Answers to HW Set #2

2.1. Buying fruit.

```
A = [3 12 1; 12 0 2; 0 2 3]
b = [2.36; 5.26; 2.77]
format bank
x = A\b
x =
0.29
0.05
0.89
```

2.2. Reduced row echelon form of the magic square of order six.

```
rref(magic(6))
```

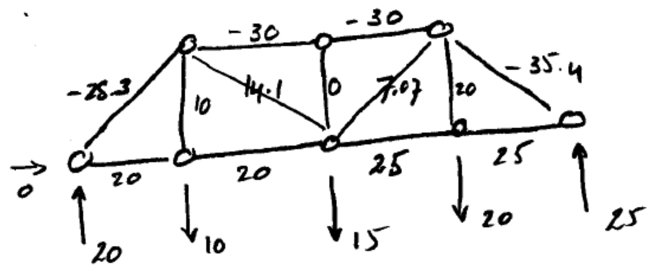
1	0	0	0	0	-2
0	1	0	0	0	-2
0	0	1	0	0	1
0	0	0	1	0	2
0	0	0	0	1	2
0	0	0	0	0	0

2.3 I would add unknown support forces at ① and ⑧ plus node equilibrium at ① and ⑧

① $H_1 + \alpha f_1 + f_2 = 0$

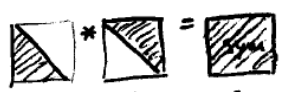
$V_1 + \alpha f_1 = 0$

⑧ $V_8 + \alpha f_{12} = 0$



2.5 a) magic(n) NO
hilb(n) YES
pascal(n) YES
eye(n,n) YES
rand(n,n) NO

b) note:



$R^T * R = A$ →

R=randn(n,n); A=R'*R YES
R=randu(n,n); A=R'+R MAYBE
R=randu(n,n); A=R'+R+n*eye(n,n) MAYBE

```
function R = mychol(A)
%MYCHOL My own version of Cholesky factorization.
%   R = mychol(A), for a positive definite matrix A,
%   is an upper triangular matrix R so that R'*R = A.

if ~isequal(A',A)
   error('Matrix must be symmetric or Hermitian')
end
[n,n] = size(A);
R = zeros(n,n);
for k = 1:n
   i = (1:k-1)';
   j = k+1:n;
   s = A(k,k) - R(i,k)'*R(i,k);
   if s > 0
      R(k,k) = sqrt(s);
   else
      error('Matrix must be positive definite')
   end
   R(k,j) = (A(k,j) - R(i,k)'*R(i,j))/R(k,k);
end
```

2.9 a) $\underline{x} = \underline{p} + t \cdot \underline{z}$ with $A\underline{p} = \underline{b}$ and $A\underline{z} = \underline{0}$
particular sol $\underline{p} = (1/3 \;\; 1/3 \;\; 0)^T$ and null space $\underline{z} = (1 \; -2 \; 1)^T$

b) finally you end up at $U(3,3) = 0$ and $c(3) = 0$ → $x(3) = \frac{0}{0}$ an arbitrary value, which is the free parameter.

c) $\underline{x} = (13/3 \;\; -23/3 \;\; 4)^T$ Good, because residual $r = b - A*x$ is small
Bad because sol is not unique!
Due to finite precision we end up with $x(3) = \frac{2*eps}{eps/2} = 4$

d) \ uses column-oriented algorithm → $x(3) = \frac{5/4*eps}{eps/2} = 5/2$!
bslashtx uses row-oriented algorithm

2.19 a) n=100; e=ones(n,1); b=(1:n)';
A=2*diag(e)-diag(e(1:n-1),-1)-diag(e(1:n-1),1);
xa=bslashtx(A,b);

b) A=spdiags([-e 2*e -e],[-1 0 1],n,n);
xb=A\b;

c) xc=tridisolve(-e,2*e,-e,b);

d) condest(A) → $5.1 \; 10^3$

2.21 U{k} is a string, the k^{th} URL.
U(k) is a 1-by-1 cell array containing the string of the k^{th} URL.
G(k,:) is nonzero for incoming nodes to the k^{th} URL.
G(:,k) is nonzero for outgoing nodes from the k^{th} URL.
U(G(k,:)) is a list of incoming URL's to the the k^{th} URL*.
U(G(:,k)) is a list of outgoing URL's from the the k^{th} URL.

*Note that this only works if G is logical.

2.22. Cliques in the harvard500 Web connectivity matrix.
U(168:180): Harvard Divinity School
U(229:248): Radcliffe Institute
U(261:281): Dana-Farber Cancer Institute
U(315:335): "Go Crimson", Harvard's athletic program

2.23. (a) For p >= 8, nnz(G^p) = 167985.
(b) nnz(G^8)/prod(size(G)) = 0.6719.
(c) for p = 1:9, subplot(3,3,p), spy(G^p), end
(d) The "Go Crimson" athletic program, nodes 46 and 315:335, has no links to the other pages in the data set.

2.25. Disconnected miniweb.

```
G =   0 0 0 1 0 0
      1 0 0 0 0 0
      1 1 0 0 0 0
      0 1 1 0 0 0
      0 0 0 0 0 1
      0 0 0 0 1 0
pagerank1(G,.85) = 0.1981
                   0.1092
                   0.1556
                   0.2037
                   0.1667
                   0.1667
```

What happens to page rank as p->1? Two possible answers here. The intuitive answer is that the graph has two disconnected subgraphs and consequently the Markov stationary probabilities are not unique. The direct solution algorithm used in pagerank certainly breaks down if p = 1. However, a second answer is that pageranksym, a symbolic version of pagerank1, produces

```
p = sym('p');
pagerank1(G,p) =
[ 1/3*(p^3+3*p^2+2*p+2)/(p^3+4*p^2+4*p+4)]
[ 1/3*(p^2+p+2)/(p^3+4*p^2+4*p+4)]
[ 1/6*(p^3+3*p^2+4*p+4)/(p^3+4*p^2+4*p+4)]
[ 1/6*(p^3+5*p^2+6*p+4)/(p^3+4*p^2+4*p+4)]
[ 1/6]
[ 1/6]
```

and

```
limit(ans,p,1) =
[ 8/39]
[ 4/39]
[ 6/39]
[ 8/39]
[ 1/6]
[ 1/6]
```

These values are 2/3 times the limiting values for the 4-by-4 subgraph and 1/3 times the values for the 2-by-2 subgraph.

-FIN-