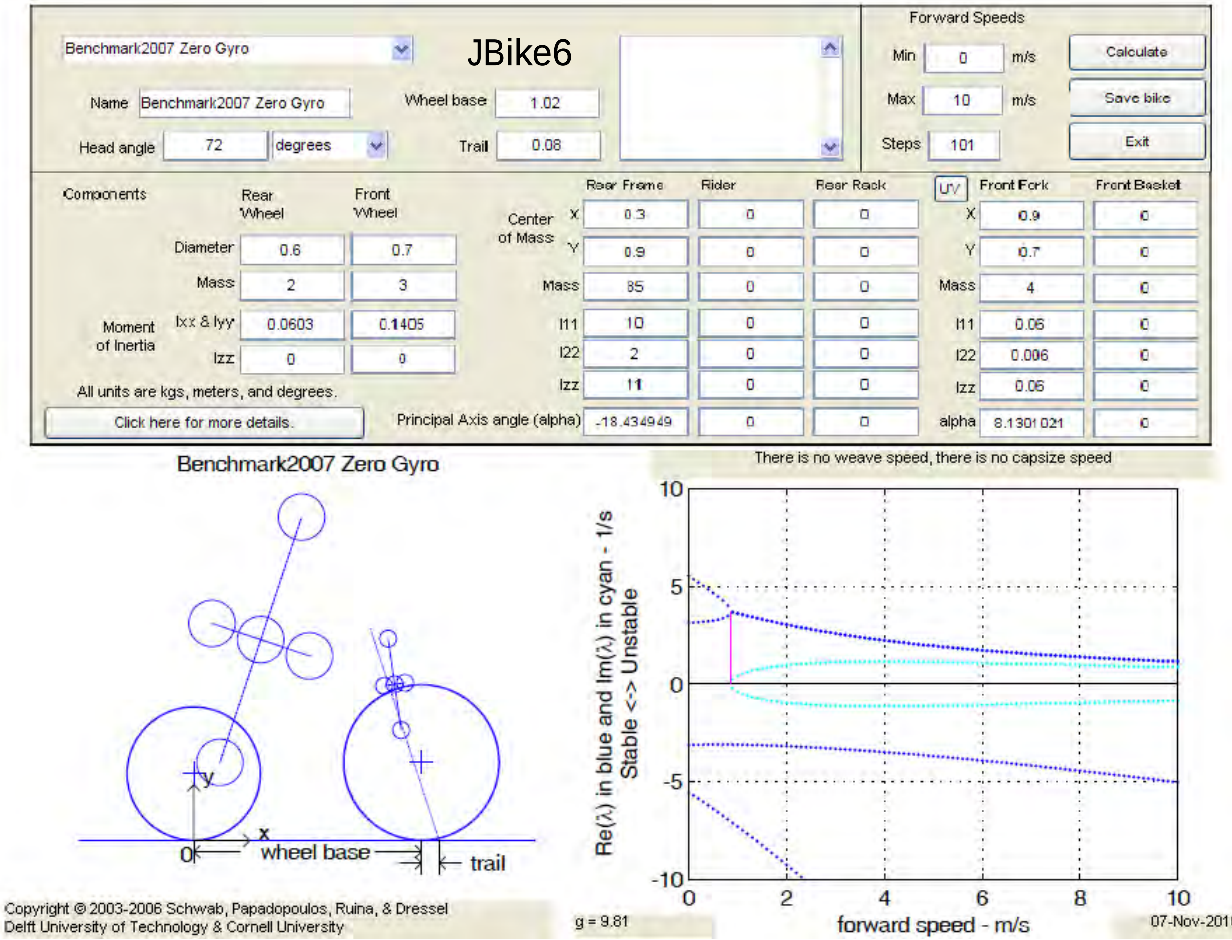
A necessary condition for a bicycle to have self-stability is that the steady turn torque applied by the rider is of the opposite sign of the handlebar angle.

End of Story?

Locked up steering, unstable

Counterexample 1

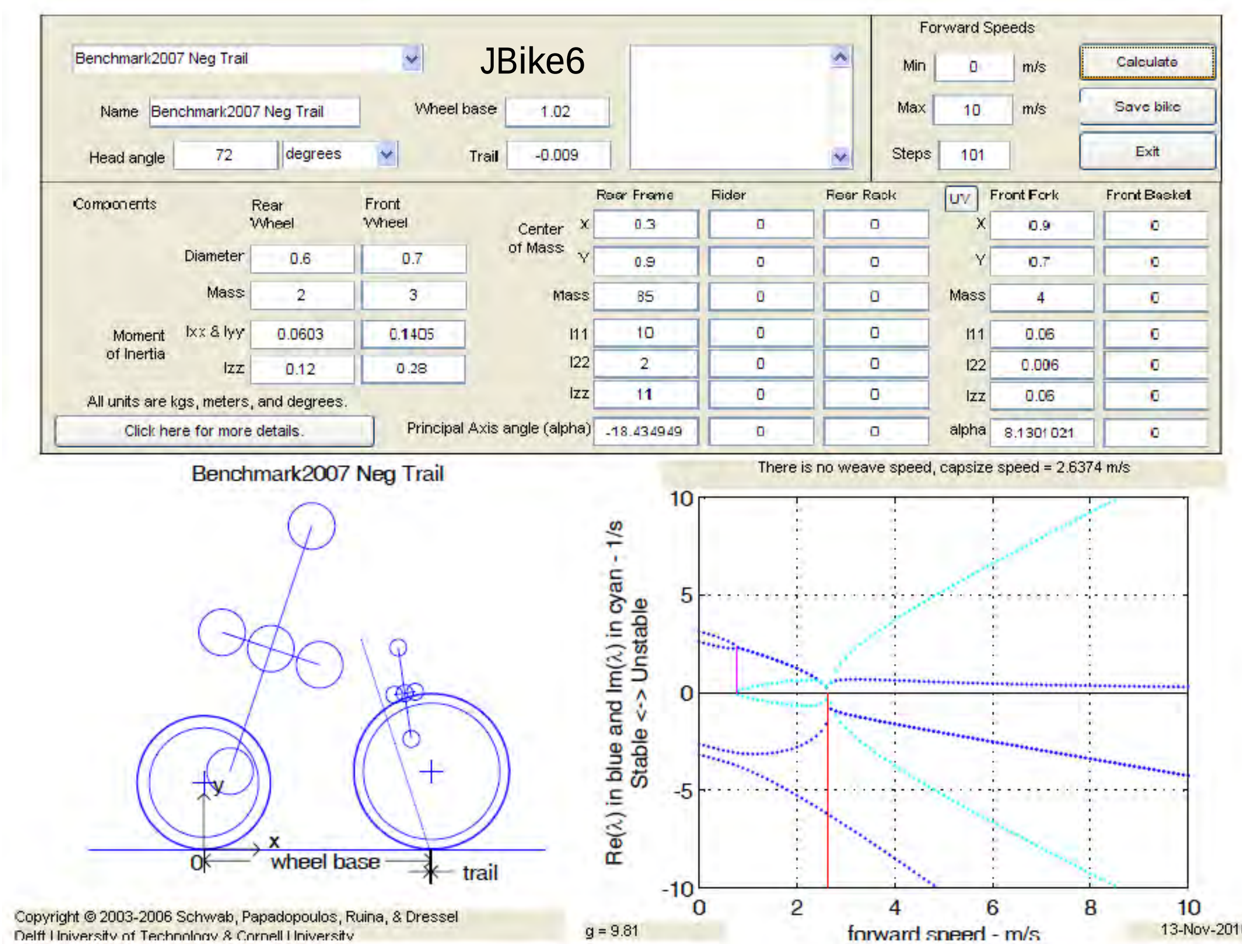
The benchmark bicycle has no stable region when the gyro is removed



A bicycle of common construction but with the gyroscopic terms eliminated.
The model is based on the benchmark bicycle where the only change that has been made is to eliminate the spin angular momentum of the wheels.
This bicycle is unstable at all forward speeds.

Counterexample 2

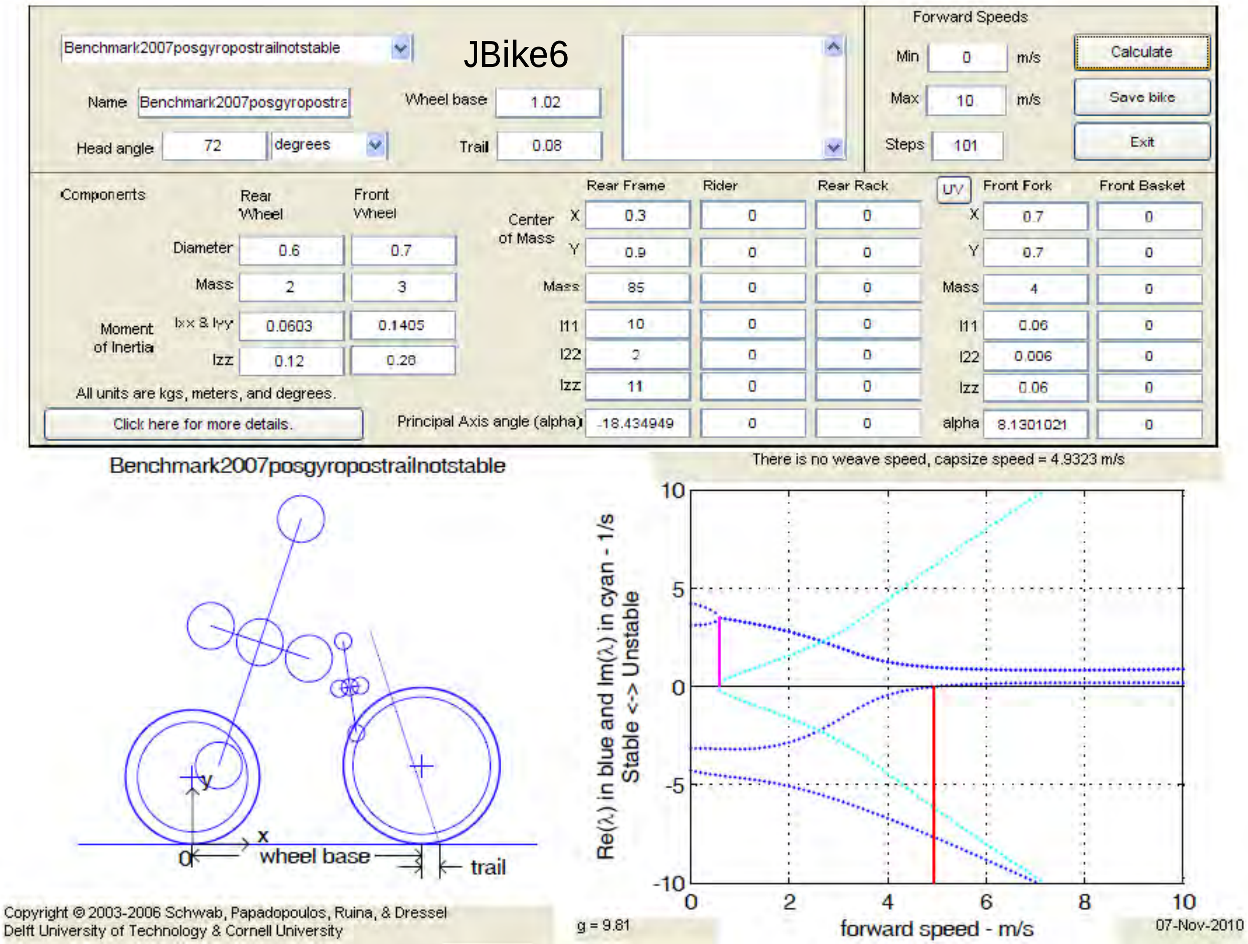
The benchmark bicycle has no stable region when the trail is made negative



A bicycle of common construction but with the trail altered.
The model is based on the benchmark bicycle where the only change that has been made is making the trail negative by displacing the steer axis backwards.
The self-stability speed range vanishes.

Counterexample 3

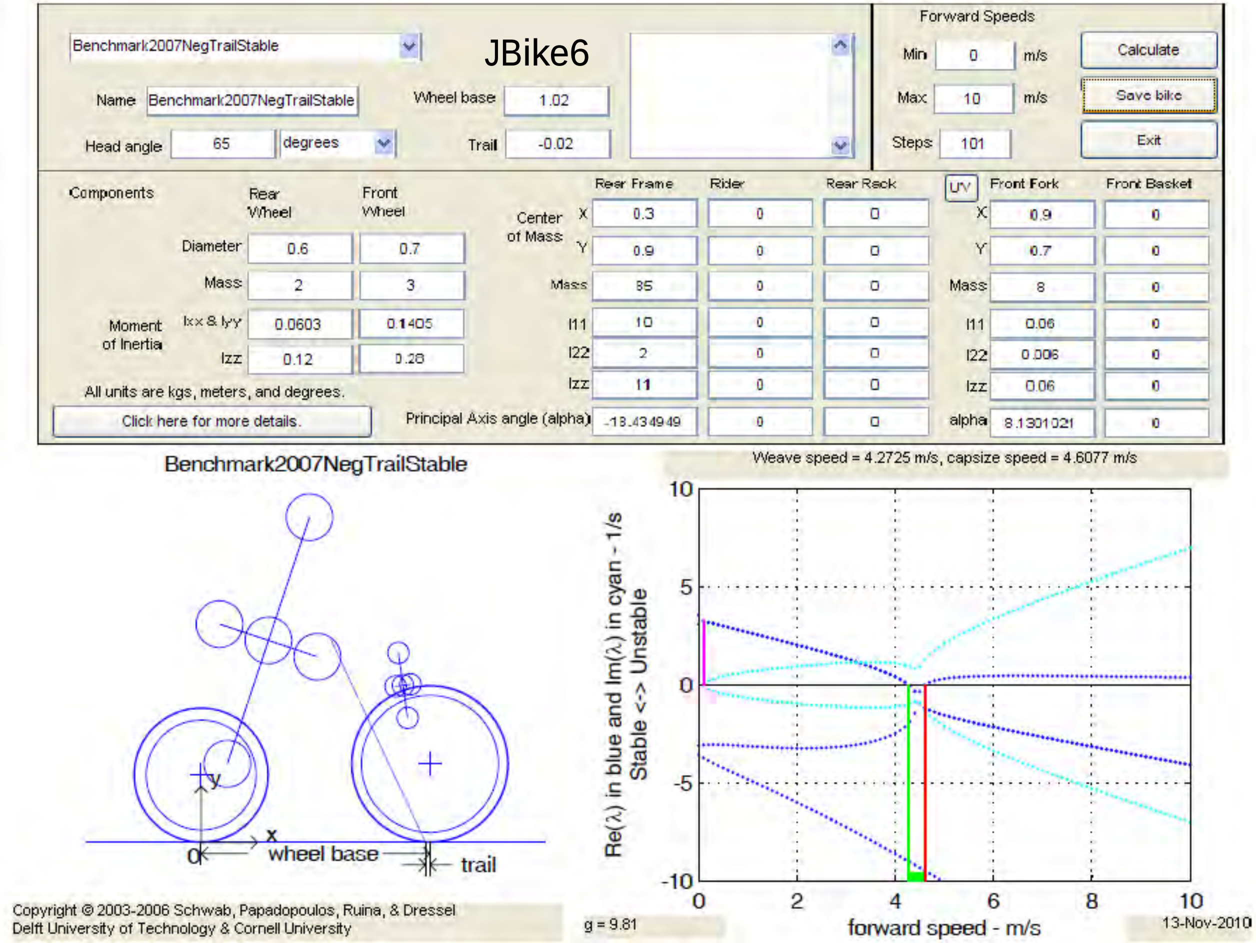
Lacks any stable speed range even with positive trail and positive gyro



A bicycle of common construction with gyroscopic action and positive trail but with no stable forward speed range.
The model is based on the benchmark bicycle where the only change that has been made is to place the center of mass of the front fork behind the steer axis instead of in front of it.
Clearly the bicycle is unstable at all forward speeds.

Counterexample 4a

Conventional bicycle displaying stability even with negative trail

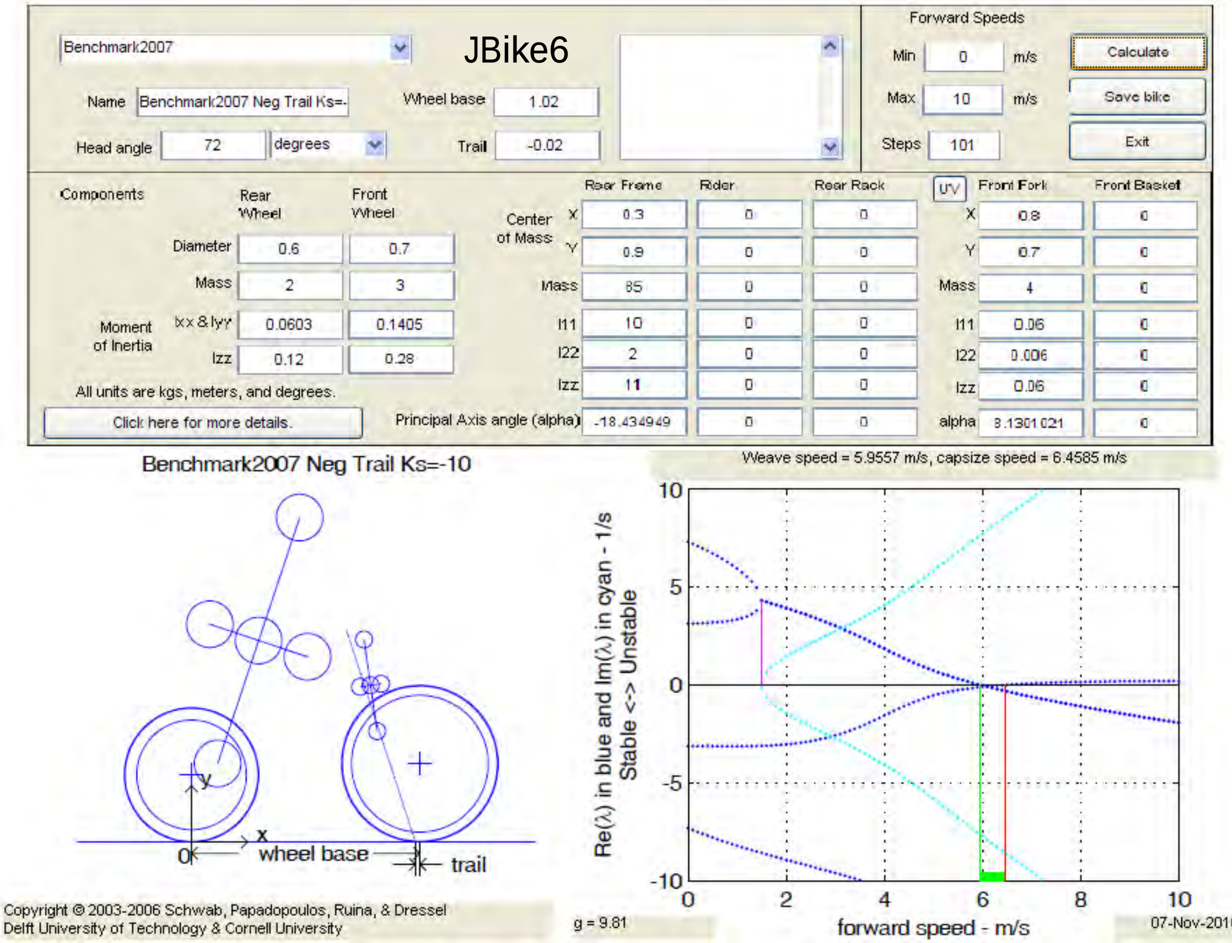


A bicycle of common construction but with negative trail, which still shows a stable forward speed range. The model is based on the benchmark bicycle but now with a negative trail of -0.02 m, an increased steer axis tilt of 25 (= 90-5) degrees and therefore with the center of mass of the front assembly more forward of the steer axis. The front frame mass was also increased.

This bicycle still shows a (small) stable forward speed range (between the vertical lines marking the weave and capsize speeds).

Counterexample 4b

Conventional bicycle displaying stability even with negative trail

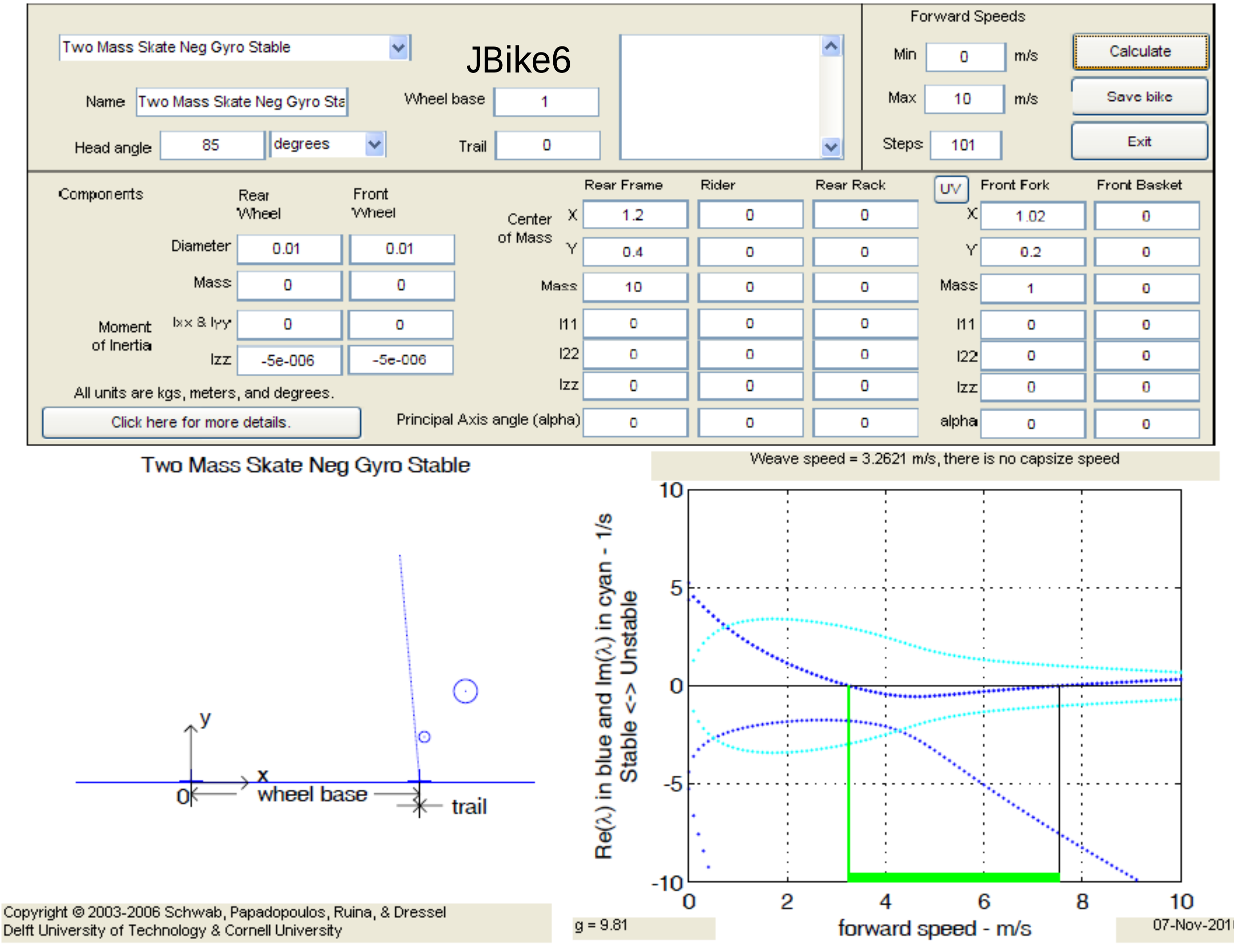


A bicycle of common construction but with negative trail, which still shows a stable forward speed range. The model is based on the benchmark bicycle but now with a negative trail and a decentering steering spring (T_δ = -kδ, with k = -10 Nm/rad).

This bicycle still shows a (small) stable forward speed range (between the vertical lines marking the weave and capsize speeds).

Counterexample 5

Stable with negative gyro

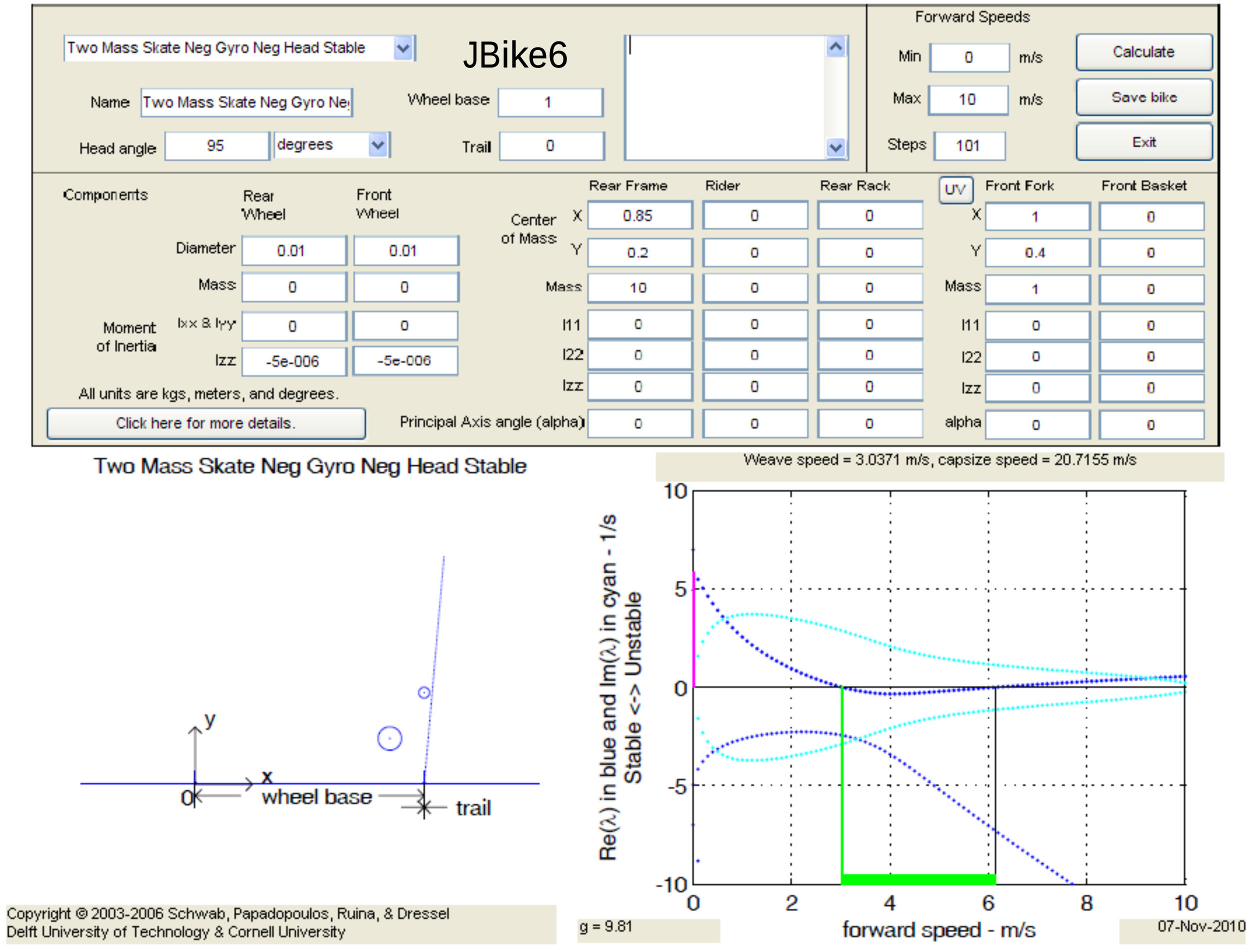


The model is based on the theoretical two-mass-skate (TMS) model but with slightly negative gyroscopic action (e.g., by counter-spinning wheels) and where the center of mass of the front fork has been lowered to 0.2 m.

This bicycle has a large stable forward speed range.

Counterexample 6

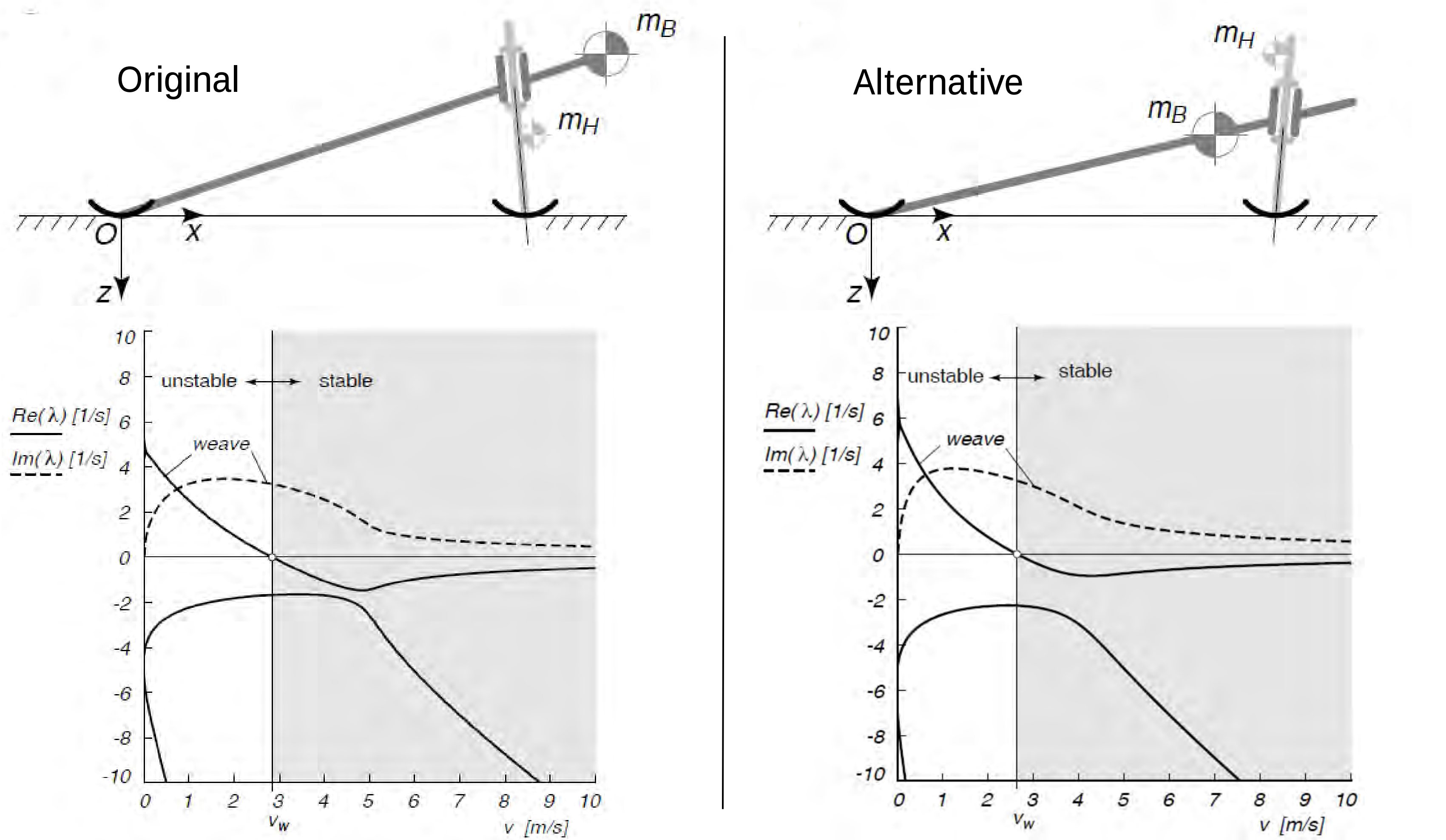
Stable speed range with a reverse tilted steer axis



A two-mass-skate (TMS) bicycle with negative gyroscopic action, reverse tilted steer axis, which shows a stable forward speed range. The model is based on the alternative theoretical two-mass-skate (TMS) model, which has a reverse tilted steer axis with added negative gyroscopic action.
This bicycle clearly shows a stable forward speed range.

Two-mass-skate (TMS) bicycle

Original and Alternative design

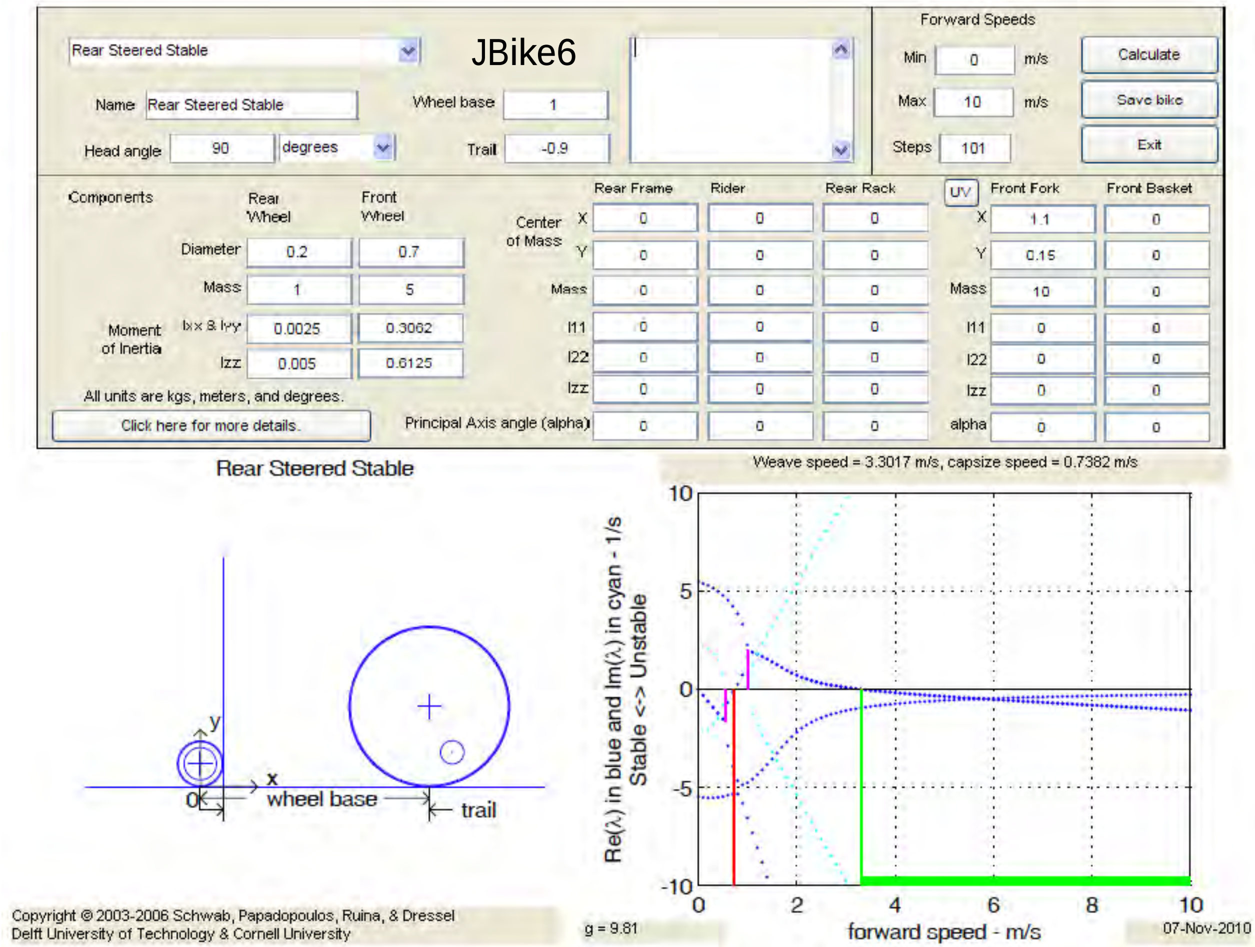


Example 7

Rolling backwards, unstable.

Counterexample 7

Stable with rear wheel steering

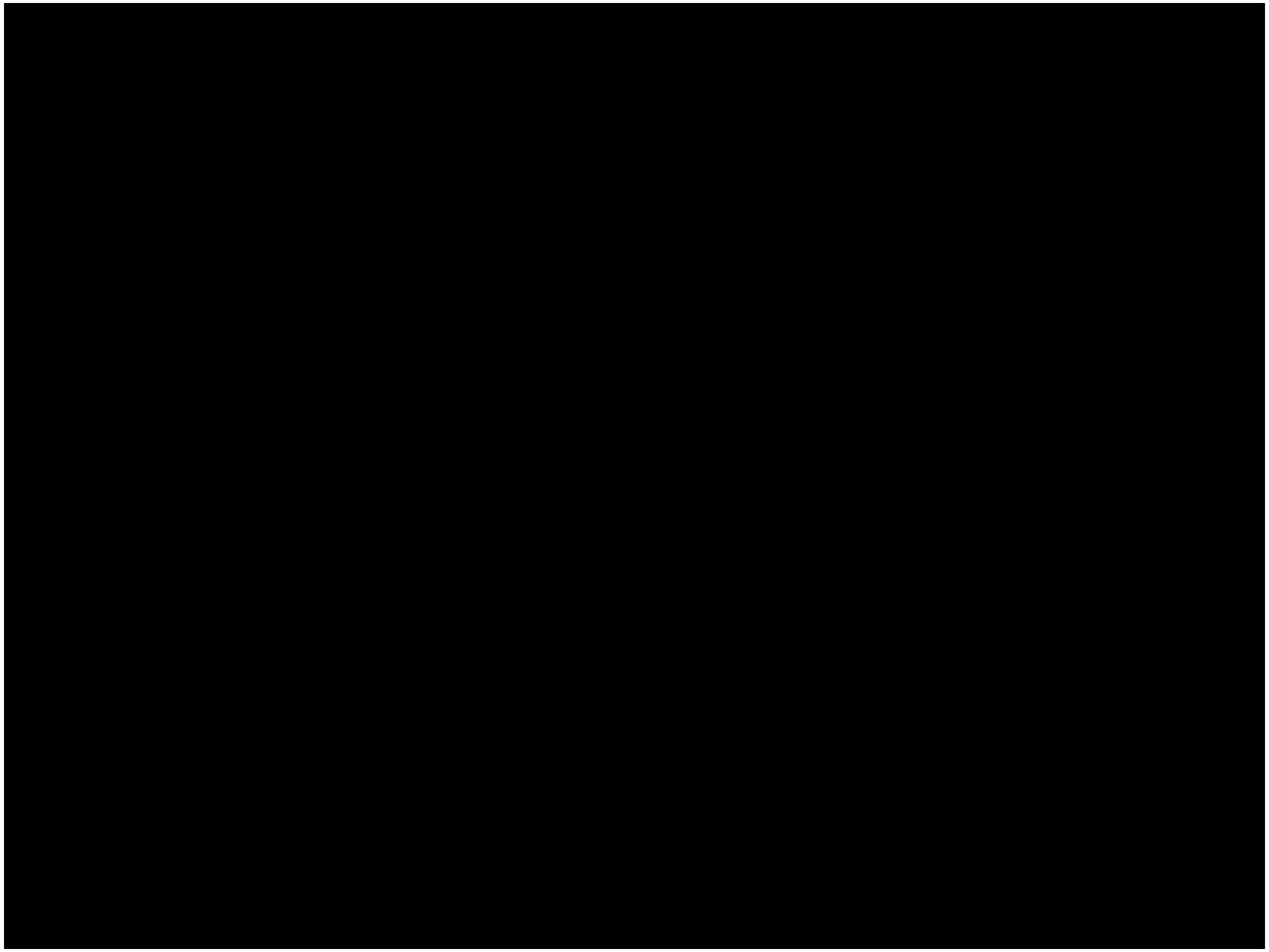


A bicycle with 'rear wheel steering' which shows a stable forward speed range. The steer axis is just in front of the rear wheel and is vertical. The heavy front assembly has a center of mass in front of the front wheel.

This rear wheel steered design has a stable forward speed range to the right of the rightmost vertical line, from 3 m/s to infinity.

Counterexample 7

Stable with rear wheel steering



Counterexample 7

Stable with rear wheel steering

Thank You

6.

Homework

Homework Assignment

Product & Motion
io2022
Masterclass Bicycle Dynamics

1th semester 2013
lecturer: Arend L. Schwab
a.l.schwab@tudelft.nl

http://bicycle.tudelft.nl/

Homework Assignment MBD

In this assignment you will investigate the handling qualities of various bicycle designs. Unfortunately, little is known about handling qualities in bicycles. However, we know some things about the uncontrolled dynamics of a bicycle with rigid rider attached. We know that the stability of an uncontrolled bicycle changes with forward speed. In a bicycle of the common construction the stability increases with increasing velocity. And we know that we can change the design of a bicycle such that it will always be unstable for any forward speed or, vice versa, very stable from a minimal speed until infinity. How does this relate to handling qualities? Well, we suspect that an unstable bicycle is hard to handle and that a stable or mildly unstable bicycle is easier to handle. Therefore we will focus on bicycle stability to predict proper handling.

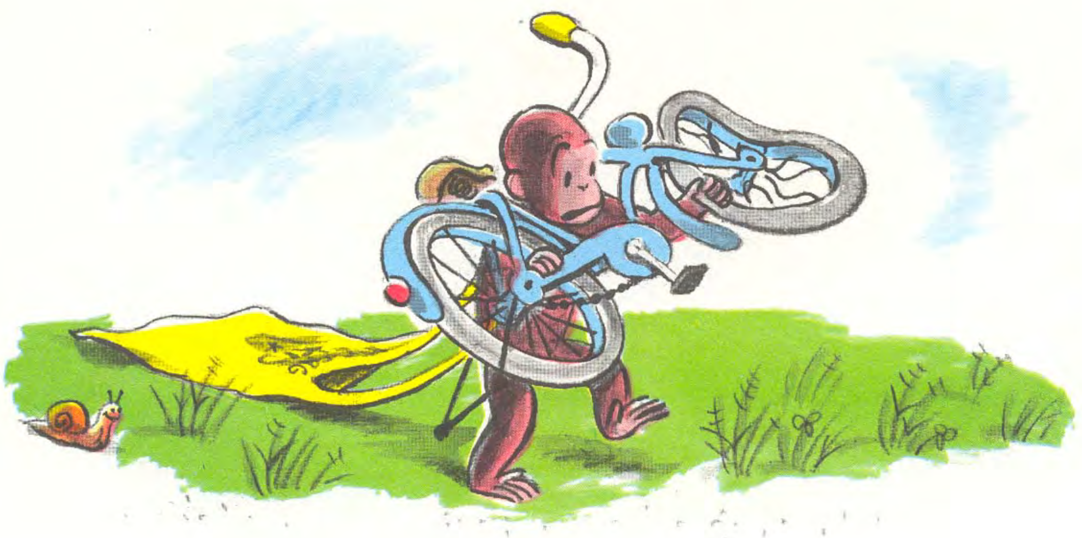
With the Whipple bicycle model [1] we are able to calculate the stability of the lateral motions of a bicycle for various designs at various forward speeds. A Matlab tool has been build which has the implementation of these equations and stability calculation, called JBike6. You can download JBike6 for free, just google "JBike6" or click on download JBike6.

Please answer the following questions:

1. For your own bicycle determine the necessary bicycle parameters like wheelbase, headangle, trail, mass and mass moments of inertia of the parts and put these in JBike6 and investigate the stability. Determining the mass and the mass moments of inertia of the individual parts is not easy. Check out the existing models in JBike6 and guess your way around for your bicycle. Mass is do-able but for mass moment of inertia you need some reference, all objects below have mass m, and I's are at the centre of mass:
   - Beam, length l: $I_{zz} = (1/12)ml^2$.

You can find this at my website:
Just google: Arend Schwab.

Thank You!



http://bicycle.tudelft.nl