

# wb1413

## Multibody Dynamics B

Spring Term 2013, Thu 15:45-17:30, room CT-CZ G, 4 ECTS credits.

### Homework assignment 8

In deriving the equations of motion for the constraint multibody system we used the principle of virtual power together with D'Alembert's principle to include inertia forces. The constraints were added to the virtual power expression by means of the Lagrangian-multiplier method.

Now read Lanczos [1], Chapter II, Section 5, *Auxiliary conditions. The Lagrangian  $\lambda$ -method*, and Chapter III, Section 1, *The principle of virtual work for reversible displacements*, and address the following items:

- a. Use the same type of reasoning as shown on pages 45–46 to come to the equations of motion for a constraint multibody system, in general.
- b. Where did we use Postulate *A* from page 76?

Note that Lanczos speaks of virtual work  $\delta W = f_i \delta x_i$ , whereas we have used the concept of virtual power  $\delta P = f_i \delta \dot{x}_i$ . Either approach will work, the difference is just a 'dot', of course my preference lies in the virtual power approach.

- c. Illustrate your reasoning by means of an example like the eqn's of motion for a 2D point mass confined to move on a slope or a circle, under the action of gravity. Solve these for the unknown accelerations and the Lagrange multiplier in a general configuration. Check the correctness of the results and discuss the physical interpretation of the Lagrange multiplier.

**Bonus Question:** Determine the determinant of the system matrix  $[\mathbf{M} \mathbf{D}^T; \mathbf{D} \mathbf{0}]$  of the double pendulum from homework assignment 2. Is this always non-zero?

### References

- [1] Lanczos, C., *The Variational Principles of Mechanics*, University of Toronto Press, Toronto, 1949.