

Answers to HW Set #1

- 1.2 ISO standard A4 paper.
 $297/210 = 1.4143 \approx \sqrt{2}$.
 Folding A4 paper in half preserves its aspect ratio.
 (A4 is A3 folded in half, A5 is A4 folded in half.)

- 1.3. How many terms in the continued fraction?

```
goldfract(24)
err = 7.9e-11.
for n = 37:39
    goldfract(n)
end
err = [-eps, eps, 0]
```

- 1.4 Use backslash to compute coefficients.

Numerically:

```
phi = (1 + sqrt(5))/2;
A = [ 1 1; phi 1-phi]
b = [ 1; 1]
c = A\b
= [0.7236; 0.2764]
```

Symbolically:

```
syms phi
A = [ 1 1; phi 1-phi]
b = [ 1; 1]
c = A\b
= 1/(2*phi-1)*[phi; 1-phi]
```

- 1.8. Repeat $X = A*X$.

$$A^n = \begin{bmatrix} f_n & f_{n-1} \\ f_{n-1} & f_{n-2} \end{bmatrix}$$

A^{1475} does not overflow, but A^{1476} does.

- 1.19. 4-by-4 magic square is singular.

```
A = magic(4)
null(A)
null(A,'r')
null(sym(A))
rref(A)
```

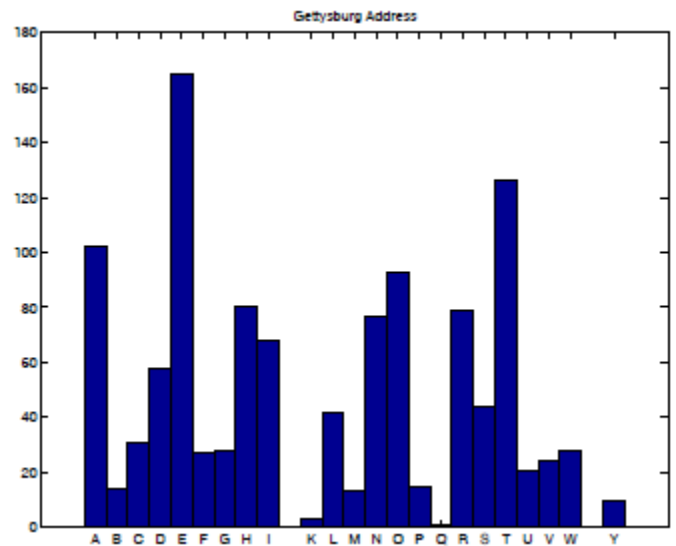
These four statements are four different ways to discover that the 4-by-4 magic square is singular and that the linear combination of its columns $A(:,1) + 3*A(:,2) - 3*A(:,3) - A(:,4)$ is the zero vector.

- 1.20. Permuting the rows and columns of a magic square preserves the row sums, the column sums, and the matrix rank, but not the diagonal sums. (Symmetric permutations, $A = A(p,p)$, also preserves the diagonal sums.)

- 1.25 Analyze the text of the Gettysburg address.

```
nchar = 1463
uniq = ,-.BFGINTWabcdefghijklmnopqrstuvw
nuniq = 35
nblank = 255
nperiod = 10
ncomma = 19
ndash = 7
most = E
missing = JXZ
```

Don't forget to re-label the x-axis with the alphabet!



- 1.35 What does each of these programs do?
`x = 1; while 1+x > 1, x = x/2, pause(.02), end`
 Exhibits roundoff Exhibits roundoff. The program produces 53 lines of output. The last two values of x are eps and $\text{eps}/2$.
`x = 1; while x+x > x, x = 2*x, pause(.02), end`
 Exhibits overflow. The program produces 1024 lines of output. The last two values of x are $2^{1023} \approx \text{realmax}/2$ and Inf .
`x = 1; while x+x > x, x = x/2, pause(.02), end`
 Exhibits underflow. The program produces 1075 lines of output. The last two values of x are $\text{eps} * \text{realmin}$ and 0 . (On computers without subnormal floating point numbers, this program would produce 1023 lines of output. The last two values would be realmin and 0).

- 1.38. Quadratic formula.
 With $b = -10^8$, you get few if any accurate digits out of $-b - \sqrt{b^2 - 4}$ unless you compute the intermediate results to very high precision. In Matlab there is no trouble with
`x1 = (10^8 + sqrt(10^6-4))/2 = 1.0000e+008`
 But
`x2 = (10^8 - sqrt(10^16-4))/2 = 7.4506e-009`
 when it should be $1.0000e-008$; you subtract two big numbers around eps apart. Clearly
`x2 = 1/x1`
 works well in this situation.
 Alternatively, `roots([1 -10^8 1])` gives two good roots, $1.0000e+008$ and $1.0000e-008$.

- 1.39. Power series for computing $\sin x$.
 The loop test in `powersin` terminates when $s+t == t$, that is when t is so small compared to s that the computed value of $s+t$ is equal to s . Change the first line of `powersin.m` to
`function [s,tmax,cnt] = powersin(x)`
 Insert these lines before the start of the while loop.
`tmax = abs(t);`
`cnt = 0;`
 Insert these lines in the loop.
`tmax = max(tmax,abs(t));`
`cnt = cnt+1;`
 Here is a table of x , $\sin(x) - \text{powersin}(x)$, $tmax$, and cnt .
- | $\pi/2$ | $11\pi/2$ | $21\pi/2$ | $31\pi/2$ |
|-------------|--------------|--------------|--------------|
| 2.2204e-016 | -2.1287e-010 | -1.3324e-004 | -5.8210e+003 |
| 1.5708e+000 | 3.0665e+006 | 1.4673e+013 | 7.9890e+019 |
| 11 | 37 | 60 | 78 |
- We see that when the largest term is about 10^p , the computed value loses about p digits. The power series is OK for x less than $\pi/2$. But as x increases, the power series requires more work and yields less accuracy. Again the problem of subtracting two big numbers less than eps apart.