Multibody Dynamics A - wb1310

Lecture 4, course 2014-2015

Arend L. Schwab
TU Delft, 3mE/BmechE

www.bicycle.tudelft.nl/schwab/wb1310fall2014/index.htm

Multibody Dynamics A
wb1310
(fall 2014, Q2)

Instructor: Arend L. Schwab

TU Delft
Delft University of Technology

Description: Multibody Dynamics A is an introductory course in applied dynamics of mechanical systems. The emphasis is on the usage of multibody dynamics software. We want you to learn enough about dynamics in 3D that you will be able to use a standard multibody dynamics software package correctly, appreciate the limitations, and say some sensible things about the model at hand.

In the course you will learn about the fundamentals of Multibody Dynamics: the description of the orientation of a rigid body in space, the Newton-Euler equations of motion for a 3D rigid body, how to add constraints to the equations of motion, and how to solve such a system of coupled equations. You will spend most of the time (80%) in doing the lab assignments. These assignment consists of a number of practical problems that have to be worked out with the software package ADAMS. Your findings are to be put down in a Lab Report.

Goal: By the end of the course you be able to make a complex model of realistic 3D mechanical system and draw some conclusions from the dynamical analysis.

Grading: The written exam is of the open book type and has the form of a questionnaire about the findings as written down in your lab report. The report serves as reference material for your exam. At the end of the exam the questionnaire together with the Lab Report are to be handed over, The final grading is 50% on the report and 50% on the written exam.

News

Hand-Outs
- The course Contents.
- The Laboratory Assignments.
- A short Introduction to ADAMS (1,169 KB).
- Tire and Road files for assignment#5: 16r26_new.tir, 18r38_new.tir, FlatRoad.rdf.

Office Hours
Instructor: Arend L. Schwab, a.l.schwab@tudelft.nl, Monday, 15-17 h., room F-0-010, phone: 015 278 2701.
TA: Sten Ponsioen, s.l.ponsioen@tudelft.nl, Monday, 13-17 h. IO-PC hall 3 (SHIFT).

Contents

Lecture	Topic	Assigment
1th-2nd	Introduction	1-Pendulum
1th-2nd	Newton-Euler eqn's of motion	2-Wheel
3rd	Modeling of Mechanical Systems	3-Crane
4th	Modeling of Mechanical Systems	4-Governer
5th	Coupled Differential And Algebraic Equations	5-Tractor/Bicycle
6th	Overview	5-Tractor/Bicycle

Accounting

Section	Hours
Lectures	7*2
Assignments (guided)	7*4
Assignments (free)	7*4
Class Prep	7*1
? **Written Exam!**	7
Total (3 ECTS)	84

Written exam: Thu 22 Jan 2015, 14-17 h.

Assignment 4

In a standard textbook on kinematics and dynamics of mechanical systems [1], the steady state analysis of a regulator is discussed as an example. A regulator is an instrument that tries to keep the speed of an engine constant, irrespective of the load. This can be traced back to James Watt and his steam engines. The literal text of the example is given as an appendix. In this assignment we will eventually make and simulate the complete model of this system in 5 steps.

1. Make a simplified model of the regulator in ADAMS. Only model the *Ground*, *Spindle*, *Ball1*, *Ball2* for the given geometry of the model.

2. Make an estimate of the equilibrium (steady state) at an angular velocity of $\omega = 105$ [rpm]. Check this by means of a simulation in ADAMS. Think of a way to damp out the transient response to get to the steady state.

3. Make the complete model of the regulator in ADAMS. Apparently the distance constraint does not exist in the ADAMS software. Replace this constraint by means of a stiff spring-damper element between the arm and collar. Make a well considered choice for the value of the stiffness and the damping (report!). Simulate this regulator at the steady state angular velocity $\omega = 105$ [rpm]. Check the position of the arms, these should make a $45°$ angle with the vertical, do they?

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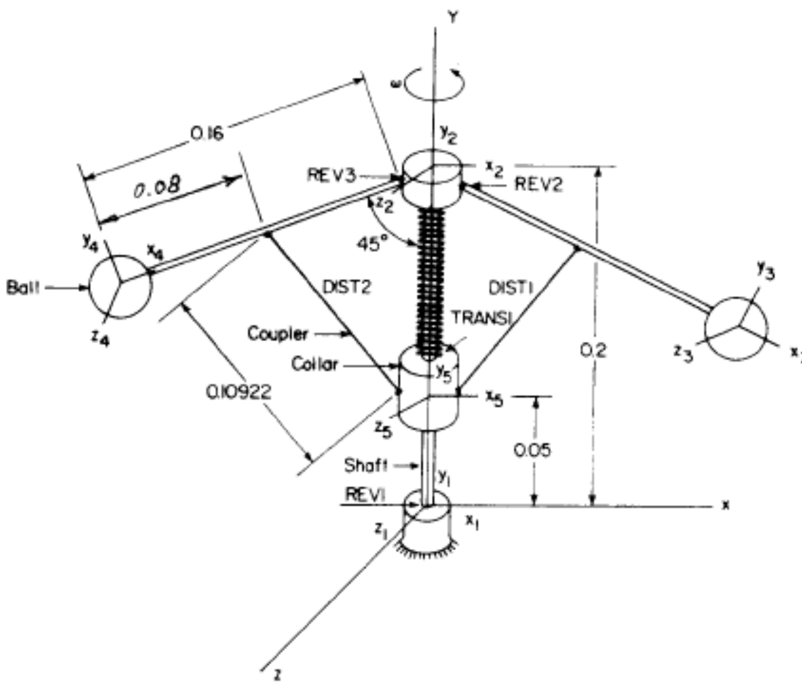


Figure 12.6.1 Governor mechanism.

collar is connected to the shaft by a translational joint. Couplers that connect the ball arms and collar are modeled as distance constraints. A TSDA element is attached between the shaft and collar. The inertia properties of the components of the system are defined in Table 12.6.1.

The intended function of the governor is to maintain a nearly constant

TABLE 12.6.1 Inertia Properties of Governor Mechanism

Body		Mass	$I_{x'}$	$I_{y'}$	$I_{z'}$	$I_{x'y'}$	$I_{y'z'}$	$I_{x'z'}$
Ground	①	1.0	1.0	1.0	1.0	0.0	0.0	0.0
Spindle	②	200.0	25.0	50.0	25.0	0.0	0.0	0.0
Ball 1	③	1.0	0.1	0.1	0.1	0.0	0.0	0.0
Ball 2	④	1.0	0.1	0.1	0.1	0.0	0.0	0.0
Collar	⑤	1.0	0.15	0.125	0.15	0.0	0.0	0.0

angular speed ω of the shaft under a varying resisting torque from the machine being driven. If the angular velocity of the shaft decreases, the balls drop and hence lower the collar. A linkage attached to the collar then opens the fuel feed to the engine, which generates an increased torque and leads to a speed-up of the shaft. As a result, the balls rise toward their nominal position as the shaft angular velocity approaches the desired value.

The elements of the kinematic model are as follows:

Bodies	
Five bodies	$nc = 35$
Constraints	
Distance constraint: DIST1	1
Distance constraint: DIST2	1
Revolute joint: REV1	5
Revolute joint: REV2	5
Revolute joint: REV3	5
Translational joint: TRANS1	5
Ground constraint	6
Normalization constraint	5
Euler parameter	
	$nh = 33$

DOF = 35 − 33 = 2.

Data for the distance constraints, revolute joints, and translational joint are tabulated in Tables 12.6.2 to 12.6.4. Data for the TSDA element are given in Table 12.6.5. Three different models (1, 2, and 3) are distinguished, with spring constants $k = 1000$, 2000, and 3000 N/m, respectively.

TABLE 12.6.2 Data for Distance Constraints

DIST1

Body \ Point		P x'	P y'	P z'	Q x'	Q y'	Q z'	R x'	R y'	R z'
Ball 1	③	−0.08	0.0	0.0	−0.08	0.0	1.0	1.08	0.0	0.0
Collar	⑤	0.0	0.0	0.0	0.0	0.0	1.0	1.0	0.0	0.0

Distance = 0.10922

DIST2

Body \ Point		P x'	P y'	P z'	Q x'	Q y'	Q z'	R x'	R y'	R z
Ball 2	④	0.08	0.0	0.0	0.08	0.0	1.0	1.08	0.0	0.0
Collar	⑤	0.0	0.0	0.0	0.0	0.0	1.0	1.0	0.0	0.0

Distance = 0.10922

TABLE 12.6.3 Data for Revolute Joints

REV1

Body \ Point		P x'	P y'	P z'	Q x'	Q y'	Q z'	R x'	R y'	R z'
Ground	①	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0
Spindle	②	0.0	−0.2	0.0	0.0	−1.2	0.0	0.0	−0.2	1.0

REV2

Body \ Point		P x'	P y'	P z'	Q x'	Q y'	Q z'	R x'	R y'	R z'
Spindle	②	0.0	0.0	0.0	0.0	0.0	1.0	1.0	0.0	0.0
Ball 1	③	−0.16	0.0	0.0	−0.16	0.0	1.0	1.0	0.0	0.0

REV3

Body \ Point		P x'	P y'	P z'	Q x'	Q y'	Q z'	R x'	R y'	R z'
Spindle	②	0.0	0.0	0.0	0.0	0.0	1.0	1.0	0.0	0.0
Ball 2	④	0.16	0.0	0.0	0.16	0.0	1.0	1.0	0.0	0.0

TABLE 12.6.4 Data for Translational Joint

Body \ Point		P x'	P y'	P z'	Q x'	Q y'	Q z'	R x'	R y'	R z'
Spindle	②	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0
Collar	⑤	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	1.0

TABLE 12.6.5 Data for Translational Spring–Damper–Actuator

Body \ Point		P x'	P y'	P z'	Q x'	Q y'	Q z'	R x'	R y'	R z'
Spindle	②	0.0	0.0	0.0	0.0	0.0	1.0	1.0	0.0	0.0
Collar	⑤	0.0	0.0	0.0	0.0	0.0	1.0	1.0	0.0	0.0

Spring constant, $k = 1000, 2000, 3000$ N/m
Damping rate, $c = 30$ kg/s
Free length of spring, $L_0 = 0.15$ m

12.6.2 Steady-State Analysis

Based on the configuration of Fig. 12.6.1, steady-state motion is determined by selecting the desired angular speed ω of the spindle. Using dynamic force balancing and $\omega = 11.0174$ rad/s, at a slope of 45° of the ball arms, the collar is to be stationary. Table 12.6.6 defines the steady-state configuration of the system.

TABLE 12.6.6 Data for Steady-State Configuration

Body		x	y	z	e_0	e_1	e_2	e_3
Ground	①	0.0	0.0	0.0	1.0	0.0	0.0	0.0
Spindle	②	0.0	0.2	0.0	1.0	0.0	0.0	0.0
Ball 1	③	0.11314	0.08686	0.0	0.9239	0.0	0.0	0.3827
Ball 2	④	−0.11314	0.08686	0.0	0.9239	0.0	0.0	0.3827
Collar	⑤	0.0	0.05	0.0	1.0	0.0	0.0	0.0

12.6.3 External Torque

An external torque T_c due to the load driven by the shaft is applied to the spindle, as shown in Fig. 12.6.2. To compensate for this torque, which tends to reduce shaft speed, the torque T_s applied to the spindle by the engine as a result of fuel fed by a collar height variation $\Delta\ell$ is modeled as

$$T_s = C \times \Delta\ell \qquad (12.6.1)$$

where C is the torque generated due to fuel fed to the engine by a unit $\Delta\ell$, which is spring deformation (vertical movement of collar). Three different values $C = 7500$, 12,500, and 17,500, corresponding to increasing engine power, are used to study the system dynamic response.

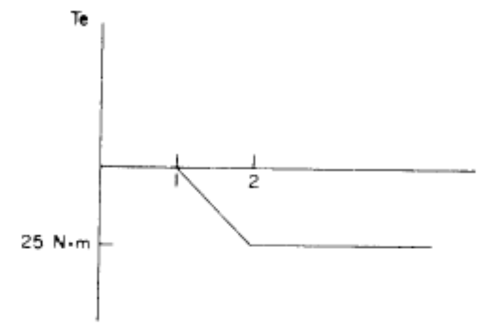
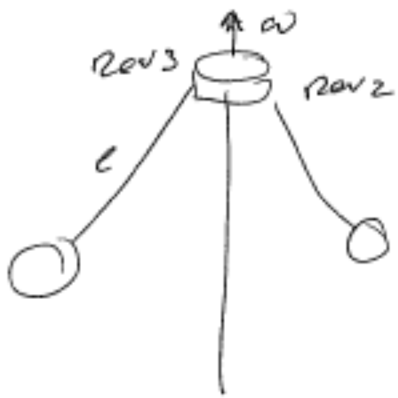


Figure 12.6.2 External torque on spindle.

"Modelleer Technieken" opgave 4 en 5 nader bekeken. (1)

(4-1) Eenvoudig Model

- ~~Waar zitten R.~~

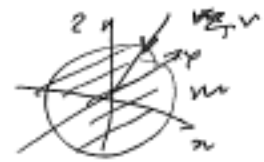


Waar zitten Rev2 en Rev3 ?
Wat is P φ R ? z.o Sheet

P = punt
φ is z'-as
R is x'-as

Hoe groot zijn de Kogels ?

$m = 1$ $\quad I = 0.1$

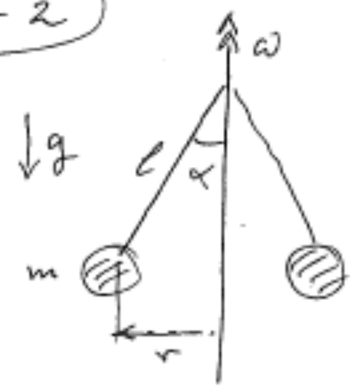


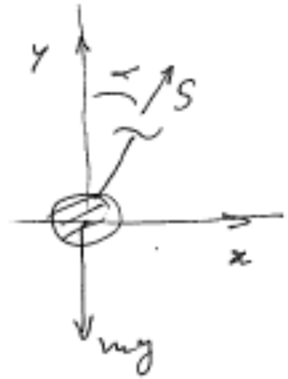
$I = \frac{2}{5} m r^2$

$0.1 = \frac{2}{3} \cdot 1 \cdot r^2 \rightarrow r = 0.5$

en $l = 0.16$ Hoe kan dat nu ??

(4-2)



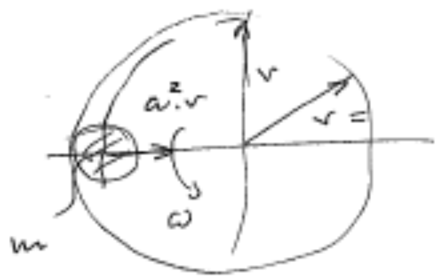


$$S \cdot \sin\alpha = m\omega^2 \cdot l \sin\alpha$$

$$S \cdot \cos\alpha - mg = 0$$

$$\Downarrow$$

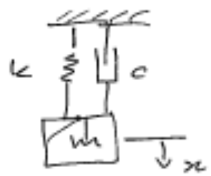
$$\omega^2 \cdot l = \frac{g}{\cos\alpha}$$



$$\cos\alpha = \frac{g}{\omega^2 \cdot l} = \frac{9.81}{\left(105 \cdot \frac{2\pi}{60}\right)^2 \cdot 0.16} = 1.039 \text{ rad}$$

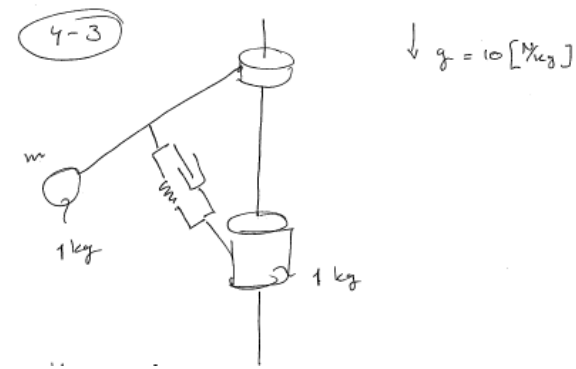
$$= 59.52^\circ$$

Hoe stijf moet de veer en hoe groot moet de demper zijn?

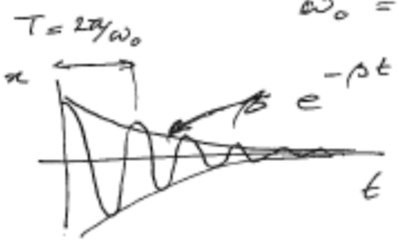


$$m\ddot{x} + c\dot{x} + kx = 0$$

$$\ddot{x} + 2\beta\omega_0\dot{x} + \omega_0^2 x = 0$$



$$\omega_0^2 = \frac{k}{m} \qquad 2\beta\omega_0 = \frac{c}{m} \qquad \beta = \frac{c}{2\sqrt{k\cdot m}}$$



~~k >> m → $\omega_0 >> 1$ → T << 1 kleine stapgrootte~~

Stijve veer → k groot → $\omega_0 >> 1$ → T << 1
kleine stapgrootte

Slappe veer → grote afwijking $\Delta l >> l_0$

$\Delta l = \frac{F}{k}$ Hoe groot is F?

$$F \approx 2\,(m_{coll}\cdot g + \underbrace{F_{veer}}_{\approx 0})$$

$\Delta l_{max} = 10^{-3}\cdot l_0 \quad\rightarrow\quad k = \frac{F}{\Delta l} = \frac{2\cdot 1\cdot 10}{10^{-3}\cdot 0.1} = 2\cdot 10^5$ [N/m]

$c = 2\beta\cdot\sqrt{k\cdot m}$ neem $\beta = 1$ en $m = 1$

$c = 2\cdot\sqrt{2\cdot 10^5\cdot 1} = 894 \rightarrow 1000$ [Ns/m]

Check Δl_{max}?

Assignment 4

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4. Next only add the feedback of the engine to your model and carry out a simulation which shows that in the absence of the disturbance the $45°$ equilibrium will be reached again.

5. Finally add the disturbance to your model and simulate the three different types of engines, take for the vertical spring a stiffness of $k = 1000$ [N/m]. Use another STEP function, as described in assignment 3, for the description of the external couple. Plot the angular velocity and the collar's vertical position and the spindle's angular velocity as a function of time and compare these results with those in [1], fig 12.6.5 and 12.6.6. Discuss this comparison.

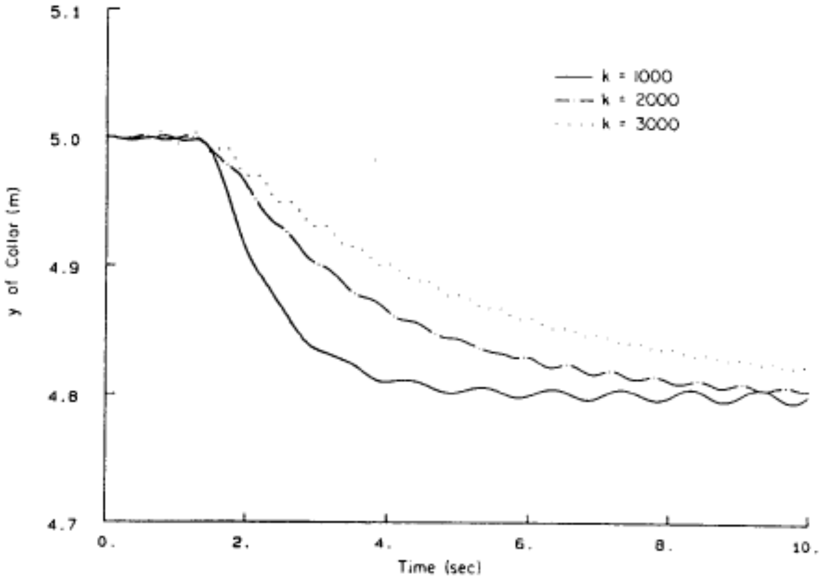

Figure 12.6.3 y of collar versus time ($C = 12{,}500$).

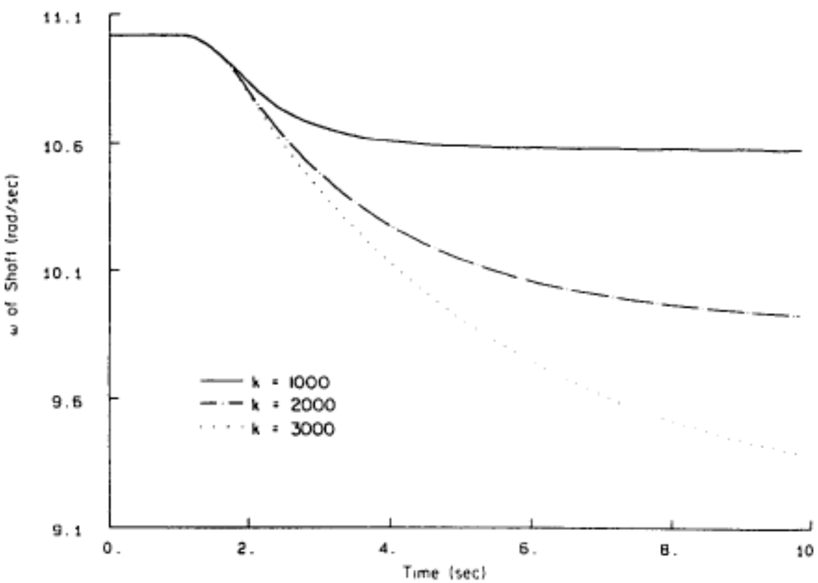

Figure 12.6.4 ω of shaft versus time ($C = 12{,}500$).

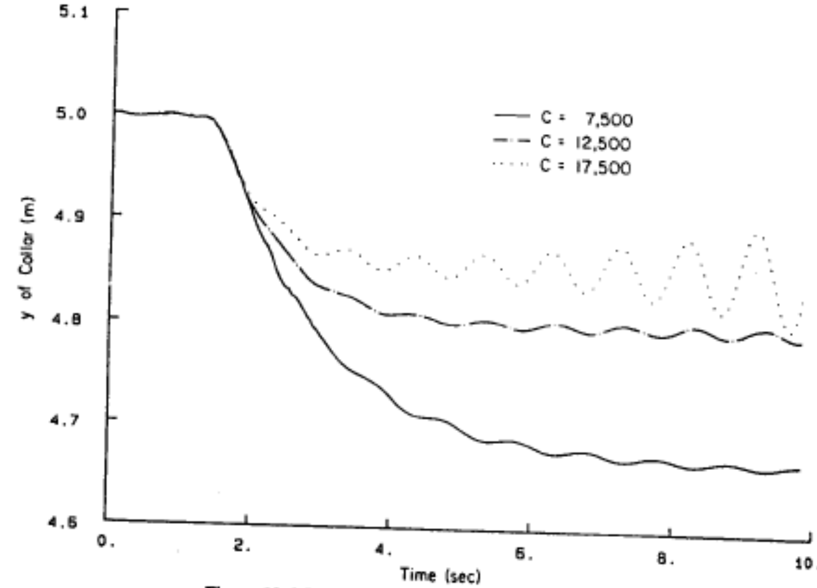

Figure 12.6.5 y of collar versus time ($k = 1000$).

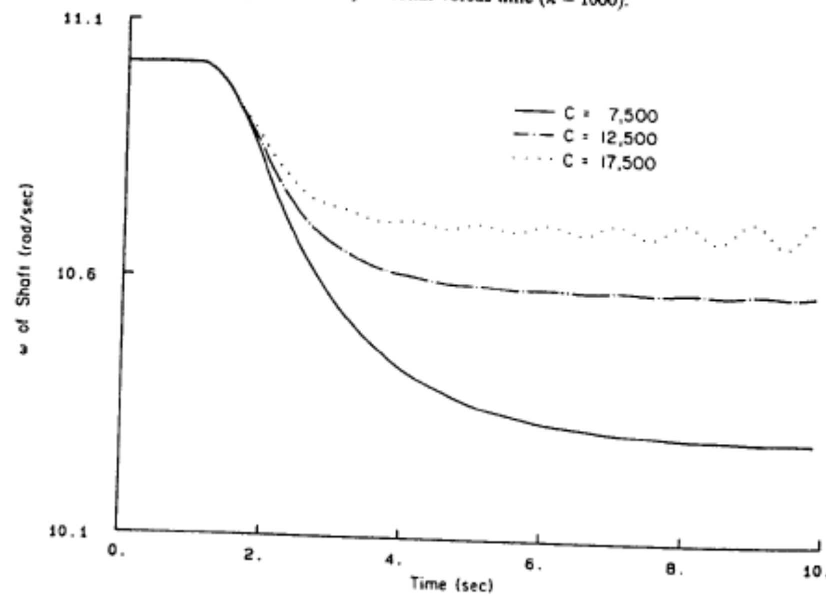

Figure 12.6.6 ω of shaft versus time ($k = 1000$).

HIGHER MECHANICS

BY

HORACE LAMB, Sc.D., LL.D., F.R.S.,

HONORARY FELLOW OF TRINITY COLLEGE, CAMBRIDGE;
LATELY PROFESSOR OF MATHEMATICS IN THE VICTORIA UNIVERSITY
OF MANCHESTER

CAMBRIDGE
AT THE UNIVERSITY PRESS
1920

Lagrange's equation

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{\phi}} - \frac{\partial T}{\partial \phi} = \Phi \quad \text{..................(19)}$$

accordingly becomes

$$C\frac{dr}{dt} - (A-B)\,pq = N. \quad \text{..................(20)}$$

Since it is indifferent which of the principal axes at the fixed point we denote by OC, the remaining equations of Euler's triad will also hold.

Ex. 2. In the steam-engine the driving power is usually controlled by some form of 'centrifugal governor.' The original type, introduced by Watt, is shewn in the annexed sketch. The spindle to which the arms carrying the two balls are hinged rotates at a rate proportional to the speed of the engine. When this rotation is uniform the balls, under the action of gravity and centrifugal force, take up a definite 'equilibrium' position depending on the speed. If the speed increases the balls diverge outwards, raising the collar c to which the lower arms are connected, and thus operating a system of levers which turn a valve so as to reduce the supply of steam. Conversely, when the speed diminishes, the collar descends, and the supply is reinforced.

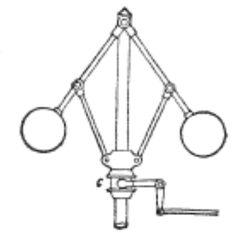

Fig. 56.

If θ denote the inclination of the upper arms to the spindle, and $\dot{\psi}$ the angular velocity about the vertical, the expression for the kinetic energy has the form

$$2T = A\dot{\theta}^2 + I\dot{\psi}^2, \quad \text{..................(21)}$$

where A and I are functions of θ. The coefficient I is supposed to include a term representing the inertia of the engine and of the train of machinery in connection with it. Lagrange's formula gives

$$\frac{d}{dt}(A\dot{\theta}) - \frac{1}{2}\frac{\partial A}{\partial \theta}\dot{\theta}^2 - \frac{1}{2}\frac{\partial I}{\partial \theta}\dot{\psi}^2 = -\frac{\partial V}{\partial \theta}, \quad \text{..................(22)}$$

and

$$\frac{d}{dt}(I\dot{\psi}) = \Psi, \quad \text{..................(23)}$$

where V is the potential energy of the governor, and Ψ represents the excess of driving power over resistance. If this excess vanishes when the valve has the position corresponding to $\theta = \alpha$ we may write, as an approximation,

$$\Psi = -\beta(\theta - \alpha). \quad \text{..................(24)}$$

For steady motion we must have $\theta = \alpha$, $\dot{\psi} = \omega$, where ω is determined by

$$\frac{1}{2}\frac{\partial I}{\partial \theta}\omega^2 = \frac{\partial V}{\partial \theta}. \quad \text{..................(25)}$$

Since this involves the above value of θ, the speed ω will vary with any permanent change in the driving power. The contrivance does not therefore maintain a constant speed independent of variations in the driving power, and it

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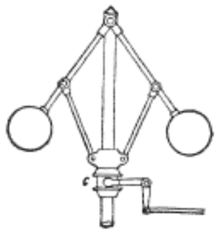

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where V is the potential energy of the governor, and Ψ represents the excess of driving power over resistance. If this excess vanishes when the valve has the position corresponding to $\theta = a$ we may write, as an approximation,

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Since this involves the above value of θ, the speed ω will vary with any permanent change in the driving power. The contrivance does not therefore maintain a constant speed independent of variations in the driving power, and it

was therefore suggested by Maxwell that it should properly be called a 'moderator' rather than a 'governor.'

To examine the effect of accidental disturbances of the steady motion, we write

$$\theta = a + x, \quad \dot\psi = \omega + y, \quad \text{.....................(26)}$$

and treat x, y as small. If we cancel the terms which refer to the steady motion, the equations (22) and (23) become

$$A\ddot x - \tfrac{1}{2}I''\omega^2 x - I'\omega y + V'' x = 0, \quad \text{.....................(27)}$$

$$I\dot y + I'\omega\dot x + \beta x = 0, \quad \text{.....................(28)}$$

where accents indicate differentiations with respect to θ, and the coefficients are supposed to have the constant values corresponding to $\theta = a$. Assuming that x and y vary as $e^{\lambda t}$, we find

$$AI\lambda^3 + \{I(V'' - \tfrac{1}{2}I'\omega^2) + I'^2\omega^2\}\lambda + I'\beta\omega = 0. \quad \text{.....................(29)}$$

It is essential of course that the governor should be stable when the speed ω is maintained constant. The condition for this is*

$$V'' - \tfrac{1}{2}I'\omega^2 > 0. \quad \text{.....................(30)}$$

This being satisfied, the coefficients in (29) are all positive. There is therefore one negative, and no positive root. Since the sum of the roots is zero, the remaining roots must be imaginary with positive real part. The complete solution of (27) and (28) therefore consists of terms of the types $e^{-2\mu t}$, $e^{\mu t}\cos\nu t$, $e^{\mu t}\sin\nu t$. The latter pair indicate an oscillation of continually increasing amplitude.

This instability is checked to some extent by the inevitable friction between various parts of the mechanism, but in order definitely to eliminate it a viscous resistance is sometimes expressly introduced, opposing variations of θ. This may be represented by inserting a term $\gamma d\theta/dt$ on the right-hand side of (22), and therefore a term $\gamma\dot x$ in (27). The resulting equation in λ is

$$AI\lambda^3 + \gamma I\lambda^2 + \{I(V'' - \tfrac{1}{2}I'\omega^2) + I'^2\omega^2\}\lambda + I'\beta\omega = 0. \quad \text{.........(31)}$$

There is obviously one negative root, as before. The condition that the remaining roots should be negative, or imaginary with negative real part, is†

$$\gamma I\{I(V'' - \tfrac{1}{2}I'\omega^2) + I'^2\omega^2\} > AII'\beta\omega, \quad \text{.....................(32)}$$

which is satisfied if the frictional coefficient γ is sufficiently great.

* In the problem of Art. 80, Ex. 1, we have

$$V = -mga\cos\theta, \quad I = ma^2\sin^2\theta,$$

and therefore

$$V'' - \tfrac{1}{2}I'\omega^2 = m\omega^2 a\sin^2\theta,$$

in the position of relative equilibrium, where $\cos\theta = g/\omega^2 a$. A closer analogy to the circumstances of Watt's governor is furnished by Ex. 3 of Art. 80.

† If α, β, γ be the roots of the cubic

$$x^3 + p_1x^2 + p_2x + p_3 = 0,$$

we have

$$(\beta + \gamma)(\gamma + \alpha)(\alpha + \beta) = p_3 - p_1p_2.$$