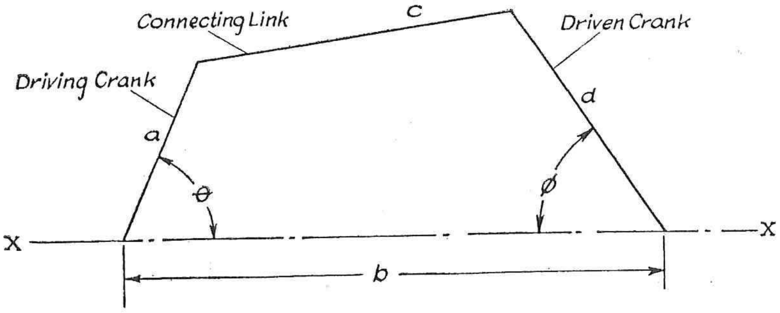
Part I—Analysis of Single 4-Bar Linkage

By Guy J. Talbourdet

Research Division

United Shoe Machinery Corporation



Calculate the values of the following factors from the geometry of the linkage:

$$K = a^2 + b^2 - c^2 + d^2$$

$$A = a \, Sin \, \theta$$

$$B = a^2 + b^2 - 2ab \, Cos \, \theta$$

$$D = K - 2ab \, Cos \, \theta$$

$$S = \sqrt{4d^2B - D^2}$$

θ = Angular displacement of driving crank.

ϕ = Angular displacement of driven crank.

$\frac{d\theta}{dt}$ and $\frac{d^2\theta}{dt^2}$ = Angular velocity and acceleration respectively of the driving crank.

$\frac{d\phi}{dt}$ and $\frac{d^2\phi}{dt^2}$ = Angular velocity and acceleration respectively of the driven crank.

Then:

$$\phi = Tan^{-1}\frac{A}{b - a \, Cos \, \theta} + Cos^{-1}\frac{D}{2d\sqrt{B}}$$

And:

$$\frac{d\phi}{dt} = \frac{d\theta}{dt}\left[\frac{a}{B}(b \, Cos \, \theta - a) - \frac{Ab}{S}\left(2 - \frac{D}{B}\right)\right]$$

And:

$$\frac{d^2\phi}{dt^2} = \frac{d^2\theta}{dt^2}\left[\frac{a}{B}(b \, Cos \, \theta - a) - \frac{Ab}{S}\left(2 - \frac{D}{B}\right)\right]$$

$$+ \left(\frac{d\theta}{dt}\right)^2\left[\frac{2A^2b^2}{BS}\left(1 - \frac{D}{B}\right) + \left(2 - \frac{D}{B}\right)\left(\frac{2A^2b^2(2d^2 - D)}{S^3} - \frac{ab \, Cos \, \theta}{S}\right)\right.$$

$$\left. - \frac{Ab}{B}\left(1 + \frac{2a(b \, Cos \, \theta - a)}{B}\right)\right]$$

When the motion of the driving crank is uniform, $\frac{d\theta}{dt} = \omega$, and $\frac{d^2\theta}{dt^2} = 0$, in which case we have:

$$\frac{d\phi}{dt} = \omega\left[\frac{a}{b}(b \, Cos \, \theta - a) - \frac{Ab}{S}\left(2 - \frac{D}{B}\right)\right]$$

And:

$$\frac{d^2\phi}{dt^2} = \omega^2\left[\frac{2A^2b^2}{BS}\left(1 - \frac{D}{B}\right) + \left(2 - \frac{D}{B}\right)\left(\frac{2A^2b^2(2d^2 - D)}{S^3} - \frac{ab \, Cos \, \theta}{S}\right)\right.$$

$$\left. - \frac{Ab}{B}\left(1 + \frac{2a(b \, Cos \, \theta - a)}{B}\right)\right]$$