

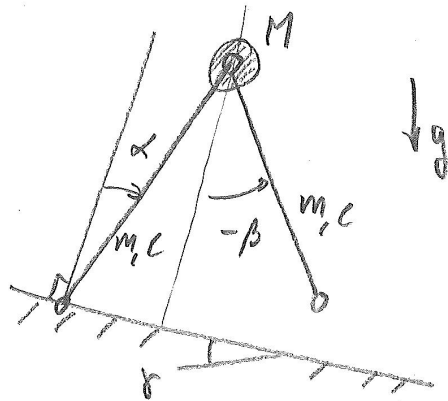
# wb1413

## Multibody Dynamics B

Spring Term 2013, Thu 15:45-17:30, room EWI-CZ C), 4 ECTS credits.

### Homework assignment 5

A simplified 2D model of a bipedal walking machine consists of two slender legs hinged at the hip. We model the system where one leg is on the ground and stays on the ground, the stance leg, and the other leg is swaying, the swing leg. The ground is tilted clockwise by an angle  $\gamma$  with respect to the downward gravity  $g$ . For generalized coordinates we take the angle  $\alpha$  of the stance leg with respect to the outer normal to the ground and the angle  $\beta$  of the swing leg also with respect to the outer normal to the ground, where clockwise is positive. Both legs are equal with length  $l$  and mass  $m$  (distributed evenly along the length of the leg). An extra point mass  $M$  is located at the hip.



- Derive the equations of motion in terms of the generalized coordinates  $q_i = (\alpha, \beta)$  by means of the Lagrange equations of motion. Write these equations in terms of a mass matrix times accelerations equals forces, as in  $\bar{M}_{ij}\ddot{q}_j = \bar{f}_i$
- Derive the equations of motion by using the principle of Virtual Power, the principle of D'Alembert and the transformation of the coordinates of the cm's of the bodies  $x_i$  in terms of the independent generalized coordinates  $q_j$  as in  $x_i = F_i(q_j)$  (sometimes referred to as TMT-method, where T stands for the transformation and M for the global mass matrix).
- Compare the results from (a) and (b).
- Discuss for large systems, where large means many bodies and many degrees of freedom, the pros and cons of the two different methods.

In deriving the equations of motion you may want to use symbolic software like Maple or the symbolic toolbox from Matlab (which is again Maple). This can reduce the number of errors.

**Bonus Question:** We like to investigate when and how fast the swing leg will hit the ground. What is the contact condition expressed in terms of the generalized coordinates and the system parameters? And derive an expression for the impact velocity (perpendicular to the ground) in terms of the generalized coordinates, the generalized velocities and the system parameters.