wb1413
Multibody Dynamics B

Spring Term 2013, Thu 15:45-17:30, room CT-CZ G, 4 ECTS credits.

Homework assignment 9

Consider the rotary motion of a torque-free satellite in deep space (no gravity, no drag). At a certain instant in time which we call $t = 0$ the orientation of the body fixed coordinate system expressed in the global xyz reference system is given by $\mathbf{e}'_x = (0.192, 0.744, 0.64)$, $\mathbf{e}'_y = (-0.856, -0.192, 0.48)$, and $\mathbf{e}'_z = (0.48, -0.64, 0.6)$.

a. Determine for $t = 0$ the rotation matrix $\mathbf{R}$ which transforms the body fixed coordinates $\mathbf{x}'$ into the reference system coordinates $\mathbf{x}$ as in $\mathbf{x} = \mathbf{R}\mathbf{x}'$.

b. Determine for $t = 0$ from this $\mathbf{R}$ the associated Euler angles (zxz) ϕ, θ, and ψ.

The initial angular velocity at $t = 0$ expressed in the global reference coordinate system are given by $\boldsymbol{\omega} = (1.92048, 7.43936, 6.4006)$ rad/s.

c. Determine for $t = 0$ the angular velocities $\boldsymbol{\omega}'$ expressed in the body fixed coordinate system.

d. Determine for $t = 0$ the rate of change of the Euler angles: $(\dot{\phi}, \dot{\theta}, \dot{\psi})$.

The satellite is modelled by a rectangular box with mass $m = 100$ kg and dimensions $l_x = 0.4$, $l_y = 1.2$ and $l_z = 0.3$ in the body fixed coordinate system.

e. Determine the mass moment of inertia matrix $\mathbf{J}'$ in the body fixed coordinate system.

Next we want to show the motion of the satellite.

f. Write down the equations of motions (Euler equations) and the state equations $\dot{\mathbf{y}} = \mathbf{f}(\mathbf{y})$. Use as state variables $\mathbf{y}$ the Euler angles (ϕ, θ, ψ) together with the angular velocities $\boldsymbol{\omega}'$ expressed in the body fixed coordinate system.

g. Show the motion of the satellite as a function of time by numerical integration of state equations for 40 seconds. Plot the Euler angles (ϕ, θ, ψ) as a function of time. Make a 3D plot of the trajectory of point $p = (l_x/2, 0, 0)$ of the body. Please discuss your results. What happens around $\theta = \pm k\pi$, $k = 0 \ldots n$?

Redo the items b,c,d,f and g but now use the Cardan angles (α, β, γ) to parameterize the rotation matrix $\mathbf{R}$. The recipe for Cardan angles is first a rotation about the z-axis by an angle α, next rotate about the rotated y-axis by an angle β, and finally rotate about the rotated x-axis by an angle γ. Note, for item g one should read: What happens around $\beta = \pi/2 \pm k\pi$, $k = 0 \ldots n$?

Bonus Question: What would be an easy check on your time series results?