

# Multibody Dynamics A - wb1310

Lecture 5, course 2014-2015

Arend L. Schwab

TU Delft, 3mE/BmechE



## Multibody Dynamics A wb1310 (fall 2014, Q2)

Instructor: [Arend L. Schwab](#)



MSC SOFTWARE

TU Delft

Delft University of Technology

**Description:** Multibody Dynamics A is an introductory course in applied dynamics of mechanical systems. The emphasis is on the usage of multibody dynamics software. We want you to learn enough about dynamics in 3D that you will be able to use a standard multibody dynamics software package correctly, appreciate the limitations, and say some sensible things about the model at hand.

In the course you will learn about the fundamentals of Multibody Dynamics: the description of the orientation of a rigid body in space, the Newton-Euler equations of motion for a 3D rigid body, how to add constraints to the equations of motion, and how to solve such a system of coupled equations. You will spend most of the time (80%) in doing the lab assignments. These assignments consist of a number of practical problems that have to be worked out with the software package ADAMS. Your findings are to be put down in a Lab Report.

**Goal:** By the end of the course you be able to make a complex model of realistic 3D mechanical system and draw some conclusions from the dynamical analysis.

**Grading:** The written exam is of the open book type and has the form of a questionnaire about the findings as written down in your lab report. The report serves as reference material for your exam. At the end of the exam the questionnaire together with the Lab Report are to be handed over. The final grading is 50% on the report and 50% on the written exam.

### News

#### Hand-Outs

- The course [Contents](#).
- The [Laboratory Assignments](#).
- A short [Introduction to ADAMS \(1,169 KB\)](#).
- Tire and Road files for assignment#5: [16r26\\_new.tir](#), [18r38\\_new.tir](#), [FlatRoad.rdf](#).

#### Office Hours

Instructor: Arend L. Schwab, [a.l.schwab@tudelft.nl](mailto:a.l.schwab@tudelft.nl), Monday, 15-17 h., room F-0-010, phone: 015 278 2701.  
TA: Sten Ponsioen, [s.l.ponsioen@tudelft.nl](mailto:s.l.ponsioen@tudelft.nl), Monday, 13-17 h. IO-PC hall 3 (SHIFT).

# Contents

Lecture	Topic	Assignment
1th-2nd	Introduction	1-Pendulum
1th-2nd	Newton-Euler eqn's of motion	2-Wheel
3rd	Modeling of Mechanical Systems	3-Crane
4th	Modeling of Mechanical Systems	4-Governer
5th	Modeling of Mechanical Systems	5-Tractor
6th	Bicycle Dynamics & Control	5-Bicycle

# Accounting

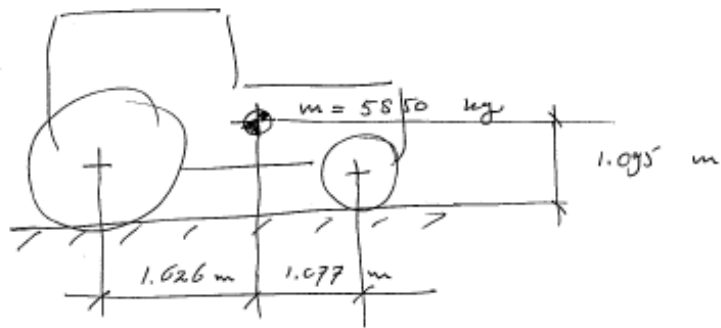
Section	Hours
Lectures	7*2
Assignments (guided)	7*4
Assignments (free)	7*4
Class Prep	7*1
? <b>Written Exam!</b>	7
Total (3 ECTS)	84

Written exam: Thu 22 Jan 2015, 14-17 h.

## Assignment 5

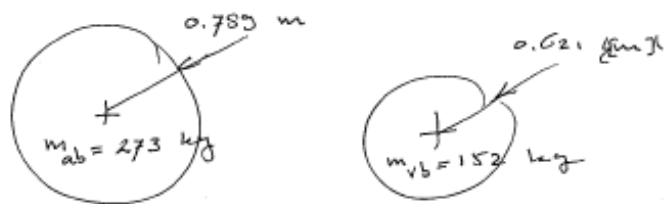
In order to examine the safety of a standard tractor driving along the public road at high speed ( $> 30$  [km/h]) [2], the vehicle has to be submitted to an ISO dual lane change, also known as the moose test. This middle range tractor (AS1) has an undamped rear axle fixed to the body and a rigid front axle attached by a swivel axis to the body at the height of the wheel axle. The steering is realized by letting both front wheels rotate by the same angle around their vertical axes. The total mass is 5850 [kg] and the mass moment of inertia around the vertical or yaw-axis 9255 [kgm<sup>2</sup>] and around the pitch-axis 7245 [kgm<sup>2</sup>]. The mass moment of inertia around the roll-axis has not been given in [2] but is estimated at 5235 [kgm<sup>2</sup>]. The centre of mass of the system is situated 1626 [mm] in front of the rear axle and vertically 1095 [mm] in relation to the ground (unloaded configuration). The wheel base is 2703 [mm] and the gauge at the front and the rear is 1900 [mm]. The rear tyres are of the 18.4R38 type with an unloaded radius of 789 [mm], a width of 467 [mm] and a total mass, tyre plus rim, of 273 [kg]. The front tyres are of the 16.9R26 type with a 621 [mm] radius, a width of 429 [mm] and a mass of 152 [kg]. The vertical stiffness of both tyres is  $2.0 \cdot 10^5$  [N/m].

1. Make a model of the tractor in ADAMS. Model the front swivel axle as a separate body with a negligible mass and mass moment of inertia. For the tyres, we will make use of the Delft-Tyre Model. The Tyre Property Files for front and rear tyres are `16r26_new.tir` and `18r38_new.tir` and the Road Data File `FlatRoad.rdf` describes a flat level road. These files can be found on the course website. Let this model drive straight ahead at a low initial speed ( $< 5$  [km/h]) in order to prove that your model is working.

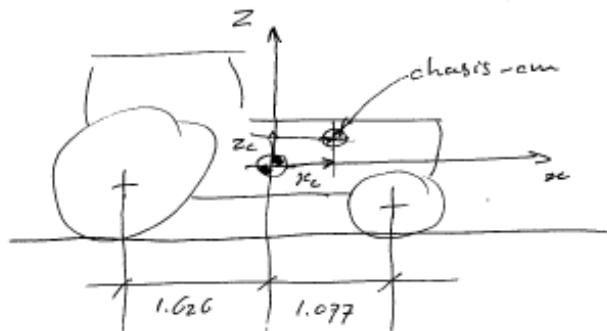


Dit is het geheel! De massa en massatraagheer van het chassis moet apart bepaald worden

$$m_{tot} = m_{ch} + 2 m_{vb} + 2 m_{ab}$$

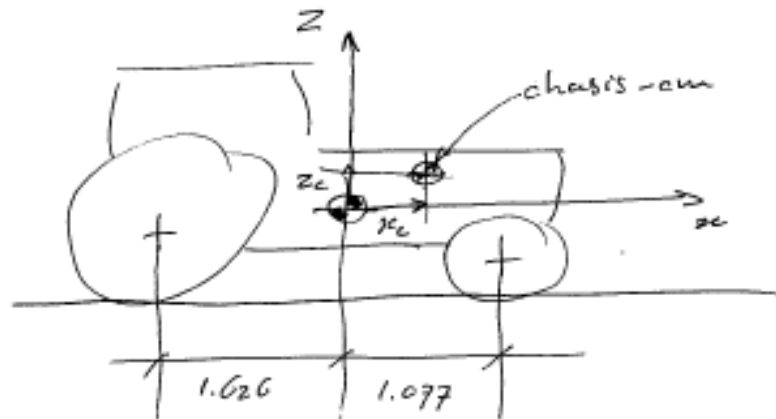


Plaats van het massamiddelpunt van het chassis volgt uit Statisch moment ~~in z~~, massaxaafstand



$$\begin{aligned} m_{chassis} &= m_{tot} - 2 m_{vb} - 2 m_{ab} \\ &= 5850 - 2 \cdot 152 - 2 \cdot 273 \\ &= \underline{\underline{5000}} \text{ kg} \end{aligned}$$

Plaats van het massamiddelpunt van het chassis volgt uit Statisch moment ~~in x-richting~~, massa x afstand



$$\begin{aligned}
 M_{\text{chassis}} &= M_{\text{tot}} - 2M_{\text{vb}} - 2M_{\text{ab}} \\
 &= 5850 - 2 \cdot 152 - 2 \cdot 273 \\
 &= \underline{\underline{5000 \text{ kg}}}
 \end{aligned}$$

Eerst Mass Moment of Inertia

Statisch moment ~~in~~ in x-richting:

begin in massamiddelpunt of geheel?

$$\begin{aligned}
 0 \cdot M_{\text{tot}} &= x_c \cdot 5000 + 2 \cdot 1.077 \cdot 152 - 2 \cdot 1.626 \cdot 273 \rightarrow \\
 x_c &= 0.1121 \text{ [m]}
 \end{aligned}$$

Statisch moment in z-richting:

begin in mm van het geheel

$$\begin{aligned}
 0 \cdot m_{\text{tot}} &= z_c \cdot 5000 - 2 \cdot (1.095 - 0.621) \cdot 152 - 2 \cdot (1.095 - 0.784) \cdot 273 \rightarrow \\
 z_c &= \underline{\underline{0.0046 \text{ [m]}}} \\
 &= 0.0046 \text{ [m]}
 \end{aligned}$$

en de y blijft hetzelfde!

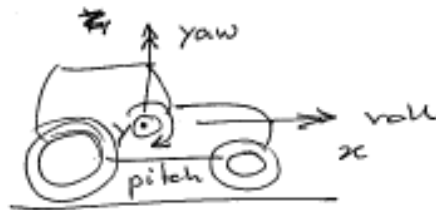
De traagheids ~~parameters~~ momenten volgen, bij benadering, uit de regel van Steiner voor vlakke structuren lichamen volgens: parallel axis theorem

$$I_A = I_{cm} + m \cdot a^2$$



$$\text{en } I_{\text{tot}} = \sum I$$

Wat is yaw-pitch en roll ?



$$I_{zz} = 9255 \text{ [kgm}^2\text{]}$$

$$I_{yy} = 7245 \text{ [kgm}^2\text{]}$$

$$I_{xx} = 5235 \text{ [kgm}^2\text{]}$$

$$I_{yy} = I_{yy}^c + m_{ch} \cdot (x_c^2 + z_c^2) + 2 \cdot m_{vb} \cdot (1.077^2 + (1.095 - 0.62)^2) + 2 \cdot m_{ab} \cdot (1.626^2 + (1.095 - 0.78)^2) \rightarrow$$

$$+ 2 \cdot \frac{1}{2} m_{vb} \cdot v_b^2 + 2 \cdot \frac{1}{2} m_{ab} \cdot v_a^2$$

$$I_{yy}^c = 9255 - 5000 \cdot (0.1121^2 + 0.0622^2) - 2 \cdot 152 \cdot (1.077^2 + 0.474^2) - 2 \cdot 273 \cdot (1.626^2 + 0.306^2) - 152 \cdot 0.61^2 - 273 \cdot 0.78^2$$

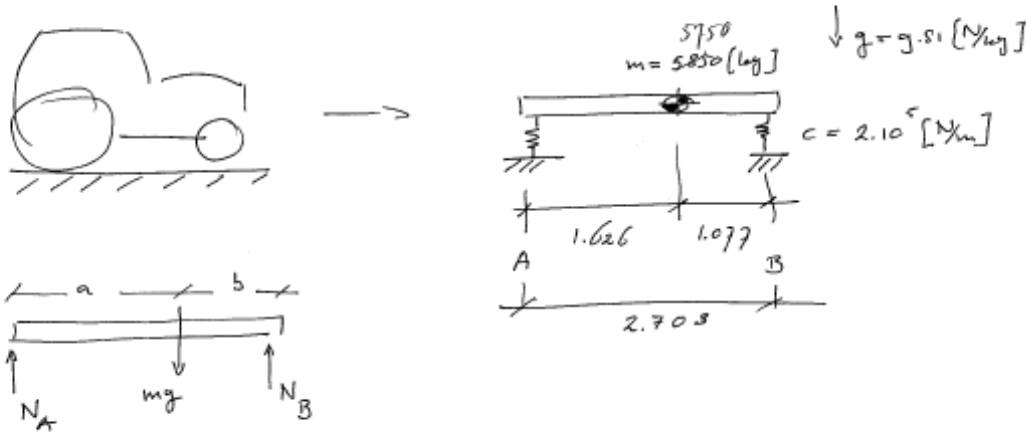
$$I_{yy}^c = 9255 - 82.2 - 420.9 - 1494.7 = 6957 \text{ [kgm}^2\text{]}$$

obusis
voor
achter
5018.7
etc.



2. Determine the vertical displacement and the pitch angle of the main body in the static equilibrium state. This static equilibrium can best be simulated by letting the tractor drive straight ahead at low speed from the initial state. This also gives the tyres a chance to spin up. Plot this vertical displacement and pitch angle as a function of time. Check these steady state values by a simple pencil-and-paper calculation.
3. Make a pencil-and-paper estimate of the eigenfrequency of the pitch movement and check this by means of a Linear analysis on your model.

# Statische Ervordichtstauer:



$$N_A + N_B = mg$$

$$a \cdot N_A = b \cdot N_B$$

$$\left(1 + \frac{a}{b}\right) N_A = mg$$

$$N_A = \frac{b}{a+b} mg = \frac{1.077}{2.703} \cdot 5850 \cdot 9.81 = 22475 \text{ N}$$

$$N_A = 22866 \text{ N}$$

$$N_B = 34522 \text{ N}$$

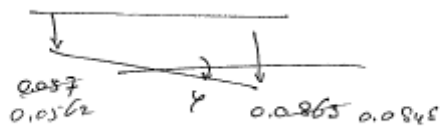
$$\Delta l_A = \frac{N_A}{2C} = \frac{22866}{4 \cdot 10^5} = 0.05716$$

$$\Delta l_B = \frac{N_B}{2C} = \frac{34522}{2 \cdot 10^5} = 0.086306$$

$$\varphi = \frac{\Delta l_B - \Delta l_A}{l} = \frac{0.086306 - 0.05716}{2.703} = 0.01078 = 0.62^\circ$$

$$V_{cm} = 0.05716 + 1.626 \cdot 0.01078 = 0.0747 \text{ cm}^3$$

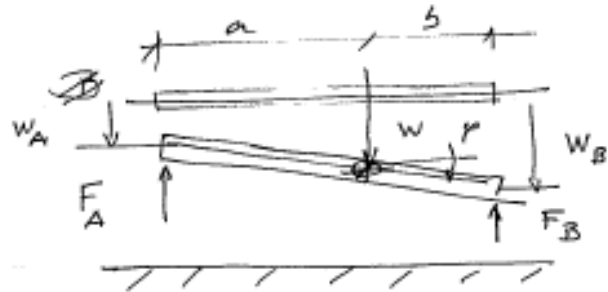
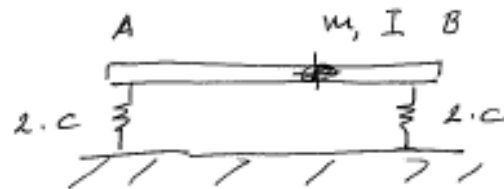
$$0.0562 + 1.626 \cdot 0.0106 = 0.0734$$



# Schätzung Eigenfreq. vd Pitch Bewegung!



balkenmodell



Newton-Euler:

$$-F_A + F_B = m \ddot{w}$$

$$F_A \cdot a + F_B \cdot b = I \cdot \ddot{\varphi}$$

$$F_A = 2c \cdot w_A$$

$$w_A = w - a \cdot \varphi$$

$$F_B = 2c \cdot w_B$$

$$w_B = w + b \cdot \varphi$$

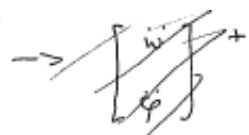
$$\begin{bmatrix} F_A \\ F_B \end{bmatrix} = \begin{bmatrix} 2c & 0 \\ 0 & 2c \end{bmatrix} \cdot 2c \begin{bmatrix} 1 & -a \\ 1 & b \end{bmatrix} \begin{bmatrix} w \\ \varphi \end{bmatrix}$$

$$\begin{bmatrix} m & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \ddot{w} \\ \ddot{\varphi} \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ a & -b \end{bmatrix} \begin{bmatrix} F_A \\ F_B \end{bmatrix}$$

DSD

$$\begin{bmatrix} m & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \ddot{w} \\ \ddot{\varphi} \end{bmatrix} + \begin{bmatrix} +1 & +1 \\ -a & +b \end{bmatrix} \begin{bmatrix} 1 & -a \\ 1 & b \end{bmatrix} \cdot 2c \cdot \begin{bmatrix} w \\ \varphi \end{bmatrix} = 0$$

$$\begin{bmatrix} m & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \ddot{w} \\ \ddot{\varphi} \end{bmatrix} + 2c \cdot \begin{bmatrix} 2c & b-a \\ b-a & a^2+b^2 \end{bmatrix} \begin{bmatrix} w \\ \varphi \end{bmatrix} = 0$$



Stel  $w = w_0 \cdot e^{i\omega t}$   
 $\varphi = \varphi_0 \cdot e^{i\omega t}$

$$\begin{bmatrix} w \\ \varphi \end{bmatrix} = \begin{bmatrix} w_0 \\ \varphi_0 \end{bmatrix} \cdot e^{i\omega t}$$

$$\left\{ -\omega^2 \begin{bmatrix} m & 0 \\ 0 & I \end{bmatrix} + c \cdot \begin{bmatrix} 4 & b-a \\ b-a & a^2+b^2 \end{bmatrix} \right\} \begin{bmatrix} w_0 \\ \varphi_0 \end{bmatrix} = 0$$

$e^{i\omega t} = \cos(\omega t) + i \sin(\omega t)$

$$\text{Det} \left( \lambda^2 \begin{bmatrix} m & 0 \\ 0 & I \end{bmatrix} + c \begin{bmatrix} 4 & b-a \\ b-a & a^2+b^2 \end{bmatrix} \right) = 0$$

Of je kan zeggen dat dit een eigenwaarde probleem volgens

$$A \underline{x} = \lambda \cdot \underline{x} \quad \rightarrow \quad (A - \lambda I) \underline{x} = 0$$

$$A = c \begin{bmatrix} 1/m & 0 \\ 0 & 1/I \end{bmatrix} \cdot \begin{bmatrix} 4 & b-a \\ b-a & a^2+b^2 \end{bmatrix} \quad \text{en} \quad \lambda = \omega^2$$

oplossen met bv Matlab  $m = 5580$   
 $I = 7245$   
 $a = 1.626$   $b = 1.077$   
 $c = 2 \cdot 10^5$

$$\rightarrow \omega_1 = 11.1 \text{ [rad/s]} \quad \begin{bmatrix} w_0 \\ \varphi_0 \end{bmatrix} = \begin{bmatrix} -0.92 \\ 0.40 \end{bmatrix}$$

(1.77 Hertz)

$$\omega_2 = 14.94 \text{ [rad/s]} \quad \begin{bmatrix} w_0 \\ \varphi_0 \end{bmatrix} = \begin{bmatrix} -0.33 \\ 0.94 \end{bmatrix}$$

(2.38 Hertz)

ADAMS ( 1.76 en 2.38 )

of toernummer  $\rightarrow 21/12 = 1.75$  Hertz

Subsequently, we would like to submit the tractor to the moose test. This test, comprehensively described in ISO/TR 3888 [3], includes an overtaking manoeuvre or a fast swerving manoeuvre around an obstacle within a track marked by pylons. You can find the description of the track as an appendix. The easiest way to steer the front wheels is by letting them rotate sine-like. The amplitude and frequency of this motion for a desired vehicle track can be estimated by remembering that the vehicle's yaw-rate  $\dot{\alpha}$  is approximately proportional to the steering position of the front wheels  $\varphi$  and the vehicle's forward speed  $v$ , and inversely proportional to the wheelbase  $l$ , summing up:  $\dot{\alpha} \approx (v/l)\varphi$ . In general the tractor will slow down as it moves along, therefore to maintain constant speed along the track you should construct a first order control system pushing and pulling the tractor forward. Do not try to drive the wheels, but let the control system work on the centre of mass of the tractor body.

4. Determine, by iterative analysis, the maximum speed  $v_{max}$  at which the tractor will safely pass the moose test. For this speed, plot the tractor's track in the horizontal plane. What is your advice on the maximal allowing speed of tractors on the public road?
5. Finally plot, as a function of time, the three tyre forces: normal, lateral and longitudinal, that the right front and rear tires are subjected to during the manoeuvre at maximum speed and compare this with the (average) tyre forces in the static equilibrium position.

## 4.2 ISO-Spurwechsel

Der in ISO/TR 3888 [87] definierte doppelte Spurwechsel, auch ISO-Spurwechsel oder -Wedeltest genannt, stellt eine sehr realitätsnahe Closed-Loop-Fahraufgabe für das Zusammenwirken Fahrer-Fahrzeug-Verkehr dar. Er simuliert einen Überholvorgang bzw. das schnelle Ausweichen vor einem Hindernis innerhalb einer vorgegebenen Fahrspur, Bild 4.24. Pkw, für die dieser Versuch konzipiert wurde, können die Spurgassen mit bis über 100 km/h durchfahren.

Der vorhandene Fahrereinfluß und dessen teilweise Rückwirkungen auf die ermittelten Größen haben bisher noch keine standardisierten Kennwerte ergeben, weshalb dieses Fahrmanöver nur in einem "Technischen Report" (TR) [ISO/TR 3888, 87] beschrieben ist. Trotzdem wird er als etabliertes Verfahren der Fahrverhaltensuntersuchungen gesehen, entweder als vergleichender Fahrleistungstest - Bestimmung der maximalen Durchfahrungs geschwindigkeit - oder zur subjektiven Beurteilung [Zomotor, 137].

In jüngerer Zeit, in der die Schnittstelle Mensch-Fahrzeug vermehrt erforscht wird, wird der ISO-Spurwechsel zur Ermittlung des Fahrer-Fahrzeug-System beschreibender vergleichender Kenngrößen, die sowohl bezüglich des objektiven Fahrzeugverhalten als auch des subjektiven Fahrereindrucks korrelieren, in Fahrversuchen sowie Simulationsrechnungen mit Fahrermodellen sehr häufig angewendet [Dibbern et al., 13 u. 14; Laermann et al., 64; Kudritzki, 61; Riedel et al., 102 u. 103], auch bei Lkw und Lastzugkombinationen [Käppler et al., 56; Köfalvi, 58; Elink Schuurman et al., 22]. Allgemein anerkannte Kennwerte wurden daraus bisher noch nicht genormt.

### 4.2.1 Versuchsdurchführung

Der doppelte Fahrspurwechsel wurde entsprechend den Maßen in ISO/TR 3888 [87] durch Pylone markiert, Bild 4.24.

Der Versuch wurde mit gestufter, jeweils konstanter Fahrgeschwindigkeit ab 20 oder 30 km/h bis zur maximalen Fahrzeuggeschwindigkeit - außer LKW - mit mindestens 3 Durchgängen pro Parameter gefahren. Die Verläufe einiger Meßgrößen zeigt beispielsweise Bild 4.24.

In der Regel wurden die Versuche mit Fahrer W durchgeführt. Der Fahrereinfluß wird anhand der Fahrzeuge AS4 und LKW dargestellt.

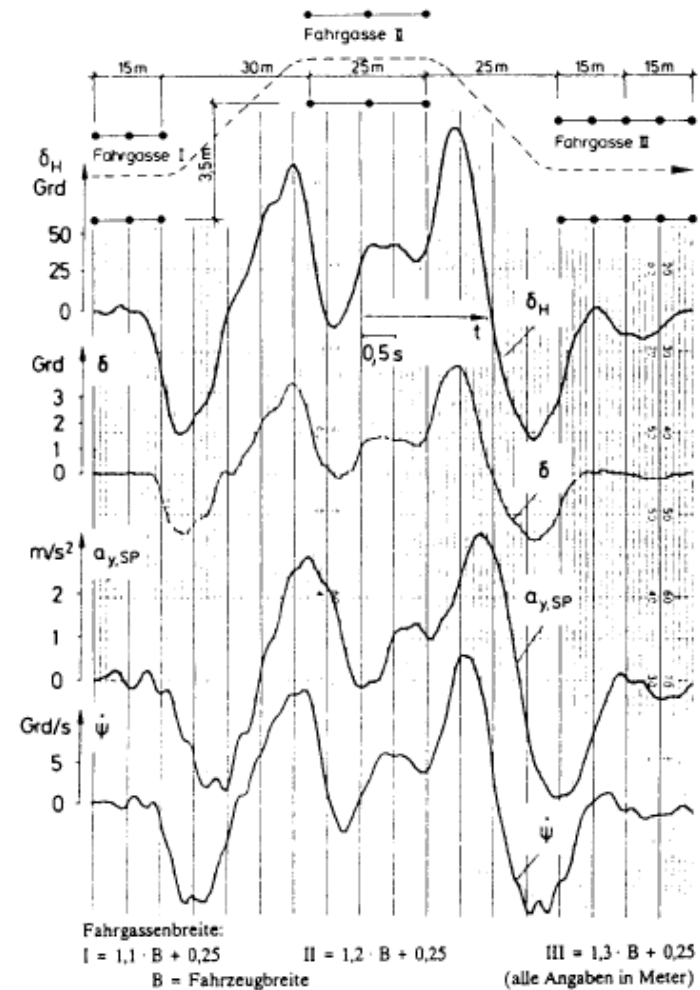


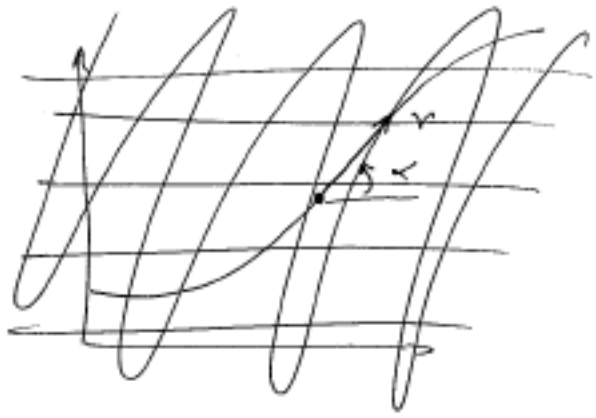
Bild 4.24: Meßgrößenverläufe beim ISO-Spurwechsel AS2,  $v = 50$  km/h [Betzler, 4]

Zur Aussage der vom Fahrer aufzuwendenden Lenkaktivität bei Einhaltung der Fahrgassen wird das "Lenkleistungsmaß" [Betzler, 4] herangezogen. Das Lenkleistungsmaß ist der über die Fahrstrecke gemittelte Lenkaufwand, s. Kap. 3.1.1, dividiert durch die Fahrzeit,

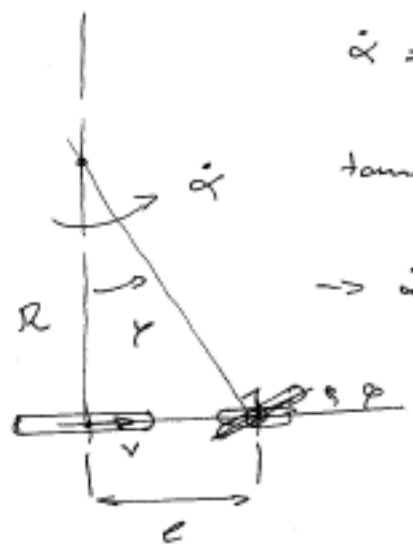
$$P_{3,H} = \frac{1}{t} \cdot \delta_{HK}$$

5-4

Hoe moet je een baan volgen?



"bicycle" model



$$\dot{\alpha} = \frac{v}{R}$$

$$\tan \varphi = \frac{l}{R}$$

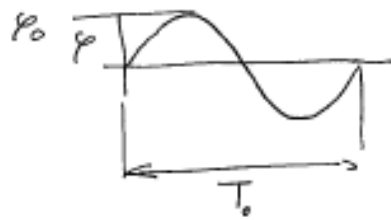
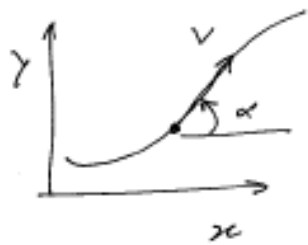
$$\rightarrow \dot{\alpha} = \frac{v}{l} \cdot \tan \varphi$$

$$\dot{\alpha} = \frac{v}{l} \cdot \varphi$$

$$\alpha = \int \frac{v}{l} \cdot \varphi \cdot dt$$

Vaste Stuurstand  
lineaire yaw-hoek  
verelies v/den l

Wella baan volg je bij sinusvarning Sturen ??



$$\varphi = \varphi_0 \cdot \sin\left(2\pi \frac{t}{T_0}\right)$$

$$\text{met } \omega_0 = \frac{2\pi}{T_0}$$

$$\varphi = \varphi_0 \cdot \sin(\omega_0 t)$$

$$\dot{\alpha} = \frac{v}{l} \cdot \varphi$$

$$\dot{\alpha} = \frac{v}{l} \cdot \varphi_0 \cdot \sin(\omega_0 t)$$

→

$$\alpha = \frac{v}{l} \cdot \varphi_0 \cdot \frac{1}{\omega_0} (1 - \cos(\omega_0 t)) \quad \text{met } t=0, \alpha=0$$

Wat is de baan in  $x$  en  $y$  componenten nu?

$$\frac{dx}{dt} = v \cdot \cos \alpha \quad \frac{dy}{dt} = v \cdot \sin \alpha$$

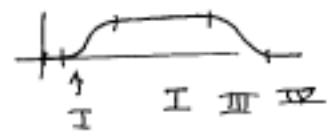
Voor kleine  $\alpha$ :

$$\dot{x} \approx v \quad \dot{y} \approx v \cdot \alpha = \frac{v^2}{l} \cdot \varphi_0 \cdot \frac{1}{\omega_0} (1 - \cos(\omega_0 t))$$

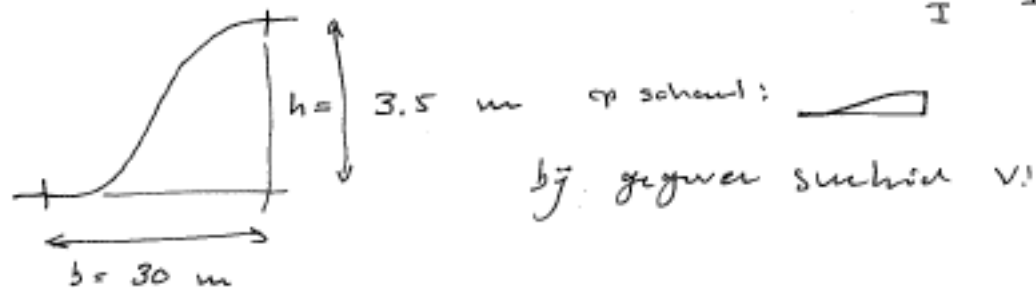
$$x = v \cdot t \quad y = \frac{v^2 \varphi_0}{\omega_0 l} \left( t - \frac{1}{\omega_0} \cdot \sin(\omega_0 t) \right)$$



$$x = v \cdot t \quad y = \frac{v^2 \varphi_0}{\omega_0 l} \left( t - \frac{1}{\omega_0} \cdot \sin(\omega_0 t) \right)$$

De Eland test baan is zoiets als 

Deel I



$$b = v \cdot T_0 \rightarrow T_0 = \frac{b}{v} \quad \omega_0 = \frac{2\pi}{T_0} \cdot v$$

$$h = \frac{v^2 \varphi_0}{2\pi \left(\frac{v}{b}\right) \cdot l} \cdot \frac{b}{v} = \frac{\varphi_0 \cdot b^2}{2\pi l} \rightarrow \varphi_0 = 2\pi \cdot \frac{h \cdot l}{b^2}$$

Voor het eerste deel wordt dit dan:

$$l = 1.626 + 1.077 = 2.703 \text{ [m]} \quad \text{b.v. } v = 50 \text{ [km/h]}$$

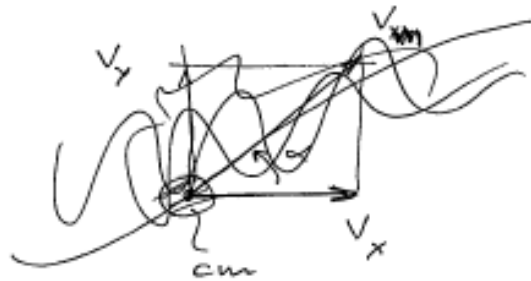
$$\varphi_0 = 2\pi \cdot \frac{3.5 \cdot 2.703}{30^2} = 0.066 \text{ [rad]} = 3.8^\circ \quad (\text{klein hé!})$$

zie streekt; kelept  $\nabla$

$$\omega_0 = 2\pi \frac{50/3.6}{30} = 2.9 \text{ rad/s} \quad T_0 = 2.16 \text{ [sec]} \quad (\text{kun wel hé})$$

En hoe blijft je New met constante snelheid rijden??

~~Voeg~~ Deur et Trek de Tractor evalue volgens



$V_0$  is de gewenste Snelheid  
 $V$  is de actuele Snelheid  
 langs de baan

$$F = m \ddot{x}$$

$$F = m \dot{v}$$

$$c(V_0 - V) = m \dot{v}$$

Alv een kracht van de de snelheid veranderen.

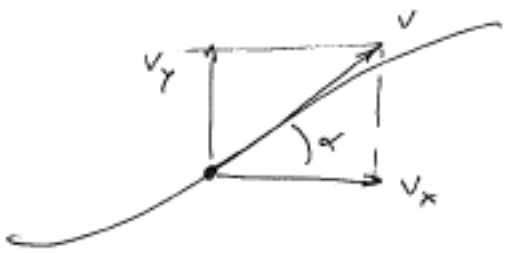
$$\text{neem } F = c(V_0 - V) \quad c(V_0 - V) \quad v \rightarrow F = \underline{\underline{\max}}$$

$$c(V_0 - V) = m \dot{v} \quad \dot{v} + \frac{c}{m} v = \frac{c}{m} V_0$$

$\sqrt{\frac{m}{c}}$  is de tijdconstante  $\tau$



10



$$v = \sqrt{v_x^2 + v_y^2}$$

$v_0 =$  gewenste snelheid langs baan

$$C \cdot (v_0 \cdot \cos \alpha - v_x) = m \dot{v}_x$$

$$C \cdot (v_0 \cdot \sin \alpha - v_y) = m \dot{v}_y$$

met  $\cos \alpha = v_x/v$   
 $\sin \alpha = v_y/v$

$$F_x = C \cdot v_x \cdot (v_0/v - 1)$$

$$F_y = C \cdot v_y \cdot (v_0/v - 1)$$

nb als  $v \rightarrow 0 \rightarrow v = 1e-3$

FIN