wb1413
Multibody Dynamics B

Spring Term 2013, Thu 15:45-17:30), 4 ECTS credits.

Homework assignment 6

Determine the motion of the double pendulum from assignment 1 by numerical integration of the equations of motion expressed in independent generalized coordinates as derived in assignment 5. The initial conditions are both bars vertically up at zero speed. We assume a gravitational field operating in the *horizontal* direction with a field strength of $g = 9.81$ N/kg. We want to determine the angle, in radians, of both bars with respect to the horizontal axis after 3.0 seconds with a maximal absolute error of 10^{-6} rad.

a. Determine the accordingly maximum step size for the following numerical integration methods:

   (1) Euler.

   (2) Heun.

   (3) Runge-Kutta 3$^{\text{rd}}$ order.

   (4) Classical Runge-Kutta 4$^{\text{th}}$ order.

   Use an error estimate method based on the method-inherent truncation error and the round-off error due to the finite precision as explained in the course. Plot for each angle in one figure the $\log_{10}$(estimated error) versus the $\log_{10}$(step size) for all four methods. In some cases it may not be feasible to find such a maximum step size. Either the method used is unstable or the step size becomes so small that it is impractical to reach the end point at $t = 3.0$ seconds. If so, please do not spend to much time on that and state your case clearly!

b. Finally use the three ODE solvers `ode23`, `ode45`, and `ode113` from Matlab. Set the error tolerance `RelTol` and `AbsTol` such that you get the same final accuracy (global error!) as above and integrate the equations of motion for 3.0 seconds.

   (1) Compare the angles of both bars at $t = 3.0$ sec with the results from above.

   (2) Determine the average step size and the total number of function evaluations (calls to the differential equation f(t,y)) as used in the three methods. Do these agree with your previous results?

Please tabulate all your results (with enough digits [for instance 15] to show convergence) and discuss.

Bonus Question: What happens when you start a simulation with initial conditions very close to those from above? Do the solutions stay close together? How do we call such a behaviour?