wb1413
Multibody Dynamics B

Spring Term 2013, Thu 15:45-17:30, room CT-CZ G, 4 ECTS credits.

Final Project 2: Dynamic Analysis of the Motion of a Bicycle

A bicycle is modelled by four rigid bodies: the rear wheel, the frame with a rigid rider attached, the fork and the handle bars assembly being the front frame, and the front wheel. The geometry of the bicycle will be described in a space fixed Cartesian coordinate system O–xyz. The rear wheel and the front wheel are in contact with a horizontal flat plane such that we assume no slip in both the longitudinal and the lateral direction of the wheel. In the initial geometry the bicycle is in the upright position with the handle bars straight ahead pointing in the x–direction. The rear wheel is connected to the frame by a revolute joint with the principal hinge axes in the y–direction. The (rear) frame is connected to the front frame by a revolute joint with the principle hinge axis in the xz–plane. The front wheel is connected to the front frame by a revolute joint with the principle axis in the y–direction. The rear wheel and front wheel are identical with a diameter of 0.7 m, a mass of 2 kg, and principal moments of inertia $\mathbf{J} = (J_{xx}, J_{yy}, J_{zz}) = (0.12, 0.24, 0.12)$ kgm^2. The wheel base is 1.2 m. The center of mass of the frame with a rigid rider attached is located at $(0.55, 0, 0.5)$ m, with a mass of 96 kg and has principal moments of inertia $\mathbf{J} = (16.8, 24, 5.2)$ kgm^2. The steering hinge is located at $(0.9, 0, 0.9)$ m pointing in the $(0.36, 0, -0.90)$ direction. The center of mass of the front frame is located at $(1.0, 0, 0.7)$ m with a mass of 1 kg and zero moments of inertia. Assume a gravity field strength of $g = 10$ N/kg.

a. Derive the equations of motion for the bicycle according to the handout 'notes on the bicycle project'.

b. Simulate the motion of the bicycle for 10 seconds. The initial conditions are an upright position with the steering straight ahead, an initial forward speed of 4.2 m/s, an initial lean angular velocity for the whole assembly of -0.5 rad/s, and zero steering velocity. Give a clear representation of the motion in a number of graphs of your own choose. The answers I got are presented in the graphs in Figure 1 on page 2.

c. Use your simulation to address one of the following questions or own of your own:

   (1) A folklore in bicycle science is that he wheel reaction forces in the contact point are in the plane of the wheel. True or not?

   (2) Below 3.5 m/s this bicycle is unstable. Is it stable for all speeds above?

   (3) Does the bicycle have any stable motions besides upright straight ahead?

   (4) By changing mass and geometry parameters, what's the slowest speed for which you can find a stable bicycle?

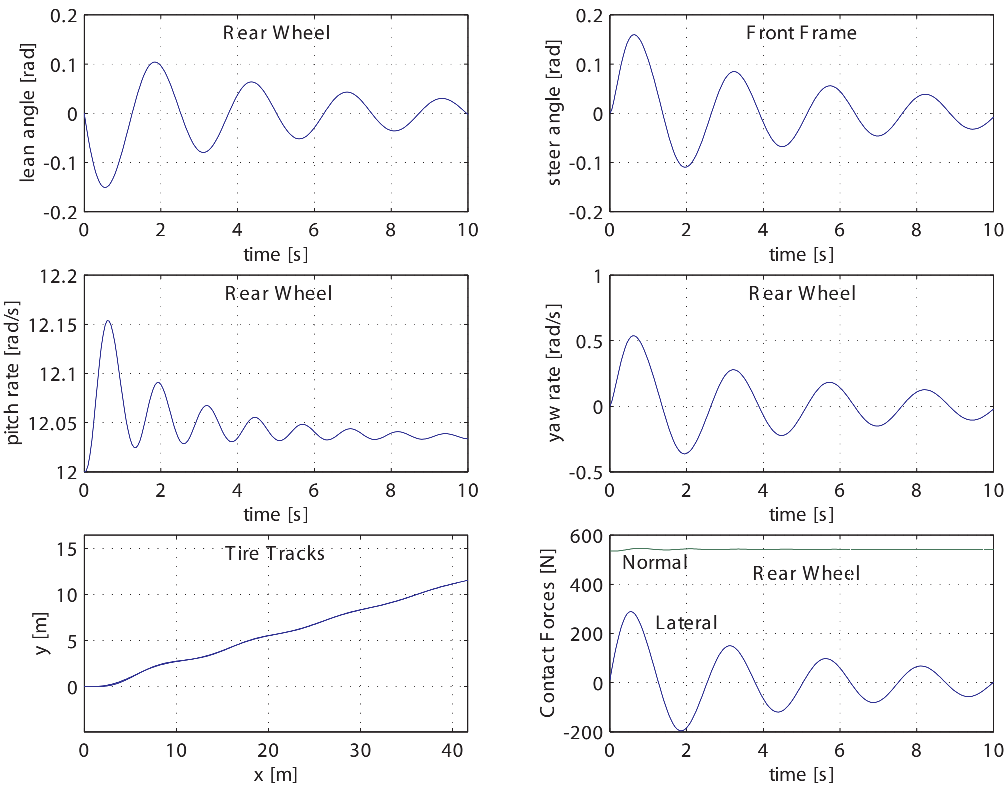

Figure 1: Time series of the simulation of the finalproject bicycle with the initial conditions: an upright position with the steering straight ahead, an initial forward speed of 4.2 m/s, an initial lean angular velocity for the whole assembly of -0.5 rad/s, and zero steering velocity.